Programme Area: Marine

Project: PerAWAT

Title: Report of Calibrated Numerical Models of Pentland Firth, including Validation Against Measured Data

Abstract:

This deliverable, a numerical model of the Pentland Firth has been developed using an ADCIRC DG code. Bathymetry data has been taken from Seazone and tidal boundary data taken from le Provost. The tidal dynamics of the Pentland Firth and surrounding area of the Orkney Isles is extremely complex. In developing the model a pragmatic approach of not trying to model every detail of the flow through the Orkney Isles but instead focusing on the modelling of the tidal dynamics in the main Pentland Firth. The model has been compared to available measurements and there is good agreement on both the amplitude and phase of water level between measurement and model.

Context:

The Performance Assessment of Wave and Tidal Array Systems (PerAWaT) project, launched in October 2009 with £8m of ETI investment. The project delivered validated, commercial software tools capable of significantly reducing the levels of uncertainty associated with predicting the energy yield of major wave and tidal stream energy arrays. It also produced information that will help reduce commercial risk of future large scale wave and tidal array developments.

Energy Technologies Institute

PerAWaT

WG3 WP6 D4A — REPORT OF CALIBRATED NUMERICAL MODELS OF PENTLAND FIRTH, INCLUDING VALIDATION AGAINST MEASURED DATA

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Revision History		
Issue / Version	Issue Date	Summary
0.1	27/1/2012	Report on Pentland Firth model for review by GH.
1.0	7/2/20112	Report on Pentland Firth model incorporating revisions from GH.

Executive summary

A numerical model of the Pentland Firth has been developed using an ADCIRC DG code. Bathymetry data has been taken from Seazone with tidal boundary data taken from le Provost.

The model has been compared to the available measurements. Given the complexity of the flow in the Pentland Firth the agreement is excellent. There is good agreement on both the amplitude and phase of water level between measurement and the model. The discrepancy between measurements and model in the dominant east/west current is small. There is substantial discrepancy in the comparatively small current in the north/south direction. The reasons for this are not understood. However, this would not significantly effect the modelling of the tidal energy resource.

Contents

Acceptance criteria

Acceptance criteria for this deliverable are set out in Table 1.

Acceptance criteria	Location in report
Report describes calibration methodology and quantifies model performance (and errors/sensitivities) of key parameters e.g. bed friction and horizontal eddy viscosity.	Whole report
Input files in format to be agreed	Input files are attached seperately

Table 1 Acceptance criteria

The attached input files (fort.14, fort.15 and fort.dg) have had the proprietary bathymetric data removed and replaced with freely available bathymetry from GEBCO. The data used for these simulations may be acquired from Seazone (now part of HR Wallingford). The forcing boundary conditions are set so that the model simulation starts on 31 December 2012.

Model set-up and parameters

Introduction

The tidal dynamics of the Pentland Firth and surrounding area is extremely complex. In developing a model for the region we have adopted a pragmatic approach of not trying to model every detail of the flow through the Orkney Isles but instead modelling the tidal dynamics in the main Pentland Firth. There are two reasons for adopting this pragmatic approach:

1. Computation time — The model will take significant time to compute. Having a fast simulation will allow more investigation of the physics both with and without energy extraction. Thus we wish to use the fewest elements that still allow us to capture the essential features of the physics. It is also important not to use elements that are too small in size, as this will necessitate a small time-step.

2. Limitations of depth-averaged models — The limitations of depth-averaged modelling are discussed at length in WG3 WP6 D2. Depth-averaged models cannot accurately reproduce all the complex flow features in the Pentland Firth. For instance vortex shedding from the islands is poorly modelled by depth integrated code (Stansby, 2006). Therefore, using a depth-averaged model to try to capture such details would simply give a spurious impression of accuracy which was not justified.

We choose not to model (effectively we just close off) any channel with a cross-sectional area less than 10,000 m^2. By comparison the cross section of the Pentland Firth is ~1,000,000 m^2. This simplifying assumption should therefore cause negligible effect on the predicted current passing through the main channel.

Mesh

The coastline was extracted from the bathymetry using a contouring algorithm in Matlab, with the results then edited by hand to address anomalies and special cases. Meshing was then carried out using SMS — again manual edits were made to the mesh to improve its characteristics. The smallest elements are ~200 m and the largest ~10 km. Small elements are used in the Pentland Firth itself with large elements in the deep water or in the Moray Firth (which is far enough from the possible location of turbines that the tidal dynamics does not need to be highly resolved).

The mesh is shown in Figure 1 with more detail in Figure 2. As part of this deliverable the mesh is supplied electronically as a fort.14 file.

