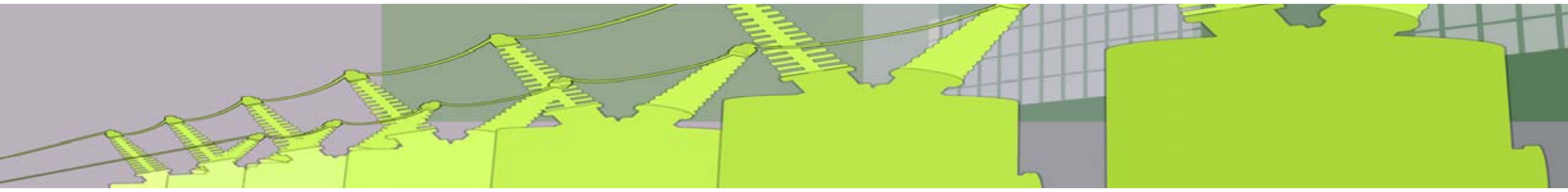

THE REBOUND EFFECT:

MICROECONOMIC DEFINITIONS, EXTENSIONS AND LIMITATIONS

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John Dimitropoulos
Steve Sorrell

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Abstract

The rebound effect results in part from an increased consumption of energy services following an improvement in the technical efficiency of delivering those services. This increased consumption offsets the energy savings that may otherwise be achieved and potentially undermines the rationale for policy measures to encourage energy efficiency.

The nature, definition and magnitude of the rebound effect are the focus of long-running disputes with energy economics. This paper brings together previous theoretical work to provide a rigorous definition of the rebound effect, clarify key conceptual issues and highlight the consequences of various assumptions for empirical estimates of the effect. The focus is on the direct rebound effect for a single energy service - indirect and economy-wide rebound effects are not discussed.

Beginning with Khazzoom's original definition of the rebound effect, we expose the limitations of three simplifying assumptions on which this definition is based. First, we argue that capital costs form an important part of the total cost of providing energy services and that the higher cost of energy efficient conversion devices will reduce the magnitude of the rebound effect in many instances.

Second, we argue that energy efficiency should be treated as an endogenous variable and that empirical estimates of the rebound effect may need to apply a simultaneous equation model to capture the joint determination of key variables.

Third, we explore the implications of the opportunity costs of time in the production of energy services and highlight the consequences for energy use of improved 'time efficiency', the influence of time costs on the rebound effect and the existence of a parallel rebound effect with respect to time.

Each of these considerations serves to highlight the difficulties in obtaining reliable estimates of the rebound effect and the different factors that need to be controlled for. We discuss the implications of these findings for econometric studies and argue that several existing studies may overestimate the magnitude of the effect.

Introduction

The rebound effect is the focus of a long-running dispute with energy economics. The question is whether improvements in the technical efficiency of energy use can be expected to reduce energy consumption by the amount predicted by simple engineering calculations.

For example, will a 20% improvement in the thermal efficiency of a heating system lead to a corresponding 20% reduction in aggregate energy consumption? Economic theory suggests that it will not. Three separate mechanisms may offset the energy savings achieved [1, 2]:

- Direct rebound effects: Improved energy efficiency for a particular energy service will decrease the effective price of that service and should therefore lead to an increase in consumption of that service. This will tend to offset the expected reduction in energy consumption provided by the efficiency improvement. For consumers, the direct rebound effect may be decomposed into a substitution and income effect, while for producers it may be decomposed into a substitution and an output effect. In both cases, the direct rebound effect is confined to the energy required to provide the relevant energy service.

- Indirect effects: The lower effective price of the energy service can lead to changes in the demand for other goods, services and factors of production that also require energy for their provision. For example, the cost savings obtained from a more efficient central heating system may be put towards an overseas holiday.

- Economy wide effects: A fall in the real price of energy services may reduce the price of intermediate and final goods throughout the economy, leading to a series of price and quantity adjustments, with energy-intensive goods and sectors gaining at the expense of less energy-intensive ones. Energy efficiency improvements may also reduce energy prices and increase economic growth, which could further increase energy consumption.

Numerous empirical studies, principally from the US, suggest that these rebound effects are real and can be significant [2]. However, while their basic mechanisms are widely accepted, their magnitude and importance are disputed. Some analysts argue that rebound effects are of minor importance for most energy services [3], while others argue that they are sufficiently important to completely offset the energy savings from improved energy efficiency [4, 5]. The policy implication is that non-price regulations to improve energy efficiency may neither reduce energy demand nor help to mitigate climate change.

Indirect and economy-wide rebound effects involve general equilibrium adjustments that are very difficult to analyse empirically. In contrast, direct rebound effects can be investigated more directly through quasi-experimental studies[1] [6] or the econometric analysis of secondary data. However, such studies raise a number of definitional and methodological issues that are inadequately discussed in the literature. The persistent disagreement over the magnitude and importance of rebound effect may result in part from lack of clarity over these basic definitions and issues. Moreover, since many empirical studies overlook key methodological issues, their estimates of the rebound effect could potentially be biased.

The paper is structured as follows. First, we present a general ‘household production’ framework for characterising the demand for energy services that helps to illustrate the different trade-offs involved. Second, we show how the direct rebound effect can be represented as an efficiency elasticity of energy demand and how may be decomposed into the sum of elasticities for the number, capacity and utilisation of energy conversion devices. Third, we show the relationship between the rebound effect and the price elasticity of the demand for useful work, as well as the price elasticity of energy demand, and show why empirical studies using these definitions provide the primary source of evidence for the rebound effect.

We then expose the limitations of these definitions, focusing on: a) the potential correlation between various input costs and improvements in energy efficiency; b) the endogeneity of energy efficiency and the implied need for simultaneous equation estimation; and c) the role of time costs and time efficiency in the production and consumption of energy services. We identify the factors that need to be controlled for to obtain accurate estimates of the rebound effect and argue that the neglect of these factors by several existing studies may lead the rebound effect to be overestimated.

The demand for energy services

The demand for energy (E) derives from the demand for energy services *(ES)* such as thermal comfort, refrigeration and motive power. These services, in turn, are delivered through a combination of energy commodities and the associated energy systems, including energy conversion devices. Consumers are assumed to derive utility (U) from consuming these services, rather than from consuming energy commodities and other market goods directly. In practice, nearly all services require energy in some form, although energy may form a much smaller proportion of total costs for some services than for others.

[1]For example, a before and after comparison of energy consumption by participants in a demand-side management scheme, with or without a control group of non-participants.

An essential feature of an energy service is the *useful work* (S) obtained, which may be measured by a variety of thermodynamic or physical indicators [7]. These indicators may, in turn, be decomposed in a variety of ways to reveal the relative importance of different contributory variables. For example, the useful work from the private cars owned by a group of households may be:

- *measured in vehicle kilometres and decomposed into the product of the number of cars and the mean driving distance per car per year:* $S = NO * UTIL$ *;*
- *measured in passenger kilometres and decomposed into the product of the number of cars (NO), the mean driving distance per car per year (UTIL) and the average number of passengers carried per car (LF):* $S = NO * UTIL * LF$ *; or*
- *measured (rather unconventionally) in tonne kilometres and decomposed into the product of the number of cars (NO), the mean driving distance per car per year (UTIL) and the mean (loaded or unloaded) vehicle weight (CAP):* $S = NO * CAP * UTIL$ *.*

In practice, the choice of indicator and associated decomposition will depend upon the objective of the analysis, the level of aggregation (e.g. household, sector, economy) and the availability of the relevant data. In much empirical work, measures of useful work are not decomposed.

It important to recognise that energy services also have broader *attributes* (A) that may be combined with useful work in a variety of ways. For example, all cars deliver passenger km, but they may vary widely in terms of features such as speed, comfort, acceleration and prestige. The combination of useful work (S) with these associated attributes (A) provides the full energy service: $ES = es(S, A)$.