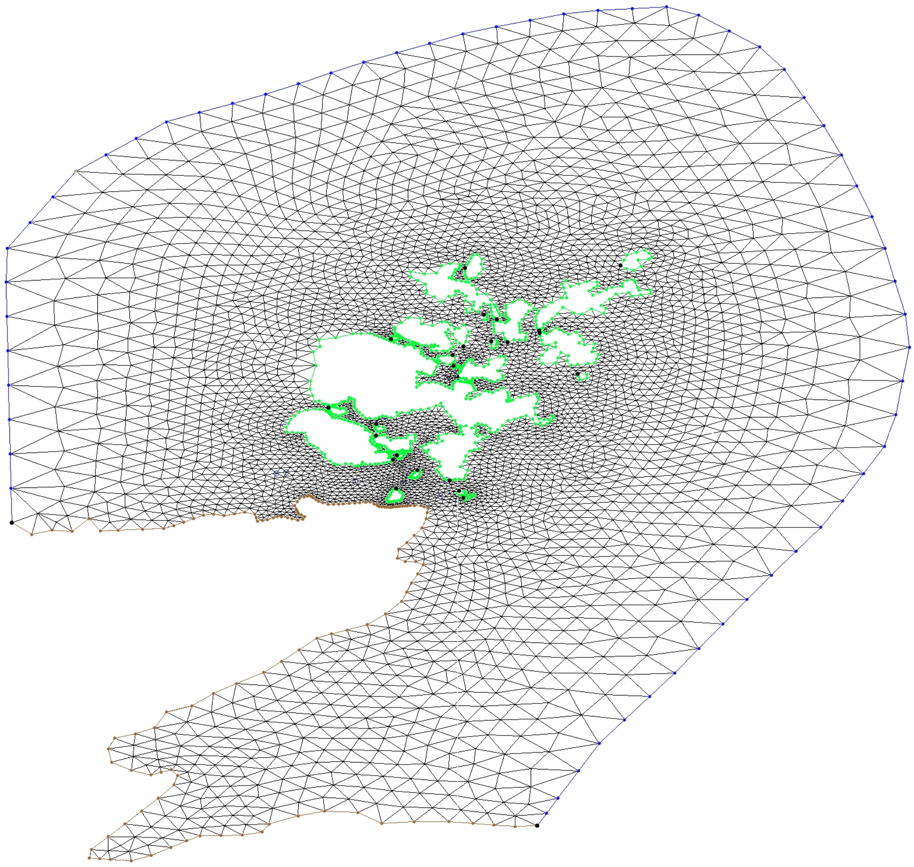

Figure 1 Mesh used for Pentland Firth model

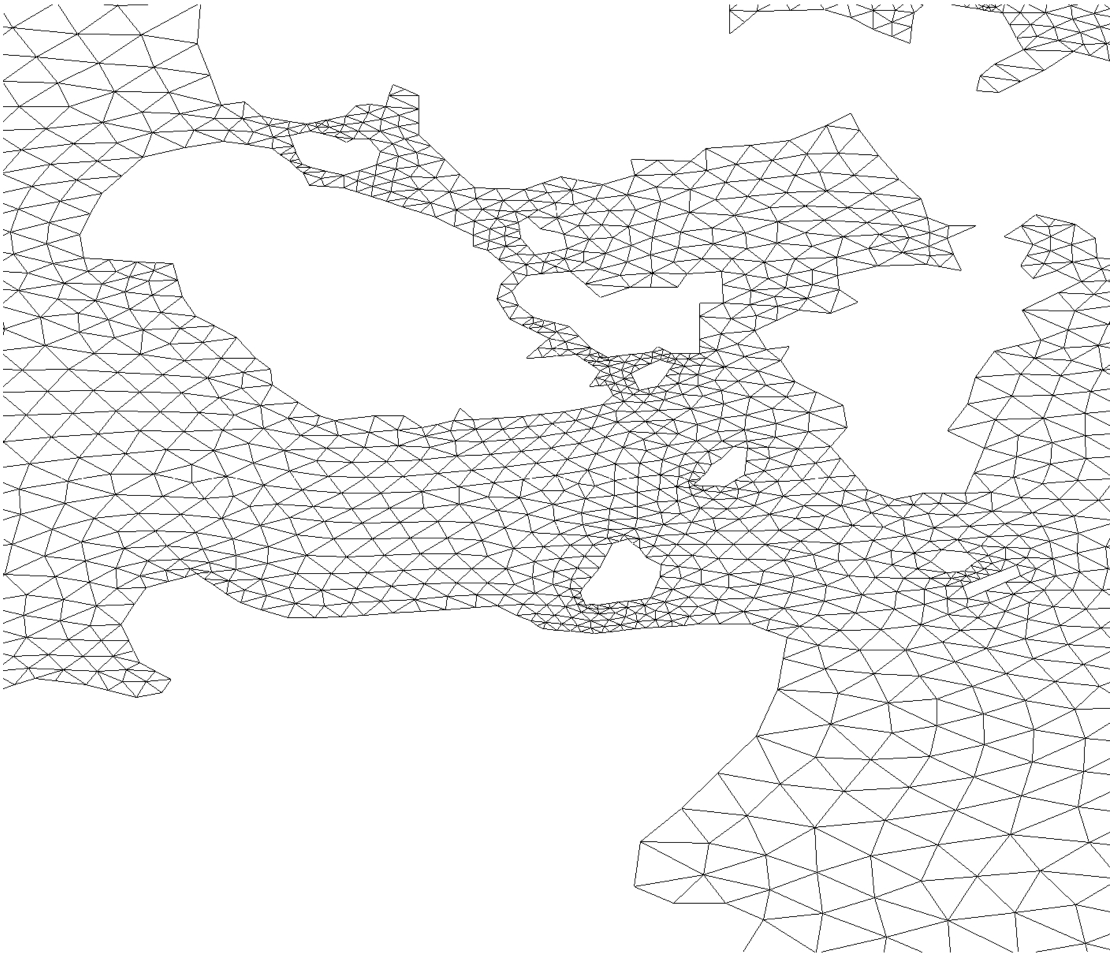

Figure 2 Detail of mesh around Pentland Firth

Boundary conditions

Tidal forcing

Numerical models of tides must be “forced” at any open boundary by specifying the amplitude and flux. These are typically derived from a lower resolution model covering a larger area. The values must then be interpolated (both spatially and temporally) for input into the model.

A number of tidal databases exist – some of which are openly available whereas others are charged for. In this work package we have opted to use the Le Provost database (Le Provost et al., 1995). Figure 3 shows the coverage and resolution of this database. The choice of the Le Provost datatbase was motivated by the existing integration of this database into the meshing software SMS (Militello & Zundel, 1999) which will be used in this work package (as demonstrated in WG3 WP6 D2).

The Le Provost database gives the values of 13 principal tidal constituents (2N2, K1, K2, L2, M2, MU2, N2, NU2, O1, P1, Q1, S2, T2): see Militello & Zundel (1999) which are sufficient for modelling the tidal dynamics. It should be noted that high-harmonics (over-tides) are significant in areas of strong current such as the Pentland Firth, which has significant M4 and M6 components. These components are generated by the local bed friction and geometry and thus do not need to be fed in at the boundary.

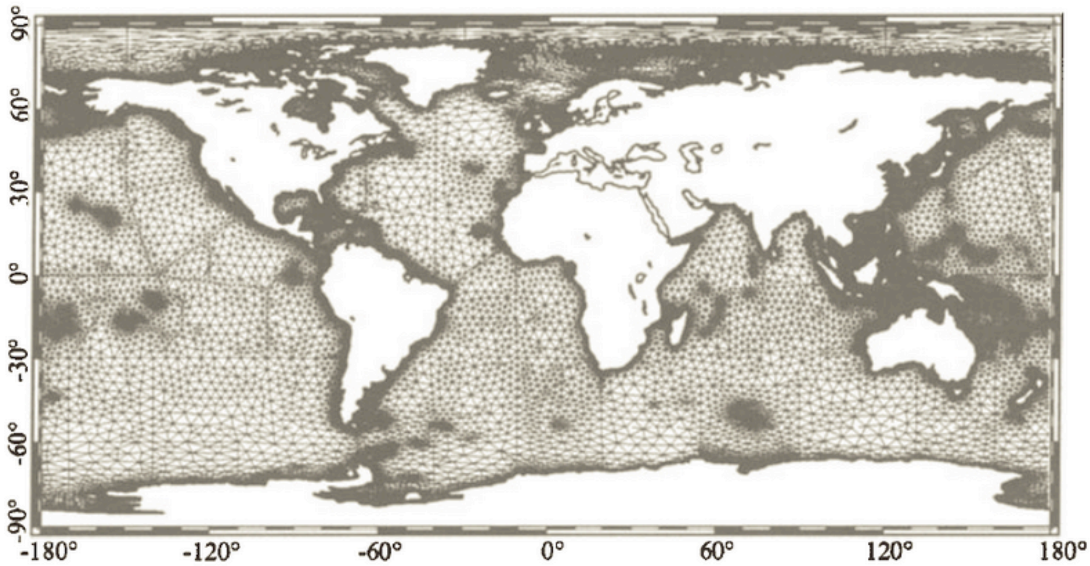

Figure 3 Coverage of Le Provost databse

Bathymetry

As set out in WG3 WP6 D3 bathymetry data has been derived from Seazone. Details of these data are given in Appendix 1. To remove the need for the computationally expensive wetting and drying a minimum depth of 2m (to still water level was applied). Very few nodes had their value modified by the application of this criterion[1]. The bathymetry was interpolated using linear interpolation. The bathymetry of the final grid is shown in Figure 4.

Due to restrictions on the onward transmission of bathymetry it has been agreed that the proprietary Seazone data will be replaced by freely available data from GEBCO (Monahan, 2008). The depth of the nodes for this is shown in Figure 5.

[1] When the same criterion was applied to the coarse GEBCO data more cells (particularly between the islands of the Orkneys) had their values modified.