Following Becker's work on 'household production' [8], individual households may be assumed to *produce* useful work (S) by combining energy, energy (E), capital (K) and other market goods (O), together with some of the household's own time (T). For example, mobility may be produced by the household through the combination of a private car (K), gasoline (E), expenditure on maintenance (O) and driving time (T). Similarly, a cooked meal may be produced through the combination of a gas cooker, natural gas, ingredients and cooking time.

The provision of useful work for a particular energy service may then be described by a production function, representing the maximum output that can be obtained from the currently available technology for a given level of energy and other inputs [9]. But the provision of broader attributes (A) for a given amount of useful work is likely to require additional inputs; or, alternatively, for a given input budget, the provision of broader attributes is likely to reduce the amount of useful work. To reflect this, the production function for the full energy service i (ES_i) may be written as:

$$ES_i = es_i[E_i, K_i, O_i, T_i \; ; A_i] \qquad (1)$$

If a household's utility is assumed to depend solely upon these services, the utility function becomes:

$$U = u[ES_1, ES_2, ES_3, ...ES_n] \tag{2}$$

The household may be assumed to be subject to the following income constraint:

$$V + T_W P_W \geq \sum_{i=1,n} (P_E E_i + P_O O_i + \delta_K K_i) \tag{3}$$

Where V represents non-wage income; P_W represents the wage rate; T_W represents the time spent in the labour market; P_E and P_O represent the unit price of energy and other goods respectively; and δ_K represents a discount factor (so $P_K = \delta_K K(A)$ gives the annualised capital costs). Households will also be also subject to a second constraint on their available time:

$$T = T_W + \sum_{t=1}^{n} T_i \tag{4}$$

Where T_i represents the time spent in producing services S_i. Becker [8] argued that, since money and time are partly interchangeable through decisions on T_W, the income and time constraints can be collapsed into a single constraint. By substituting $T_W = T - \sum_{t=1}^{n} T_i$ into the budget constraint and rearranging , we obtain:

$$V + P_W T \geq \sum_{t=1}^{n} (P_E E_i + P_O O_i + \delta_K K_i + P_W T_i) \tag{5}$$

Versions of Becker's 'household production' model form the basis of a substantial volume of empirical research [10, 11]. This includes numerous applications to energy use, although these studies frequently (and importantly) neglect the time inputs to energy services [12-14]. The model rests upon a set of behavioural and other assumptions that may be criticised on a variety of grounds [11, 15].[2] Nevertheless, it offers a number of advantages over conventional models of household demand (especially for energy) and predictions from the model appear broadly confirmed by empirical research [11]. Its primary contribution in the present context is to emphasise that consumption of an energy service involves three basic trade-offs:

- *between consumption of useful work versus consumption of other attributes of an energy service;*
- *between energy, capital, other market goods and time into the production of an energy service; and*
- *between consumption of different types of energy service.*

[2] Including: the assumption that each market good or allocation of time is dedicated to the production of a single service; the notion that households are indifferent to the allocation of time, except as an input into the production of services; difficulties in defining what a service actually is (e.g. travel by car for a visit or the visit itself); the neglect of the fact that utility may be a function of producing as well as consuming a service; the implicit assumption of constant returns to scale in production; the difficulty in operationalising the model; the lack of good data on time use patterns; and the usual difficulties associated with models that assume 'hyper-rational', utility maximising individuals.

This general framework forms the foundation for what follows.

The rebound effect as an efficiency elasticity

The energy efficiency (ε) of an energy system may be defined as $\varepsilon = S/E$, where *E* represents the energy input required for a unit output of useful work (however measured).[3] For example, a car may require ten litres of gasoline to drive one hundred kilometres. The *energy cost* of useful work (P_S) is then given by $P_S = P_E/\varepsilon$, where P_E represents the price of energy. This is one component of the *total cost* of useful work, which also includes other input costs, such as annualised capital costs, maintenance costs and time costs.

Consider a situation where the energy efficiency of an energy system is improved ($\Delta\varepsilon > 0$), but the costs of non-energy inputs and the consumption of other attributes of the energy service remain unchanged. In the absence of a rebound effect, the demand for useful work would remain unchanged ($\Delta S = 0$) and energy demand would be reduced in proportion to the improvement in energy efficiency ($\Delta E/E = -\Delta\varepsilon/\varepsilon$). But the efficiency improvement lowers the energy cost per unit of useful work ($\Delta P_S < 0$) and hence also the total cost. Assuming that the energy service is a normal good with non-zero price elasticity, consumers will demand more useful work ($\Delta S > 0$) and the proportional change in energy consumption will be less than the proportional change in energy efficiency ($\Delta E/E < -\Delta\varepsilon/\varepsilon$).

The change in demand for useful work following a small change in energy efficiency may be measured by the *efficiency elasticity of the demand for useful work* ($\eta_\varepsilon(S)$):

$$\eta_\varepsilon(S) = \frac{\partial S}{\partial \varepsilon}\frac{\varepsilon}{S} \qquad (6)$$

In a similar manner, the change in energy demand following a small change in energy efficiency may be measured by the *efficiency elasticity of the demand for energy* ($\eta_\varepsilon(E)$):

$$\eta_\varepsilon(E) = \frac{\partial E}{\partial \varepsilon}\frac{\varepsilon}{E} \qquad (7)$$

Substituting $E = S/\varepsilon$ in the equation for $\eta_\varepsilon(E)$ and taking partial derivatives we can derive the following relationship between these two elasticities:[4]

[3] The appropriate measure of energy efficiency depends upon how useful work is defined and is generally a property of the energy system, rather than just the energy conversion device. For example, if the average internal temperature is taken as the appropriate measure of useful work from a household heating system, energy efficiency will depend upon both the thermal efficiency of the boiler and the level of thermal insulation.

[4] See the Annex for derivations of this and subsequent definitions and formulae.

Definition 1: $\eta_{\varepsilon}(E) = \eta_{\varepsilon}(S) - 1$

The efficiency elasticity of the demand for useful work ($\eta_{\varepsilon}(S)$) has been commonly taken as a direct measure of the rebound effect [16]. The actual saving in energy consumption will only be equal to the predicted saving from engineering calculations when this elasticity is zero ($\eta_{\varepsilon}(S) = 0$). Under these circumstances, the efficiency elasticity of demand for energy ($\eta_{\varepsilon}(E)$ is equal to minus one. A positive rebound effect implies that $\eta_{\varepsilon}(S) > 0$ and $|\eta_{\varepsilon}(E)| < 1$. For example, a positive rebound effect for car travel implies that improvements in vehicle fuel efficiency increase the demand for vehicle kilometres, with the result that the savings in energy consumption are less than predicted from engineering calculations alone. If the demand for the energy service is inelastic ($0 < \eta_{\varepsilon}(S) < 1$) improvements in energy efficiency should reduce energy demand ($0 > \eta_{\varepsilon}(E) > -1$). But if the demand for the energy service is elastic ($\eta_{\varepsilon}(S) > 1$), improvements in energy efficiency will actually increase energy consumption. This somewhat perverse outcome is termed 'backfire' in the literature [4].

Technological improvements in energy efficiency may lead to an increase in the number of energy conversion devices (*NO*), their average size (*CAP*), their average utilisation (*UTIL)* and/or their average load factor (*LF*). For example, people may buy more cars, buy larger cars, drive them further and/or share them less. Similarly, people may buy more washing machines, buy larger machines, use them more frequently and/or reduce the size of the average load. The equation for the efficiency elasticity of energy demand may therefore be decomposed in a variety of ways, depending upon data availability and the choice of measure for useful work (*S*). For example, if useful work is defined as the product of the number, capacity and utilisation of energy conversion devices, the equation becomes:

Definition 2: $\eta_{\varepsilon}(E) = [\eta_{\varepsilon}(NO) + \eta_{\varepsilon}(CAP) + \eta_{\varepsilon}(UTIL)] - 1$

The relative importance of these variables may vary widely between different energy services and over time. For example, technological improvements in the energy efficiency of new refrigerators are unlikely to increase the average utilisation of the refrigerator stock (measured in hours/year) but could lead to an increase in both the number and average size of refrigerators over time (since the cost per m^3 of refrigeration has reduced).