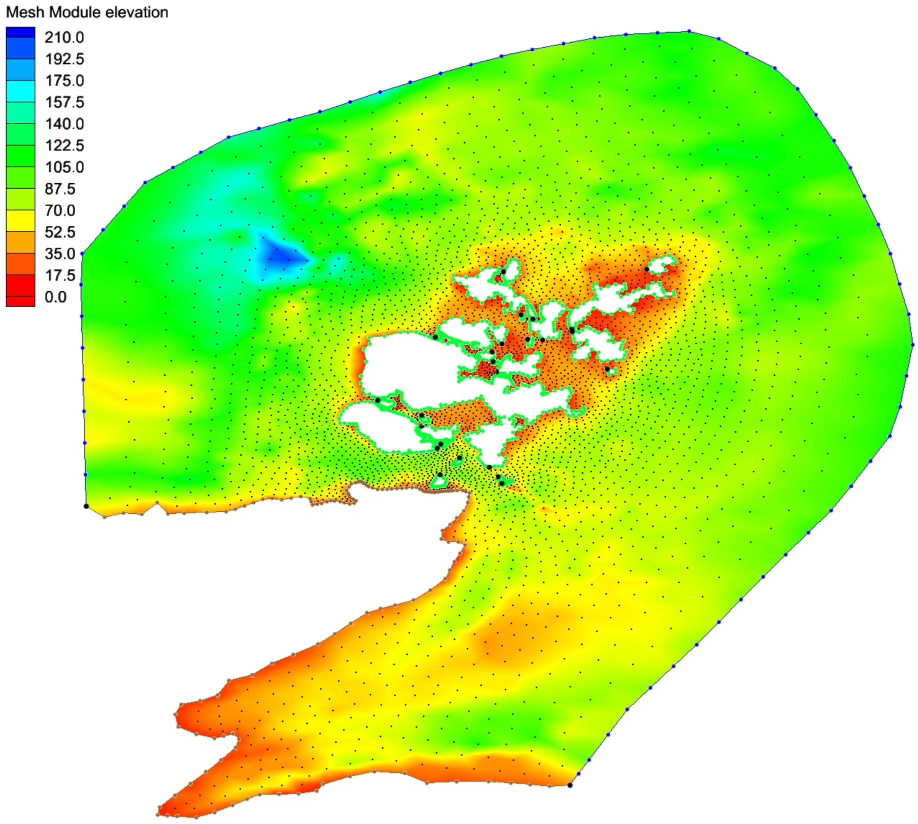

Figure 4 Depth of the nodes (m). Nodes shown with black dots

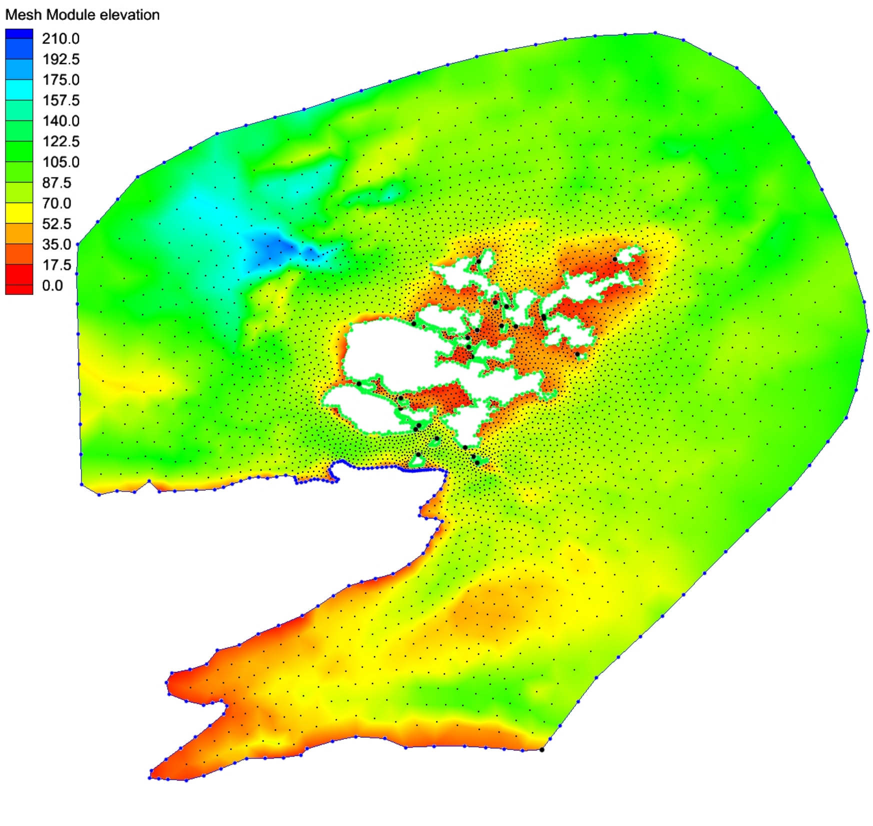

Figure 5 Depth of nodes on GEBCO mesh (delivered to ETI). Nodes shown with black dots

Model parameters

Several parameters are needed in the specification of the model. Some of these have been varied so as to "tune" the model to reproduce measured data. The final model configuration used the following parameters.

Parameter	Value	Notes
Time-step	1 s or 0.25 s	Larger values than 1.0 s led to instability in the code. Smaller values gave results negligibly different. Value required depends on the order of elements being used.
Element order	1 or 2	The results presented in this report were run using first order elements. Second order elements give near identical results.
Numerical method	4th order Runge-Kutta	
Initial ramp	0.1 days	Ramping function at boundary
Eddy viscosity coefficient	3 kg m/s	Standard value adopted for tidal modelling.
Non-linear bed friction coefficient	0.0025	See discussion below
Coriolis	Automatic	Derived from latitude and longitude
Harmonics used	2N2, K1, K2, L2, M2, MU2, N2, NU2, O1, P1, Q1, S2, T2, 2N2	Standard components available from database. Only M2, S2 and N2 are important at this location.

Wetting and drying	Off	The Pentland Firth area has a very small intertidal zone. Including this would not significantly improve the model but would greatly increase computation times.

Table 2 Parameters used in final model run

The crucial parameter which can be varied is the bed friction. Soulsby (1998) gives a range of suggested values for different sea-bed conditions and Salter (2009) gives some estimates of bed friction for the Pentland Firth. An example of the difference that different values of bed-friction make is shown in Figure 6. The difference in current through the main part of the channel is approximately 0.4 m/s. Consideration of the phase and amplitude of measurements led us to use the value given in Table 2.

It should be noted that there is no "right" answer to the bed friction parameter. In reality, the bed friction parameter varies spatially. Indeed the appropriate value may well be anisotropic (varying with direction) and also may not be constant with flow velocity. It would be possible to vary the parameter locally in the model in an attempt to obtain better agreement with measurements. However, given the limited measurements and the uncertainty in these data we choose not to use a variable bed friction. We find sufficiently good agreement using a constant bed friction value and prefer the simplest model which gives adequate results. There is some evidence that analyses with variable bed friction can be problematic (Richard Soulsby, personal communication).

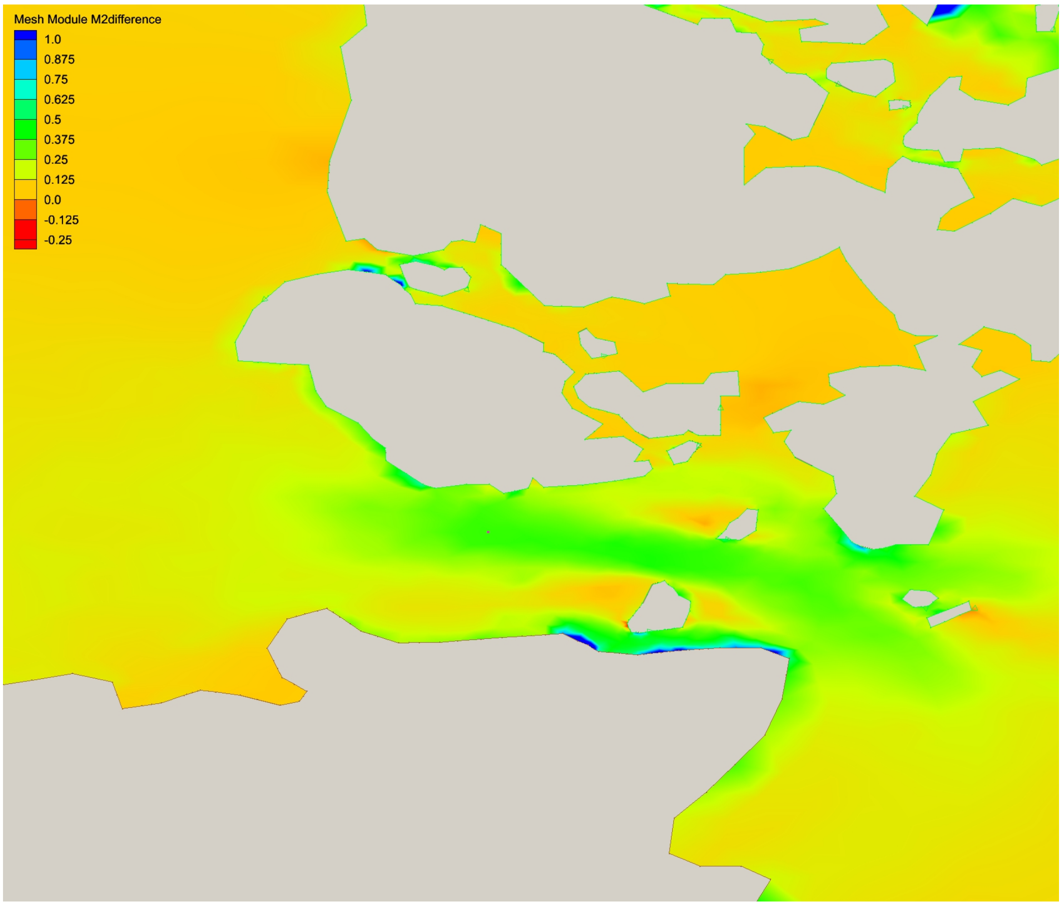

Figure 6 Difference in M2 tidal stream velocity for bed friction values of 0.0025 and 0.005

Model results

Figure 7 and Figure 8 show both water surface (relative to mean sea level) and depth-averaged current for typical flood and ebb tides respectively. The loss of head across the Pentland Firth is clearly evident.