The majority of empirical estimates of the rebound effect relate to travel by private cars, where useful work is commonly measured in terms of total vehicle kilometres travelled and decomposed into the product of vehicle numbers and the mean distance travelled per car per year [17, 18].

One consequence of this is that increases in average vehicle weight as a result of energy efficiency improvements (e.g. more SUVs) as well as decreases in average load factor (e.g. less car sharing) are commonly overlooked.[5]

The marginal utility of energy service consumption is likely to decline with increased demand, which should reduce the (direct) rebound from energy efficiency improvements. For example, rebound effects from improvements in the energy efficiency of household heating systems should decline rapidly as indoor temperatures exceed 22°C. One implication, frequently observed in the policy evaluation literature, is that rebound effects will be higher among low income groups, since these are much further from satiation in their consumption of energy services [19].

The rebound effect as a price elasticity

Since $P_S = P_E / \varepsilon$, raising (lowering) energy efficiency (ε) when energy prices (P_E) are constant should have the same effect on the energy cost of useful work (P_S) as falling (rising) energy prices when energy efficiency is constant. Under the ceteris-paribus assumptions given above, the effect on the total cost and hence the demand (S) for useful work should be symmetrical. If other inputs are held constant, we can write the demand for energy solely as a function of P_E and ε, namely: $E = s(P_E / \varepsilon)/\varepsilon$. Assuming that energy prices are exogenous (i.e. P_E does not depend upon ε), we can differentiate this equation with respect to energy efficiency to give an alternative definition of the rebound effect:

Definition 3: $\eta_\varepsilon(E) = -\eta_{P_S}(S) - 1$

Hence, under these assumptions, the efficiency elasticity of energy demand ($\eta_\varepsilon(E)$) is equal to the *energy cost elasticity of the demand for useful work* ($\eta_{P_S}(S)$), minus one. Effectively, the negative of the energy cost elasticity for useful work ($\eta_{P_S}(S)$) is being used as a proxy for the efficiency elasticity of useful work ($\eta_\varepsilon(S)$), which in turn is the primary definition of the rebound effect. If useful work is a normal good, we expect that $\eta_{P_S}(S) \leq 0$. For example, if the elasticity of vehicle km (S) with respect to energy cost per kilometre (P_S) is estimated as -0.10, then the elasticity of gasoline demand with respect to fuel efficiency can be estimated from Definition 3 as –0.90. This implies that the demand for gasoline will fall by only 9% if the fuel

[5] The first of these rebound effects could be captured if useful work for private travel was measured in unloaded tonne kilometres rather than vehicle kilometres. This would be possible if data was available on the composition of the vehicle stock and the average unloaded weight of different types of vehicle. The second effect could be captured if useful work was measured in passenger kilometres rather than vehicle kilometres. This would require data on the average load factor of different types of vehicle. To capture both of these rebound effects, useful work would need to be measured in *loaded* tonne kilometres.

efficiency of vehicles improves by 10% - or, alternatively, that 10% of the potential savings in gasoline consumption will be 'taken back' by increased vehicle use.

A version of this expression is derived by Khazzoom [20], Berkhout *et al* [16], Binswanger [21] and Greene *et al* [22] and is generally used in preference to Definition 1 in empirical estimates of the rebound effect [1]. For many energy services, the available data provides only limited variation in the independent variable for Definition 1 (ε) while at the same time requiring energy prices to be controlled for. In contrast, the data provides much greater variation in the independent variable for Definition 3 (P_S) since this reflects both variations in energy efficiency and variations in energy prices.

For many energy services, the historical and cross-sectional variations in the relevant energy commodity prices tend to be much greater than the corresponding variations in the energy efficiency of the relevant energy systems. Given the assumption that consumers respond in the same way to increases (decreases) in energy prices as to decreases (increases) in energy efficiency, Definition 3 provides a means to estimate the potential magnitude of rebound effects from efficiency improvements even in circumstances where the available data provides little or no variation in energy efficiency.

Empirical studies based upon Definition 3 require accurate measures of both the demand for useful work (S) for the relevant energy service and the energy cost per unit of useful work (P_S). The latter, in turn, depends upon energy commodity prices and the energy efficiency of the relevant energy system. But, depending upon how it is defined, the measurement of useful work for many types of energy service can be problematic. For example, the useful work from a domestic heating system could be defined as the average internal temperature of the house and measured directly using field thermometers or indirectly from thermostat settings.

But such measurements are notoriously inaccurate and can be a poor proxy for the thermal comfort of the occupants, which depends upon other variables such as humidity and airflow. The reason that travel by private car (in the United States) is the most widely studied area for the rebound effect is that relatively good data is available on vehicle kilometres as a measure of useful work, while fuel costs per kilometre is easily estimated by combining data on gasoline prices and vehicle fuel efficiency [23].

While obtaining measures of useful work (S) can be difficult, data is more commonly available on the energy demand (E) for the relevant energy service. For example, data may be available on the demand for gas for household heating (although the use of gas for cooking could provide a complication).

If we assume that that energy efficiency is constant, the symmetry argument implied by the ratio $P_S = P_E / \varepsilon$ leads to an alternative definition for the rebound effect based upon the own price elasticity of energy demand:

Definition 4: $\eta_{\varepsilon}(E) = -\eta_{P_E}(E) - 1$

It is this expression, rather than Definition 2, that was originally put forward by Khazzoom and is also used by Wirl [9] in his comprehensive analysis of the economics of energy efficiency. Definition 4 shows that under certain assumptions, the rebound effect may be approximated by the own price elasticity of *energy* demand for the relevant energy service. Note that this definition is only meaningful when the energy demand in question relates to a single energy service (e.g. refrigeration). In practice, available measures of energy demand frequently apply to a collection of energy services (e.g. household electricity use), although in some cases the proportion of demand attributable to an individual service can be estimated [24].[6] Following this rough approximation, a number of authors have used new or existing estimates of the own-price elasticity of energy demand as approximate indicators of the magnitude of the rebound effect [16, 25-27]. This opens up a very large evidence base, as reviewed, for example by Espey [28] and Dahl [29].

Most studies suggest that energy demand is relatively inelastic, with typical values ranging from -0.3 to –0.4 in the long run. Applying Definition 4, these figures suggest that some 30-40% of energy savings deriving from energy efficiency improvements may be 'taken-back' by the direct rebound effect. However, elasticity estimates vary widely between different energy commodities, end-uses, sectors, countries and levels of aggregation; as well as being larger in the long run than in the short run and increasing proportionately with the price level [16]. Of particular interest is the fact that elasticities tend to be higher for periods with rising energy prices than for those with falling energy prices.

The primary explanation for this appears to be the irreversibility of energy efficiency investment [30]. When prices increase, producers and consumers invest in more efficient equipment, such as thermal insulation and this investment tends to remain in place when energy prices fall [31]. As a result, estimates of the rebound effect based upon time series data are likely to vary according to whether energy prices were rising, falling (or both) over the period in question [32]. Since the appropriate proxy for improvements in energy efficiency are *reductions* in energy prices, empirical estimates based upon periods of rising energy prices are likely to overestimate the size of the effect.

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[6] For example, Haas and Biermayr (1997) unbundled energy use for space heating from that for water heating by assuming that the latter was constant over the year, while the former depended upon external temperature.