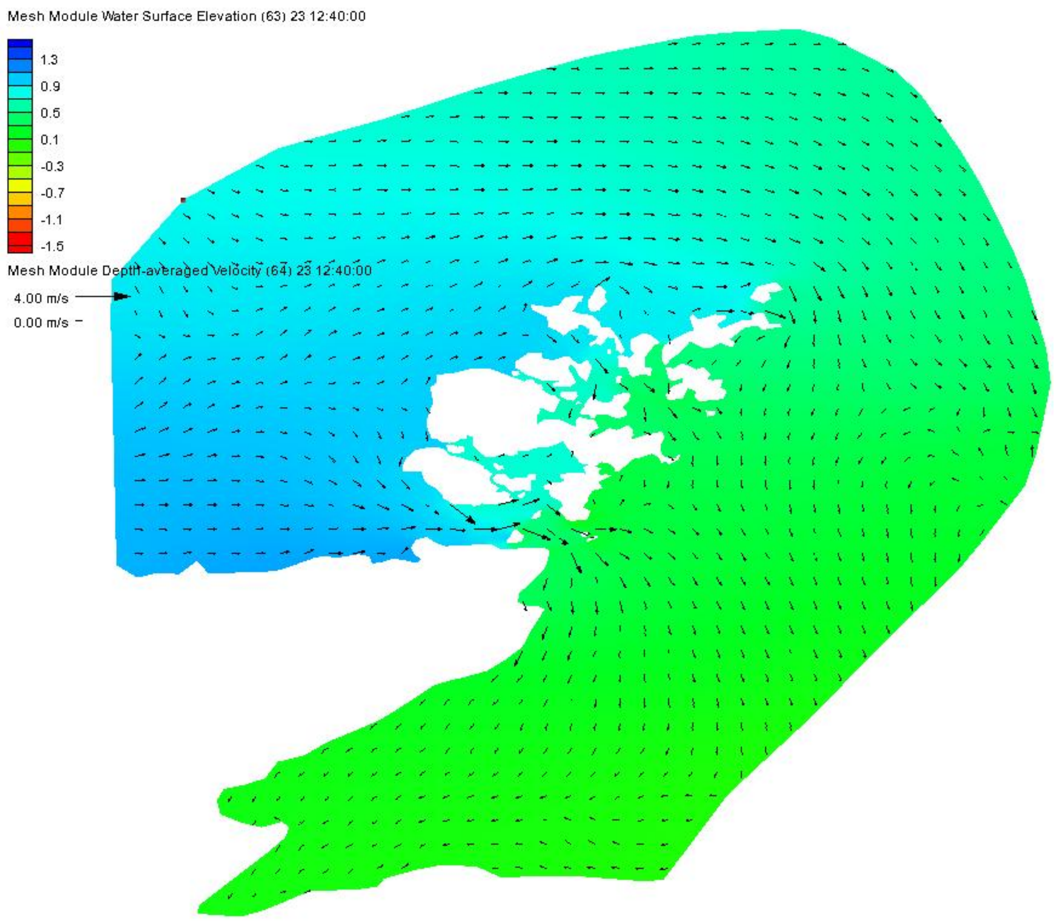

Figure 7 Model prediction for a typical flood tide

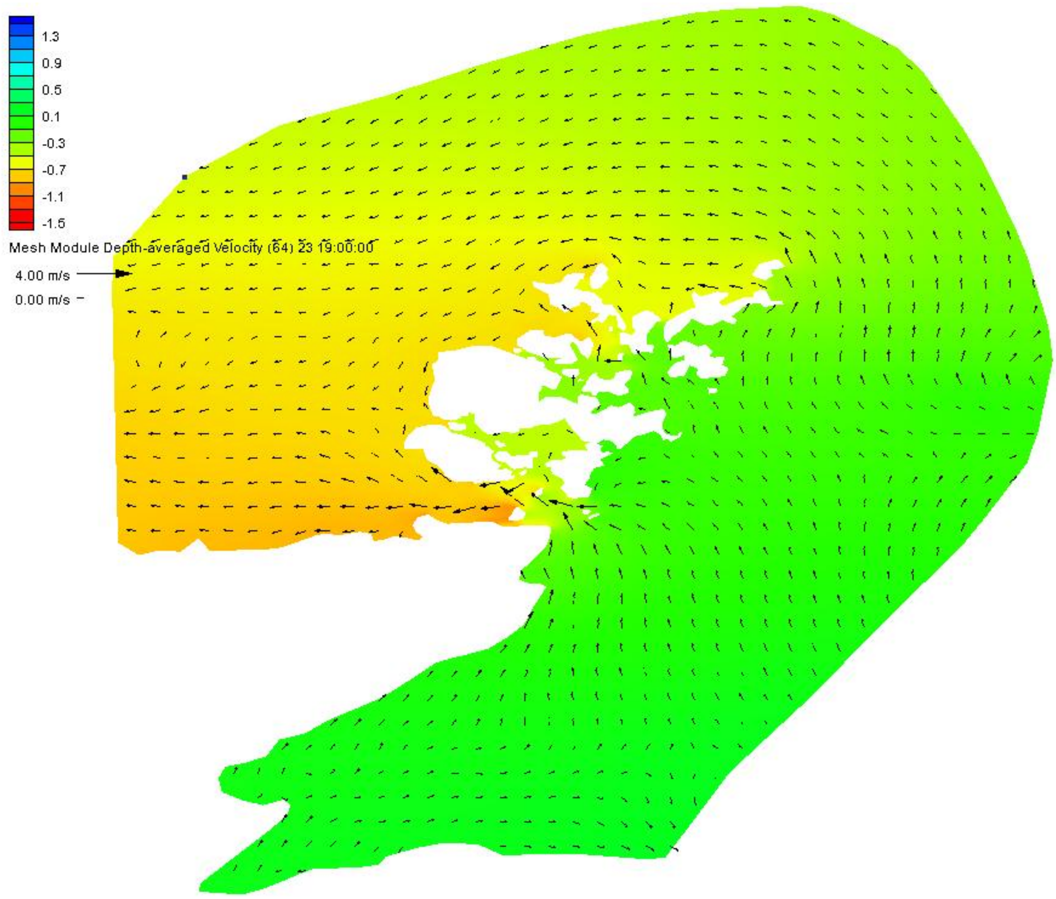

Figure 8 Model prediction for typical Ebb tide

It is also possible to isolate the main tidal harmonics by way of harmonic analysis (Doodson, 1921). In Figure 9, Figure 10 and Figure 11 the predicted M2, S2 and N2 water level amplitudes are plotted. Although with different amplitude scales, these all exhibit broadly similar. The phase relative to the start of the simulation shown in Figure 12, Figure 13 and Figure 14. These appear to behave in agreement with the expectation that the tidal wave propagates around the north of Great Britain and into the North Sea. The plots visually show good agreement with other maps of water level amplitude, for example Pingree & Griffiths (1981a) for M2 and N2 components, and Pingree & Griffiths (1981b) for the S2 tidal amplitude component.

In Figure 15, Figure 16 and Figure 17 we plot the M2, S2 and N2 current amplitudes — zooming in on the Pentland Firth. Again all the plots appear similar. The areas where eddies form behind the islands show the expected small velocities due to the separation of the flow. The eddy structure that forms behind these islands is not expected to be reproduced correctly, as noted in the introduction. Very high velocities are recorded in some of the shallow areas around the headlands. In reality there are very high velocities in these areas where tidal races occur (Admiralty, 2006).