Since econometric estimates based upon Definitions 3 and 4 are the primary source of evidence for the rebound effect, the assumptions behind these definitions (and particularly the symmetry argument) require careful scrutiny. The following three sections explore the limitations of these basic definitions in more detail, focusing on:

- *the correlation between energy efficiency and other input costs, notably capital costs;*
- *the endogeneity of energy efficiency and the implied need for simultaneous equation estimation; and*
- *the role of time costs and time efficiency in the production and consumption of energy services.*

Correlation between energy efficiency and other input costs

For an individual energy service, changes in energy commodity prices are unlikely to be correlated with changes in other input costs or with changes in the broader attributes of the energy service. But the same cannot be said about changes in energy efficiency. In practice, energy efficient conversion devices will frequently have a higher capital cost than the inefficient models that they replace (i.e. ε and K are positively correlated). For example, UK building regulations now require high efficiency condensing boilers to be used when installing or replacing a domestic central heating system and these typically cost some £200-300 more than a conventional boiler.

Khazzoom [20] assumed this problem away by arguing that a more efficient appliance does not necessarily entail a greater initial cost and citing the lower cost of smaller and more fuel-efficient cars as an example. But in this case, the improvement in energy efficiency is likely to have been achieved at the expense of other attributes of the energy service such as carrying capacity and legroom (i.e. ε and A are negatively correlated). In general, improvements in energy efficiency could result from technological change, substitution between energy other inputs or substitution between useful work and other output attributes. In practice, many energy services have multiple attributes (e.g. size, comfort, reliability, speed) and each attribute may have non-zero elasticity with respect to the energy cost of useful work. As Einhorn [33] has argued, the long-term response to a reduction in energy costs will depend upon the trade-offs between useful work and these multiple attributes.

Khazzoom's neglect of capital costs has been challenged by several authors [33-36] who argue that it may lead empirical studies that rely upon Definitions 3 and 4 to overestimate the rebound effect. Henly *et al* [36] illustrate this clearly by including annualised capital costs (P_K) in the equation for energy demand. Assuming that capital costs are a function of energy efficiency, the basic identity becomes: $E = s[P_E/\varepsilon, P_K(\varepsilon)]/\varepsilon$. We can then derive the following alternative definition of the efficiency elasticity of energy demand:[7]

Definition 5: $\eta_\varepsilon(E) = -1 - \eta_{P_S}(S) - [\eta_{P_K}(S)\eta_\varepsilon(P_K)]$

Compared to Definition 3 (and Definition 4), there is an additional term in square brackets. This is the product of the elasticity of demand for useful work with respect to capital costs ($\eta_{P_K}(S)$) and the elasticity of capital costs with respect to energy efficiency ($\eta_\varepsilon(P_K)$). We expect the first of these to be negative: higher capital costs should reduce the demand for useful work, largely because they should reduce the number of energy conversion devices ($\eta_{P_K}(NO) \le 0$) and/or their average size ($\eta_{P_K}(CAP) \le 0$ - assuming that capital costs are proportional to size). Under the assumption that energy efficient equipment is more expensive, the second term will be positive, making the product of these two expressions negative. The net result will be to reduce the absolute magnitude ($|\eta_\varepsilon(E)|$) of the efficiency elasticity of energy demand. Hence, if energy efficient equipment is more expensive, the rebound effect may be *smaller* than implied by Definitions 3 and 4. This implies that empirical estimates based upon these definitions and relying primarily upon historical or cross-sectional variations in energy prices may overestimate the rebound effect. The size of this upward bias will depend upon the relative magnitude of the three separate elasticities.[8]

The correlation between energy efficiency and capital costs may be expected to vary between energy services and over time. In areas such as computing, for example, improvements in energy efficiency have long been associated with both improvements in service attributes and *reductions* in capital costs [37, 38]. Also, higher capital costs will only reduce the rebound effect if the consumer faces the full cost of the purchase decision. If, for example, the additional cost of energy efficient conversion devices is fully subsidised, the higher initial cost should not affect the purchase decision. Furthermore, if government subsidies make energy-efficient devices *cheaper* than inefficient models, it is possible that the rebound effect will be amplified (i.e. if both $\eta_{P_K}(S)$ and $\eta_\varepsilon(P_K)$ are negative, their product will be positive). Empirical support for this is provided by Roy's study of rebound effects for rural lighting in India [26].

[7] This definition, with the own price elasticity of energy demand, instead of the own price elasticity of the demand for useful work, appeared originally in [36].

[8] Henly *et al* make the additional observation that mandatory energy efficiency standards may disproportionately affect low income households that would otherwise have purchased cheaper and less efficient appliances. Such households tend to be relatively insensitive to changes in operating costs, but relatively sensitive to changes in capital costs when making purchase decisions (i.e. $|\eta_{P_S}(S)| \le |\eta_{P_K}(S)|$).

Consideration of the role of capital costs further highlights the importance of distinguishing between the number, capacity and utilization of energy service devices when estimating rebound effects. Once an appliance is purchased, the capital cost is sunk and hence should be irrelevant to the utilisation decision. But higher capital costs may lead to the purchase of fewer, smaller and/or different conversion devices, depending upon the trade-offs between different categories of input costs and between useful work and other output attributes. Holding output attributes constant, efficient conversion devices allow their owners to enjoy a greater consumer surplus in each time period, owing to the higher demand for useful work [33]. But if the more eefficient appliance is also more expensive than the inefficient alternative, it will only be purchased if the present value of the discounted stream of additional consumer surplus exceeds the present value of the additional capital cost.

It is possible that improvements in energy efficiency will be associated with changes in other input costs, such as operation and maintenance (O&M) costs. If, for example, more efficient conversion devices are less reliable and more costly to maintain, the rebound effect will be smaller. However, the evidence for a positive correlation between energy efficiency and O&M costs is absent for most energy services, and for some the correlation may be negative. In general, the magnitude and direction of the bias in estimating the rebound effect using Definitions 3 and 4 will depend upon the degree and sign of the correlation between energy efficiency and all other categories of input costs. If they are positively correlated, the bias will be negative and the rebound effect will be overestimated, while if they are negatively correlated the bias will be positive and the rebound effect underestimated.

Even if improvements in energy efficiency are not associated with changes in other input costs, certain types of rebound effect may be constrained by the real or opportunity costs associated with increasing the demand for useful work. Two important examples are the opportunity cost of space (e.g. increasing refrigerator size is not the best use of available space) and the opportunity to cost of time (e.g. driving longer distances is not the best use of available time). Both of these reflect an absolute physical constraint on the demand for useful work by individual households. However, space constraints may become less important over time if technological improvements reduce the average size of conversion devices per unit of useful work (e.g. computing) or if rising incomes lead to an increase in average living space (e.g. compare refrigerator sizes in the US and the UK) [39]. In contrast, while technological improvements may reduce the time requirements per unit of useful work, the opportunity cost of time will *increase* with rising incomes.

The relationship between time constraints and energy service consumption appears particularly important and is discussed further below.

Endogenous energy efficiency

Definitions 1 and 3 assume that energy efficiency is independent of the values of other independent variables – in other words, that it is *exogenous*. This follows naturally from Khazzoom's original focus, namely the effect of mandatory energy efficiency standards for household appliances. In practice, however, the level of energy efficiency is likely to be influenced by one or more of the other dependent variables – in other words, energy efficiency must be considered partly *endogenous*. In particular, energy efficiency may be expected to be a function of current and historical energy prices: $\varepsilon(P_E)$ [17, 18].[9]

If energy efficiency depends upon energy prices, the demand for energy for the relevant energy service can be represented as: $E = s[P_E / \varepsilon(P_E)] / \varepsilon(P_E)$. If we differentiate this expression with respect to energy prices and substitute the resulting expression for $\eta_{P_S}(S)$ into Definition 3, we obtain an alternative definition of the rebound effect that takes into account price-induced energy efficiency improvements:

Definition 6: $\eta_{\varepsilon}(E) = -\left[\dfrac{\eta_{P_E}(E) + \eta_{P_E}(\varepsilon)}{1 - \eta_{P_E}(\varepsilon)}\right] - 1$

Previous versions of this equation have appeared in Blair *et al* [40], Mayo and Mathis [41] and Small and Van Dender [18]. In principle, Definition 6 provides an alternative method of estimating the rebound effect. Rather than estimating the energy cost elasticity of useful work, one could separately estimate the own price elasticity of energy consumption for the relevant energy service ($\eta_{P_E}(E)$) *and* the elasticity of energy efficiency with respect to energy prices ($\eta_{P_E}(\varepsilon)$). The resulting calculated value for the energy cost elasticity of useful work ($\eta_{P_S}(S)$) could then used to estimate the rebound effect.

It is clear from Equation (7) that the energy cost elasticity of the demand for useful work ($\eta_{P_S}(S)$) will only be equal to the own price elasticity of the demand for energy for the relevant energy service ($\eta_{P_E}(E)$) if the energy price elasticity of energy efficiency is equal to zero ($\eta_{P_E}(\varepsilon) = 0$). This is unlikely to be the case in practice.

[9] In the short term, increases in energy commodity prices may encourage consumers to utilise existing equipment in more energy efficient ways – such as increasing average load factor (e.g. car sharing), or adopting energy efficient operating practices (e.g. avoiding excessive speed). In the longer term, consumers may choose to purchase more energy efficient conversion devices, while producers may choose to devote expenditure to developing, improving and marketing such devices.

Hanley *et al* [42] have derived an expression for the relative magnitude of different price elasticities that should hold for all econometric estimates:

$$\left|\eta_{P_E}(S)\right| \leq \left|\eta_{P_S}(S)\right| \leq \left|\eta_{P_E}(E)\right| \leq \left|\eta_{P_S}(E)\right| \tag{8}$$

This relationship provides a useful point of reference for the results from individual studies and is supported by evidence from recent surveys [28, 42, 43]. It suggests that the elasticity of the demand for useful work with respect to energy costs should be smaller than the elasticity of energy demand with respect to energy prices. This shows that, relative to Definition 3, Definition 4 is likely to overestimate the magnitude of the rebound effect due to the neglect of price-induced energy efficiency improvements.

It seems likely that energy efficiency will also be a function of other endogenous or exogenous variables in ways that could bias the results of empirical studies [18]. In particular, if consumers expect to have a high demand for useful work, they may be more likely to choose an energy-efficient conversion device in order to minimise the energy cost of useful work. For example, drivers may choose to purchase a more fuel-efficient car if they expect to drive long distances.[10] This may create a positive correlation between S and ε that is in addition to the positive correlation created by the direct rebound effect. If this is not corrected for in empirical studies, the magnitude of the rebound effect will again be overestimated. Moreover, as pointed out by Small and Van Dender, [18] this type of endogeneity makes the logic behind Definition 3 circular: the demand for useful work (S) depends upon the energy cost of useful work (P_S), which in turn depends upon energy efficiency (ε) which in turn depends upon the demand for useful work (S).

This simultaneous determination of an endogenous variable (ε) with another endogenous variable (S) can be captured with a simultaneous equation model. This starts with a set of n equations for n an endogenous variables, with each equation representing either a causal relationship or an equilibrium condition. Such models could be formulated in a variety of ways, depending upon data availability. Small and Van Dender [18], for example, established separate equations for the number (*NO*) of private cars, their total annual mileage (S) and the average fuel efficiency of the car fleet (ε).

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[10] This is a hypothesis to be tested. A counter argument could be that drivers will purchase larger cars if they expect to drive long distances, since these are more comfortable. As larger cars tend to be less fuel-efficient, this may lead to a negative correlation between S and ε.

They base their model upon the following generic assumptions regarding consumer choices:

- *The total demand for useful work (S) is influenced by the number of energy conversion devices (NO), the energy cost of useful work (P_E/ε) and a number of exogenous variables (X_S).*
- *The number of energy conversion devices (NO) is influenced by the capital cost of those devices (P_K), the anticipated demand for useful work (S), the energy cost of useful work (P_E/ε) and a number of exogenous variables (X_N).*
- *The efficiency of the stock of conversion devices (ε) is influenced by the price of energy (P_E), the anticipated demand for useful work (S), regulatory standards on the energy efficiency of new devices (R_ε) and a number of exogenous variables (X_ε).*

This leads to the following set of 'structural' equations:[11]

$$
\begin{aligned}
S &= s(NO,(P_E/\varepsilon),X_S) \\
NO &= no(P_K,S,(P_E/\varepsilon),X_{NO}) \\
\varepsilon &= \varepsilon(P_E,S,R_\varepsilon X_\varepsilon)
\end{aligned}
\tag{9}
$$

It is an empirical question as to whether a simultaneous equation model is appropriate for a particular energy service. In some cases, the joint dependence of some or all of the variables may either not hold or be sufficiently weak that it can be ignored. For example, Johansson and Schipper [44] assumed that mean driving distance per vehicle was a function of the number of vehicles and their average fuel efficiency, but argued that the latter did not depend upon mean driving distance because: '…..one chooses what distance to drive for a given vehicle stock with different characteristics, and not the other way round'[44]. In contrast, Small and Van Dender [45], Greene *et al* [17] and Wheaton [46] all formulate models in which energy efficiency is a function of the number of cars and distance driven and each find the relevant coefficients to be statistically significant.[12]

The key point, however, is that *if* joint dependence is relevant, the equations need to be estimated through an appropriate simultaneous equations technique, such as two stage least squares (2SLS). If, instead, one or more of the individual equations are estimated through ordinary least squares (OLS), the resulting coefficients will be biased and inconsistent.[13]

[11] For time series or panel data, these equations are also likely to be autoregressive, in that the magnitude of the endogenous variables in one year may depend on their magnitude in the previous year (for example, vehicle numbers will not adjust instantly to reductions in vehicle prices). To reflect this, the equations should be modified to include a one-period lag of the dependent variables. For cross-sectional data, this does not apply.

[12] However, Small and Van Dender find no support for the endogeneity implied by the second of the equations in (9), since the coefficients on P_S and *S* are not significant.

[13] The difficulty arises because the endogenous variables used as regressors in each equation are correlated with the error term. For example, an increase in the error term for the first of equations (11) will directly increase *S*, indirectly increase *NO* (through *S*) and thereby indirectly increase *S* (through *NO*). Hence, *NO* and the error term of this equation are correlated. The standard solution to this type of problem is to replace *NO* with an instrumental variable that is correlated with *NO* but not with the error term. For the latter to be the case, the instrumental variable should be uncorrelated with *S*.

As an illustration, Small and Van Dender [45] found that the use of OLS in their model overestimated the short and long-run rebound effects for car travel by 88% and 53% respectively (although factors other then endogeneity may have been involved).

The use of a simultaneous equation model provides a clearer understanding of the implications of changes in energy efficiency, whether induced by regulatory intervention, energy price increases or other factors. For example, a mandatory standard for the energy efficiency of new conversion devices (e.g. cars) will have a *direct* effect on the energy efficiency of the stock, through the third of the equations in (9). However, improvements in energy efficiency will also tend to increase the number of conversion devices, which in turn will increase the total demand for useful work. Improvements in energy efficiency should also increase the demand for useful work by reducing the associated energy costs. The net increase in the demand for useful work will in turn encourage higher energy efficiency. Hence, a change in an exogenous variable such as regulatory standards for energy efficiency triggers a complex set of changes within the system until a new equilibrium is reached. If the behavioural assumptions given above hold, the total change in energy efficiency following the regulatory intervention will be greater than the direct change, as will the total change in energy service demand.

The structural equations may be solved to allow each of the endogenous variables to be written solely as functions of the exogenous variables, giving so-called 'reduced form' equations. However, many empirical estimates of the rebound effect use neither a structural equation system nor their reduced form solution.