Figure 9 Predicted M2 water level amplitude

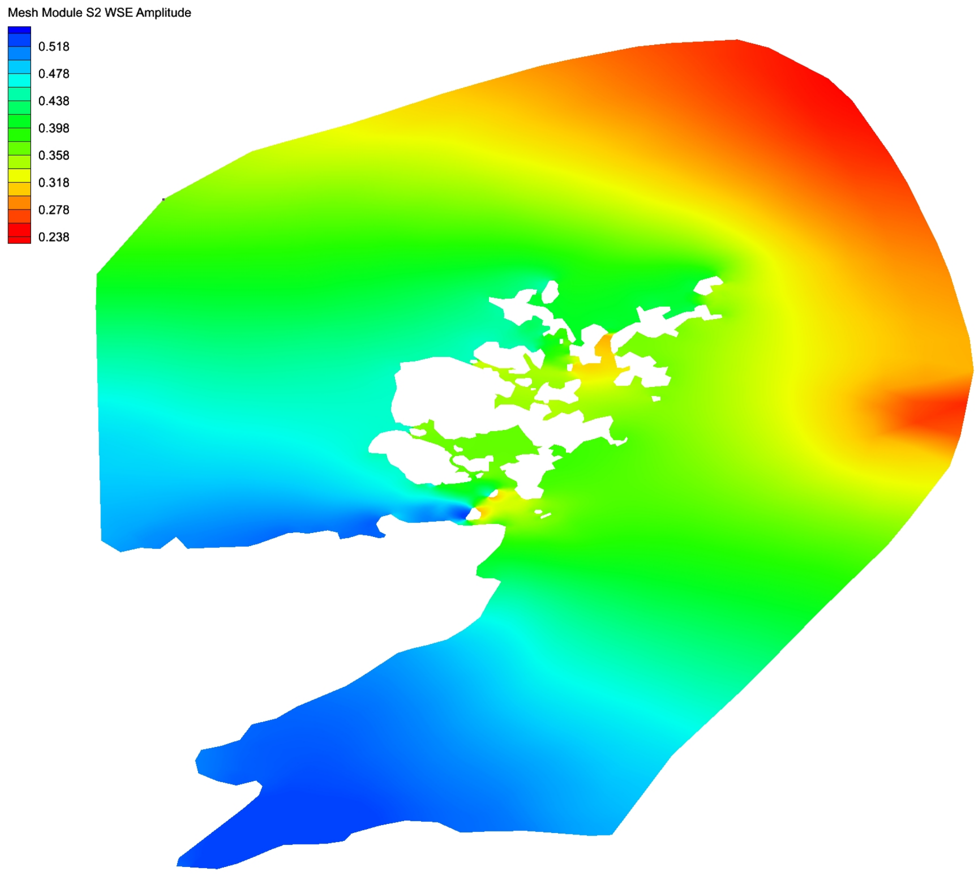

Figure 10 Predicted S2 water level amplitude

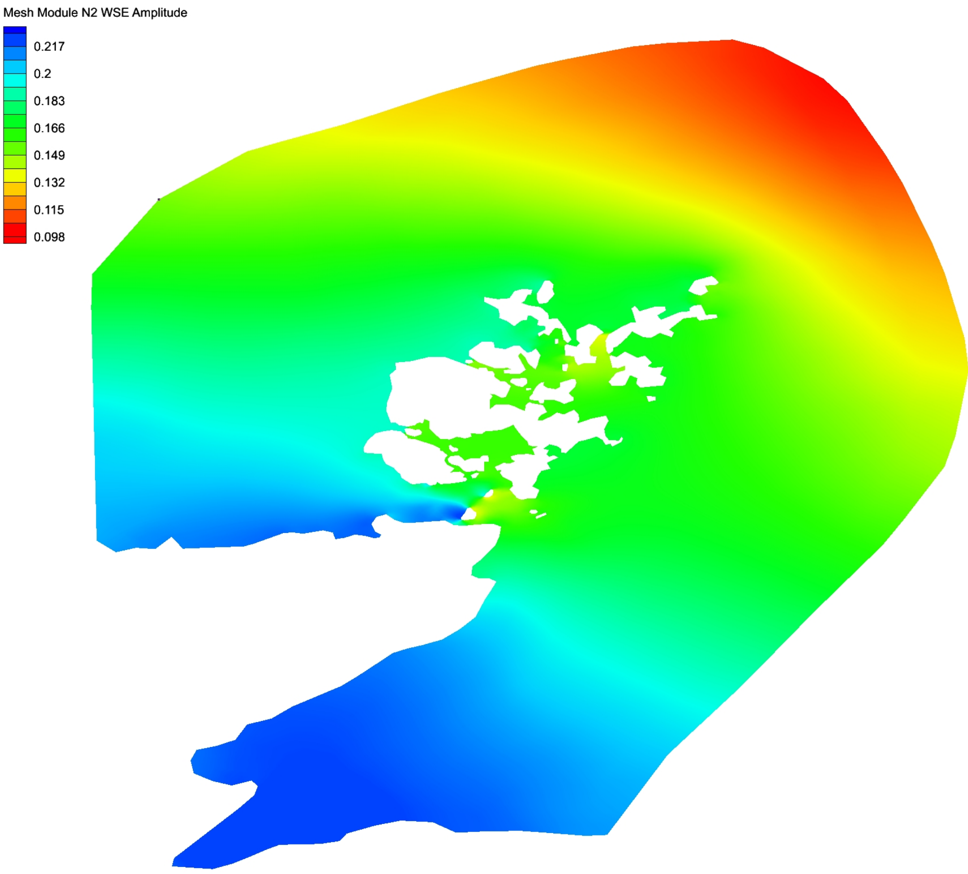

Figure 11 Predicted N2 water level amplitude