Instead, they employ what Small and Van Dender [45] term a 'partially reduced form' equation for S, denoted here by the symbol $\hat{s}$. This includes energy efficiency indirectly via the energy cost of useful work, but does not include the number of conversion devices:

$$S = \hat{s}(P_K, (P_E / \varepsilon), X_{NO}, X_S) \qquad (10)$$

Since energy efficiency is endogenous, estimation of this equation by OLS is likely to lead to biased estimates of the rebound effect. Moreover, the bias will be compounded if (as is commonly the case), capital costs (P_K) or other input costs are correlated with either S or ε, but are omitted from the equation owing to lack of data.

Energy efficiency and time costs

The household production model represented by Equation (5) is based upon Becker's work on the economics of time [8]. As Binswanger [21] has argued, time costs and the efficiency of time use have important implications for energy use in general and the rebound effect in particular. However, empirical work in this area remains in its infancy [47].

For consumers, time is a necessary input to the production and enjoyment of energy services. For example, it takes time to drive from one place to another; to purchase food; to prepare a meal; to wash, dry and iron clothes and so on. The total cost of time for a particular energy service will depend upon the opportunity cost of time and the amount of time required per unit of useful work. In the household production model, the cost of time is conventionally measured by the average hourly wage for the household (P_W) and hence should vary from one household to another.[14] The amount of time required per unit of useful work may be measured by the efficiency of time use (θ), which depends upon the technology used. For example, a microwave oven is more time efficient than a conventional oven; a car is more time efficient than a bike;[15] an aircraft is more time efficient than a ship; and so on. The relationship between useful work and time consumption for a particular energy services may then be expressed as: $S = \theta T$, while the time cost per unit of useful work may be expressed as $P_T = P_W / \theta$. These expressions are entirely analogous to those used for energy consumption for a particular energy service (namely $S = \varepsilon E$ and $P_S = P_E / \varepsilon$).

Under these assumptions, the contribution of time costs to the full cost of an energy service should be inversely proportional to the time efficiency of the relevant technology and proportional to the wage rate. Similarly, the contribution of energy costs should be inversely proportional to the energy efficiency of the relevant technology and proportional to the energy price. Consumers should be able to choose between technologies with different combinations of energy and time efficiency in the provision of a particular energy service, and also between energy services with different levels of time and energy efficiency. The relative price of time and energy should influence the direction of technological innovation and encourage higher or lower levels of time/energy efficiency for individual energy services, as well shifts towards the development of more or less time/energy efficient services.

These considerations suggest that an increase in the cost of time (i.e. wages) relative to energy prices should induce a substitution away from time and toward energy in the production of individual services, as well as a substitution away from time-intensive services and towards energy intensive services.[16] Since wages appear

[14] And also from one household member to another and from one time to another (e.g. weekdays versus weekends), but this is often overlooked.
[15] Assuming no road congestion. As with energy efficiency, time efficiency is a function of the overall energy system, which could have multiple users. While congestion is given for individual decisions, it is an endogenous variable for the system as a whole.
[16] Note that traditional consumer theory would only capture the second of these effects and that the model implies that increases in non-wage income would not encourage either type of substitution.

to have grown faster than energy prices within developed countries over the last few decades, this appears to be a fair characterisation of recent trends [21]. With time costs forming a significant and increasing proportion of the total cost of many energy services, consumers and producers have sought ways to improve the time efficiency, rather than the energy efficiency, of service provision. So travel by private car has replaced walking, cycling and public transport; automatic washing machines have replaced washing by hand; fast food and ready meals have replaced traditional cooking; supermarkets (and more recently e-shopping and home delivery) have replaced the trip up the high street; email has replaced letters; and so on. Increases in aggregate energy consumption could therefore have been driven as much by the substitution of energy for time as by the overall increases in income.

The relative importance of time and energy costs may be expected to vary over time and between different energy services. One area where time costs are particularly important and relatively well researched is transport. For example, figures presented by Small [48] suggest that the average time costs for US car travel were more than three times total running costs, implying that they were more than six times the total fuel costs. If the value of time is proportional to the average wage, this ratio will be higher for high-income groups and may be expected to increase over time. For other energy services, such as household heating, time costs might be a less significant determinant of demand.

However, time costs for this service may have been much greater in the past when coal or wood fires were the norm, since time was required for preparing and lighting the fuel. In many developing countries, the time required to collect fuelwood remains an enormous burden.

The relationship between energy and time may be represented by setting time efficiency as a function of energy efficiency ($\theta(\varepsilon)$) or vice versa ($\varepsilon(\theta)$). By taking the first of these, we may write the energy demand for a particular energy services as: $E = s[P_S(\varepsilon), P_T(\theta(\varepsilon))]/\varepsilon$. This leads to an alternative definition of the rebound effect that takes into account the associated changes in time costs:

Definition 7: $\eta_\varepsilon(E) = -1 - \eta_{P_S}(S) + [\eta_{P_T}(S)\eta_\theta(P_T)\eta_\varepsilon(\theta)]$

Again, as compared to Definitions 3 and 4 there is an additional term in square brackets. This is the product of the elasticity of demand for useful work with respect to time costs ($\eta_{P_T}(S)$), the elasticity of time costs with respect to time efficiency ($\eta_\theta(P_T)$) and the elasticity of time efficiency with respect to energy efficiency ($\eta_\varepsilon(\theta)$). We expect the first of these to be negative (higher time costs should reduce the demand for useful work) and the second to be positive (higher time efficiency should reduce time costs). However, the sign of the last elasticity is ambiguous: while substitution between energy and time implies that energy efficiency is negatively correlated with time efficiency, technological improvements may sometimes improve both (e.g. microwave ovens).

However, in many cases greater energy (time) efficiency is likely to be achieved at the expense of lower time (energy) efficiency (i.e. θ and ε are negatively correlated). For example, a sports car is less energy efficient than a Smart car; aircraft are less energy efficient than ships; washing machines are less energy efficient than hand-washing; and so on. In these circumstances, the resulting increase in time costs will offset the saving in energy costs leading to a smaller rebound effect. For example, while greater fuel efficiency may make driving cheaper, consumers may not be willing to spend the time driving greater distances. This again suggests that empirical estimates based upon Definitions 3 and 4 and relying primarily upon historical or cross-sectional variations in energy prices may overestimate the magnitude of the rebound effect.

As with capital costs, the size of this upward bias will depend upon the relative magnitude of the different elasticities. One notable implication is that rebound effects from improved energy efficiency may be expected to decrease over time, since GDP growth should increase average wages and make time costs relatively more important in the total cost of energy services. One of the few studies to show evidence for this is Small and Van Dender [18], although their methodology was subsequently criticised by Harrison *et al* [49],

If improvements in energy efficiency are associated with changes in both time *and* capital costs, the appropriate expression for the rebound effect becomes:

Definition 8: $\eta_{\varepsilon}(E) = -1 - \eta_{P_S}(S) - [\eta_{P_K}(S)\eta_{\varepsilon}(P_K)] + [\eta_{P_T}(S)\eta_{\theta}(P_T)\eta_{\varepsilon}(\theta)]$

As pointed out by Binswanger [21], the analogy between time and energy efficiency also suggests that there should be a parallel *rebound effect with respect to time*. Since improvements in the time efficiency associated with a particular service lower the cost of that service, there should be a corresponding increase in service demand that will offset the potential time savings. Again, transport provides a particularly good example: the potential time savings from faster modes of transport may be partly or wholly taken back by travelling greater distances. Similar patterns are likely to apply to other services (e.g. washing clothes more often), but may be less noticeable if time costs form a smaller proportion of total costs, or if the assumptions of the simple Becker model (e.g. no joint production) do not apply.

The rebound effect with respect to time may be defined as an efficiency elasticity ($\eta_{\theta}(T) = \eta_{\theta}(S) - 1$) or as a price elasticity ($\eta_{\theta}(T) = \eta_{P_T}(S) - 1$) in a similar manner to the conventional rebound effect. Empirical investigation of this effect would similarly need to take into account the potential correlation between improvements in time efficiency and other input costs (including capital and energy costs); and the potential endogeneity of time efficiency (e.g. consumers may choose a more time efficient technology if they anticipate a high demand for the service). But in the absence of good data on time use patterns and time efficiency, such considerations remain academic.

Both the substitution of energy for time in the production and consumption of energy services, and the subsequent rebound effect with respect to time should act to increase overall energy consumption. Indeed, it is possible that these processes have had a more important influence upon aggregate energy consumption than the conventional rebound effect with respect to energy efficiency. Moreover, if wages continue to increase faster than energy prices, the first two effects may be expected to increase in importance, while the conventional rebound effect decreases in importance. To date, however, analytical and political attention has focused disproportionately on the former.

Summary

This paper has sought to clarify and bring together a number of definitions of the direct rebound effect and identify their underlying assumptions. It has clarified the relationship between the 'engineering' definition of the rebound effect as an efficiency elasticity and the more common definition in the empirical literature as a price elasticity. It has discussed a number of factors that need to be taken into account when developing such empirical estimates and emphasised the trade-offs between both the different categories of input costs and between useful work and other attributes of an energy service. It has also shown how different measures of useful work (together with differing ways of decomposing those measures) may lead to different conclusions regarding the nature and size of rebound effects.

Most empirical estimates are based upon price elasticities and rely primarily upon historical or cross-sectional variations in energy prices. The paper has argued that such studies could potentially overestimate the magnitude of the rebound effect. Factors contributing to this include: the irreversibility of energy efficiency investment and the consequent asymmetry of price elasticity estimates; the positive correlation between energy efficiency and capital costs; the role of price induced efficiency improvements; the endogeneity of energy efficiency; and the negative correlation between energy efficiency and time efficiency. Different studies address these factors in different ways and to a greater or lesser extent, with some of the best examples being recent US studies of travel by private car [17, 45]. Those studies that use the own-price elasticity of energy demand as a proxy for the rebound effect appear to be particularly flawed.

Perhaps the greatest area of neglect is the time costs associated with energy service provision. This may be largely due to the lack of adequate data in this area. However, the substitution of energy for time in the provision of energy services, together with the parallel 'rebound effect with respect to time' are likely to be important drivers of increases in aggregate energy consumption. Both of these deserve further research.

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Annex

Derivation of Definition 1

Given $S = \varepsilon E$

$$\eta_\varepsilon(E) = \frac{\partial(\frac{S}{\varepsilon})}{\partial\varepsilon}\left(\frac{\varepsilon}{S/\varepsilon}\right) = \left(-S\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon}\frac{\partial S}{\partial\varepsilon}\right)\left(\frac{\varepsilon^2}{S}\right) = \frac{\partial S}{\partial\varepsilon}\frac{\varepsilon}{S} - 1$$

Or: $\eta_\varepsilon(E) = -\eta_\varepsilon(S) - 1$

Derivation of Definition 2

Given $S = \varepsilon E$ and $S = NO*CAP*UTIL$

$$\eta_\varepsilon(E) = \frac{\varepsilon}{E}\left[-\frac{(NO*CAP*UTIL)}{\varepsilon^2} + \frac{1}{\varepsilon}\left((NO*CAP)\frac{\partial UTIL}{\partial\varepsilon} + (NO*UTIL)\frac{\partial CAP}{\partial\varepsilon} + (CAP*UTIL)\frac{\partial NO}{\partial\varepsilon}\right)\right]$$

Substituting $E = (NO*CAP*UTIL)/\varepsilon$ and cancelling terms:

$$\eta_\varepsilon(E) = -1 + \left(\frac{\varepsilon}{UTIL}\frac{\partial UTIL}{\partial\varepsilon} + \frac{\varepsilon}{CAP}\frac{\partial CAP}{\partial\varepsilon} + \frac{\varepsilon}{NO}\frac{\partial NO}{\partial\varepsilon}\right)$$

Or: $\eta_\varepsilon(E) = [\eta_\varepsilon(NO) + \eta_\varepsilon(CAP) + \eta_\varepsilon(UTIL)] - 1$

Derivation of Definition 3

Given $E = S(P_S)/\varepsilon$ and $P_S = P_E/\varepsilon$ and assuming that P_E is exogenous, we have:

$$\eta_\varepsilon(E) = \frac{\partial E}{\partial\varepsilon}\frac{\varepsilon}{E} = \frac{\varepsilon}{E}\left[-\frac{S}{\varepsilon^2} + \frac{\partial S}{\partial P_S}\frac{\partial P_S}{\partial\varepsilon}\right] = \frac{\varepsilon}{E}\left[-\frac{S}{\varepsilon^2} - \frac{1}{\varepsilon}\frac{P_E}{\varepsilon^2}\frac{\partial S}{\partial P_S}\right]$$

$$= -\frac{S}{\varepsilon E} - \frac{P_E}{\varepsilon^2 E}\frac{\partial S}{\partial P_S} \quad = -1 - \frac{P_S}{S}\frac{\partial S}{\partial P_S}$$

Or: $\eta_\varepsilon(E) = -\eta_{P_S}(S) - 1$

Derivation of Definition 4

Given $E = S(P_S)/\varepsilon$ and $P_S = P_E/\varepsilon$ we have:

$$\eta_{P_S}(S) = \frac{\partial S}{\partial P_S}\frac{P_S}{S} = -\frac{\partial(\varepsilon E)}{\partial(P_E/\varepsilon)}\frac{P_E/\varepsilon}{\varepsilon E}$$

But if energy efficiency is held constant the above relationship becomes:

$$\eta_{P_S}(S) = \frac{\partial E}{\partial P_E}\frac{P_E}{E} = \eta_{P_E}(E)$$

Or: $\eta_\varepsilon(E) = -\eta_{P_E}(E) - 1$

Derivation of Definition 5

Including the capital costs of new equipment (P_K), the basic identity becomes:

$$E = s[P_S(\varepsilon), P_K(\varepsilon)] / \varepsilon$$

Taking derivatives with respect to energy efficiency, we have:

$$\frac{\partial E}{\partial \varepsilon} = -\frac{S}{\varepsilon^2} + \frac{1}{\varepsilon}\frac{\partial S}{\partial \varepsilon} = -\frac{S}{\varepsilon^2} + \frac{1}{\varepsilon}\left[\frac{\partial S}{\partial P_S}\frac{\partial P_S}{\partial \varepsilon} + \frac{\partial S}{\partial P_K}\frac{\partial P_K}{\partial \varepsilon}\right] = -\frac{S}{\varepsilon^2} - \frac{P_E}{\varepsilon^3}\frac{\partial S}{\partial P_S} + \frac{1}{\varepsilon}\frac{\partial S}{\partial P_K}\frac{\partial P_K}{\partial \varepsilon}$$

Multiply through by ε / E to obtain $\eta_\varepsilon(E)$:

$$\frac{\partial E}{\partial \varepsilon}\frac{\varepsilon}{E} = -\frac{S}{\varepsilon E} - \frac{P_E}{\varepsilon^2 E}\frac{\partial S}{\partial P_S} + \frac{1}{E}\frac{\partial S}{\partial P_K}\frac{\partial P_K}{\partial \varepsilon} \quad = \quad -1 - \frac{P_E/\varepsilon}{\varepsilon\; E}\frac{\partial S}{\partial P_S} + \frac{1}{E}\frac{\partial S}{\partial P_K}\frac{\partial P_K}{\partial \varepsilon}$$

$$\frac{\partial E}{\partial \varepsilon}\frac{\varepsilon}{E} = -1 - \frac{P_S}{S}\frac{\partial S}{\partial P_S} + \frac{\varepsilon}{S}\frac{\partial S}{\partial P_K}\frac{\partial P_K}{\partial \varepsilon}$$

Multiplying numerator and denominator of the last term with P_K, we have:

$$\frac{\partial E}{\partial \varepsilon}\frac{\varepsilon}{E} = -1 - \frac{P_S}{S}\frac{\partial S}{\partial P_S} + \left(\frac{P_K}{S}\frac{\partial S}{\partial P_K}\right)\left(\frac{\varepsilon}{P_K}\frac{\partial P_K}{\partial \varepsilon}\right)$$

Or: $\eta_\varepsilon(E) = -1 - \eta_{P_S}(S) + [\eta_{P_K}(S)\eta_\varepsilon(P_K)]$

Derivation of Definition 6

If energy efficiency depends upon the energy prices, the basic identity can be written as follows:

$$E = \frac{S}{\varepsilon} = \frac{S(P_S)}{\varepsilon(P_E)} = \frac{s[P_E/\varepsilon(P_E)]}{\varepsilon(P_E)}$$

Use the product and chain rules to differentiate this with respect to energy commodity prices:

$$\frac{\partial E}{\partial P_E} = -\frac{S}{\varepsilon^2}\frac{\partial \varepsilon}{\partial P_E} + \frac{1}{\varepsilon}\left[\frac{\partial S}{\partial P_S}\frac{\partial P_S}{\partial P_E} + \frac{\partial S}{\partial P_S}\frac{\partial P_S}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial P_E}\right]$$

$$\frac{\partial E}{\partial P_E} = -\frac{S}{\varepsilon^2}\frac{\partial \varepsilon}{\partial P_E} + \frac{1}{\varepsilon}\left[\frac{\partial S}{\partial P_S}\frac{1}{\varepsilon} - \frac{\partial S}{\partial P_S}\frac{P_E}{\varepsilon^2}\frac{\partial \varepsilon}{\partial P_E}\right] = -\frac{S}{\varepsilon^2}\frac{\partial \varepsilon}{\partial P_E} + \frac{1}{\varepsilon^2}\frac{\partial S}{\partial P_S} - \frac{P_E}{\varepsilon^3}\frac{\partial S}{\partial P_S}\frac{\partial \varepsilon}{\partial P_E}$$

Multiplying both sides by P_E/E to switch into elasticity forms:

$$\frac{\partial E}{\partial P_E}\frac{P_E}{E} = -\frac{S}{\varepsilon^2}\frac{P_E}{\varepsilon}\frac{\partial \varepsilon}{\partial P_E} + \frac{1}{\varepsilon^2}\frac{P_E}{\varepsilon}\frac{\partial S}{\partial P_S} - \frac{P_E}{\varepsilon^3}\frac{P_E}{E}\frac{\partial S}{\partial P_S}\frac{\partial \varepsilon}{\partial P_E}$$

Noting that $S = \varepsilon E$ and $P_S = P_E / \varepsilon$, we can simplify:

$$\frac{\partial E}{\partial P_E}\frac{P_E}{E} = -\frac{P_E}{\varepsilon}\frac{\partial \varepsilon}{\partial P_E} + \frac{P_S}{S}\frac{\partial S}{\partial P_S} - \frac{P_S}{S}\frac{\partial S}{\partial P_S}\frac{P_E}{\varepsilon}\frac{\partial \varepsilon}{\partial P_E}$$

Expressing each term as an elasticity, we obtain:

$$\eta_{P_E}(E) = \eta_{P_S}(S) - \eta_{P_E}(\varepsilon)\left[1 + \eta_{P_S}(S)\right]$$

Or alternatively: $\eta_{P_S}(S) = \dfrac{\eta_{P_E}(E) + \eta_{P_E}(\varepsilon)}{1 - \eta_{P_E}(\varepsilon)}$

Derivation of the relative magnitude of price elasticities

Starting with the identity $E = \frac{S[P_E / \varepsilon(P_E)]}{\varepsilon(P_E)}$, the energy cost elasticity of the demand for useful work may be expressed as:

$$\eta_{P_S}(S) = \frac{P_S}{S}\frac{\partial S}{\partial P_S} = \frac{P_S}{S}\left[\varepsilon\frac{\partial S}{\partial P_S} + E\frac{\partial \varepsilon}{\partial P_S}\right] = \frac{P_S}{E}\frac{\partial E}{\partial P_S} + \frac{P_S}{\varepsilon}\frac{\partial \varepsilon}{\partial P_S}$$

Or:

$$\eta_{P_S}(E) = \eta_{P_S}(S) - \eta_{P_S}(\varepsilon)$$

We expect that $\eta_{P_S}(\varepsilon) \geq 0$ (higher costs for useful work encourages higher energy efficiency). In contrast, we expect that $\eta_{P_S}(S) \leq 0$ (higher prices reduce demand). Hence we expect that:

$$\left|\eta_{P_S}(E)\right| \geq \left|\eta_{P_S}(S)\right|$$

By a very similar process we can show:

$$\eta_{P_E}(E) = \eta_{P_E}(S) - \eta_{P_E}(\varepsilon)$$

And hence we can argue that:

$$\left|\eta_{P_E}(E)\right| \geq \left|\eta_{P_E}(S)\right|$$

Rearranging Definition 6 we have:

$$\eta_{P_E}(E) = \eta_{P_S}(S)\left[1 - \eta_{P_E}(\varepsilon)\right] - \eta_{P_E}(\varepsilon)$$

In most cases we would expect $1 \geq \eta_{P_E}(\varepsilon) \geq 0$ and $0 \geq \eta_{P_S}(S) \geq -1$. This implies that:

$$\left|\eta_{P_S}(S)\right| \leq \left|\eta_{P_E}(E)\right|$$

Combining the above three relationships, we obtain:

$$\left|\eta_{P_E}(S)\right| \leq \left|\eta_{P_S}(S)\right| \leq \left|\eta_{P_E}(E)\right| \leq \left|\eta_{P_S}(E)\right|$$

Derivation of Definition 7

Including time costs and assuming time efficiency (θ) is a function of energy efficiency (ε) we have:

$$E = s[P_S(\varepsilon), P_T(\theta(\varepsilon))]/\varepsilon$$

Taking derivatives with respect to energy efficiency, we have:

$$\frac{\partial E}{\partial \varepsilon} = -\frac{S}{\varepsilon^2} + \frac{1}{\varepsilon}\left[\frac{\partial S}{\partial P_S}\frac{\partial P_S}{\partial \varepsilon} + \frac{\partial S}{\partial P_T}\frac{\partial P_T}{\partial \theta}\frac{\partial \theta}{\partial \varepsilon}\right] = -\frac{S}{\varepsilon^2} - \frac{P_E}{\varepsilon^3}\frac{\partial S}{\partial P_S} + \frac{1}{\varepsilon}\frac{\partial S}{\partial P_T}\frac{\partial P_T}{\partial \theta}\frac{\partial \theta}{\partial \varepsilon}$$

Multiply through by ε / E to obtain $\eta_\varepsilon(E)$:

$$\frac{\partial E}{\partial \varepsilon}\frac{\varepsilon}{E} = -\frac{S}{\varepsilon E} - \frac{P_E}{\varepsilon^2 E}\frac{\partial S}{\partial P_S} + \frac{1}{E}\frac{\partial S}{\partial P_T}\frac{\partial P_T}{\partial \theta}\frac{\partial \theta}{\partial \varepsilon}$$

Multiply the third term by ($\theta P_T / \theta P_T$) and rearrange:

$$\frac{\partial E}{\partial \varepsilon}\frac{\varepsilon}{E} = -1 - \frac{P_S}{S}\frac{\partial S}{\partial P_S} + \left(\frac{P_T}{S}\frac{\partial S}{\partial P_T}\right)\left(\frac{\theta}{P_T}\frac{\partial P_T}{\partial \theta}\right)\left(\frac{\varepsilon}{\theta}\frac{\partial \theta}{\partial \varepsilon}\right)$$

Or: $\eta_\varepsilon(E) = -1 - \eta_{P_S}(S) + [\eta_{P_T}(S)\eta_\theta(P_T)\eta_\varepsilon(\theta)]$

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