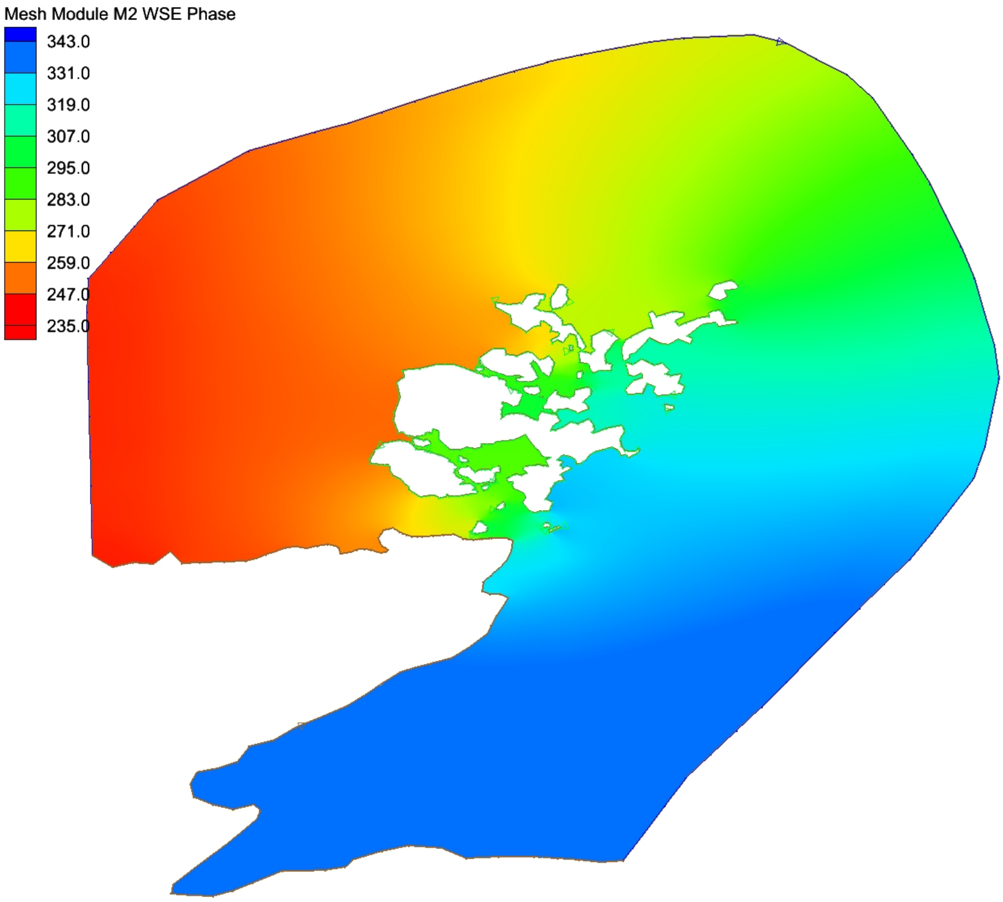

Figure 12 Phase of M2 water level

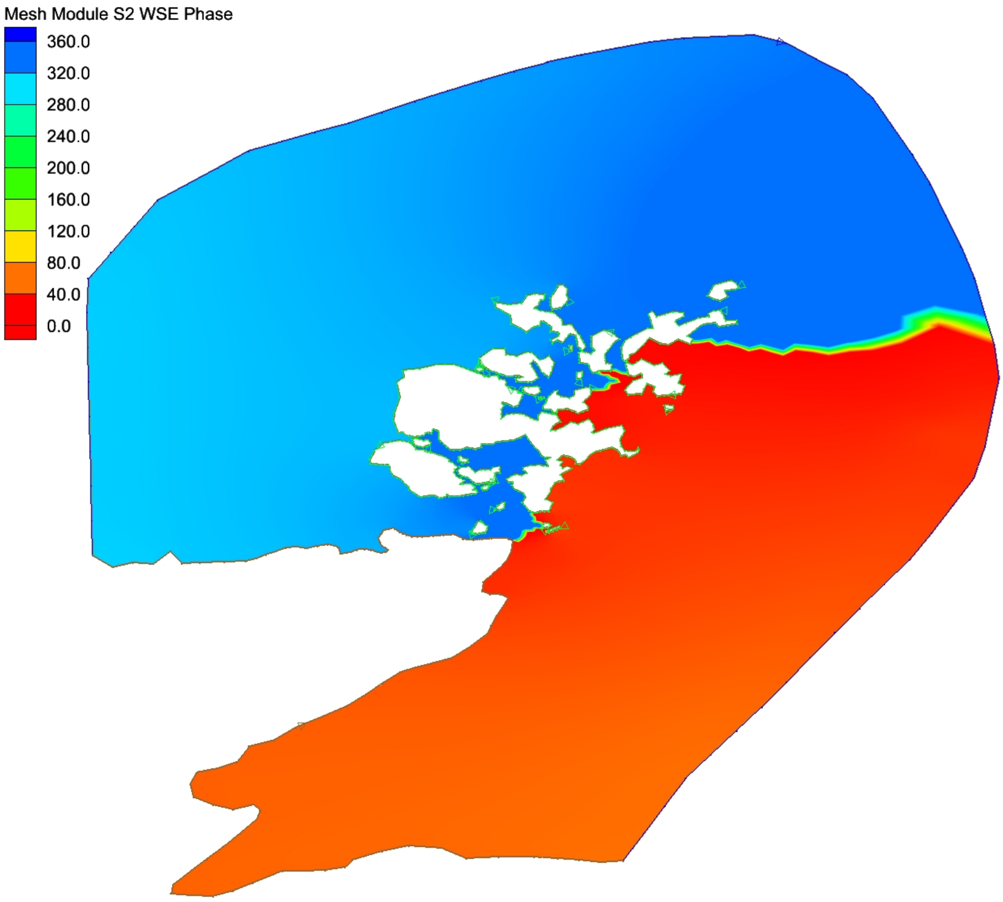

Figure 13 Phase of S2 water level

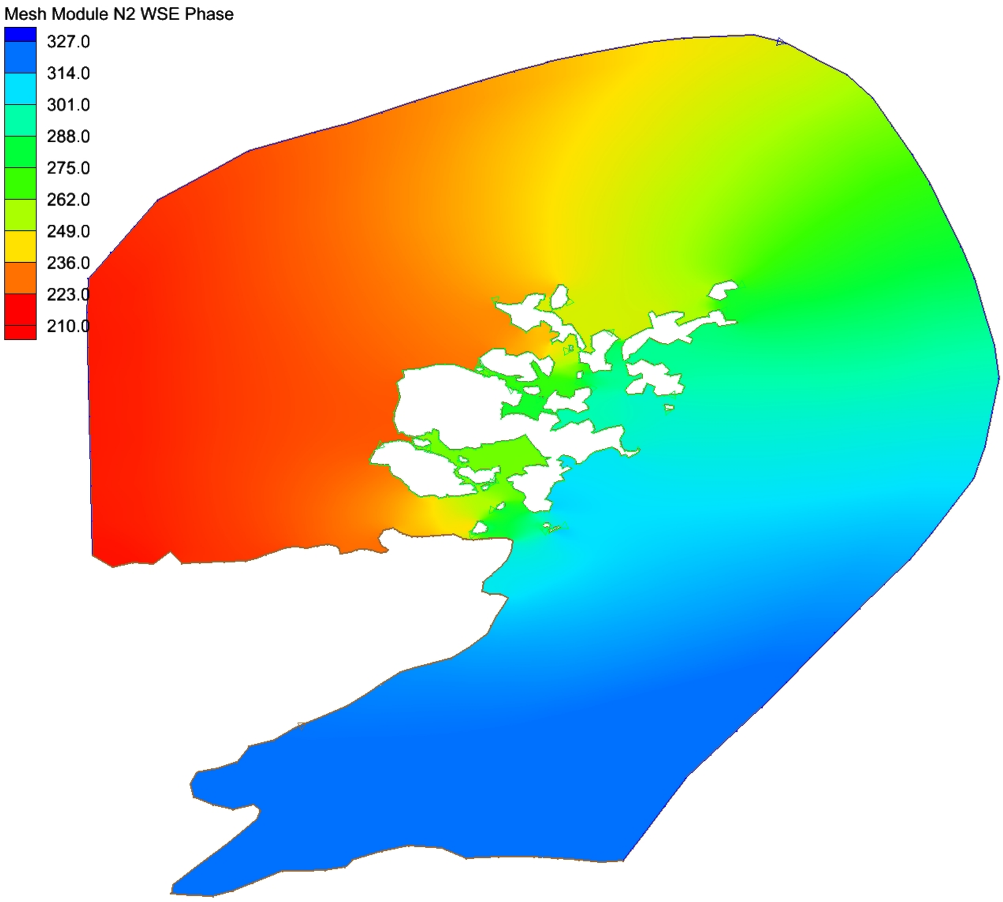

Figure 14 Phase of N2 water level

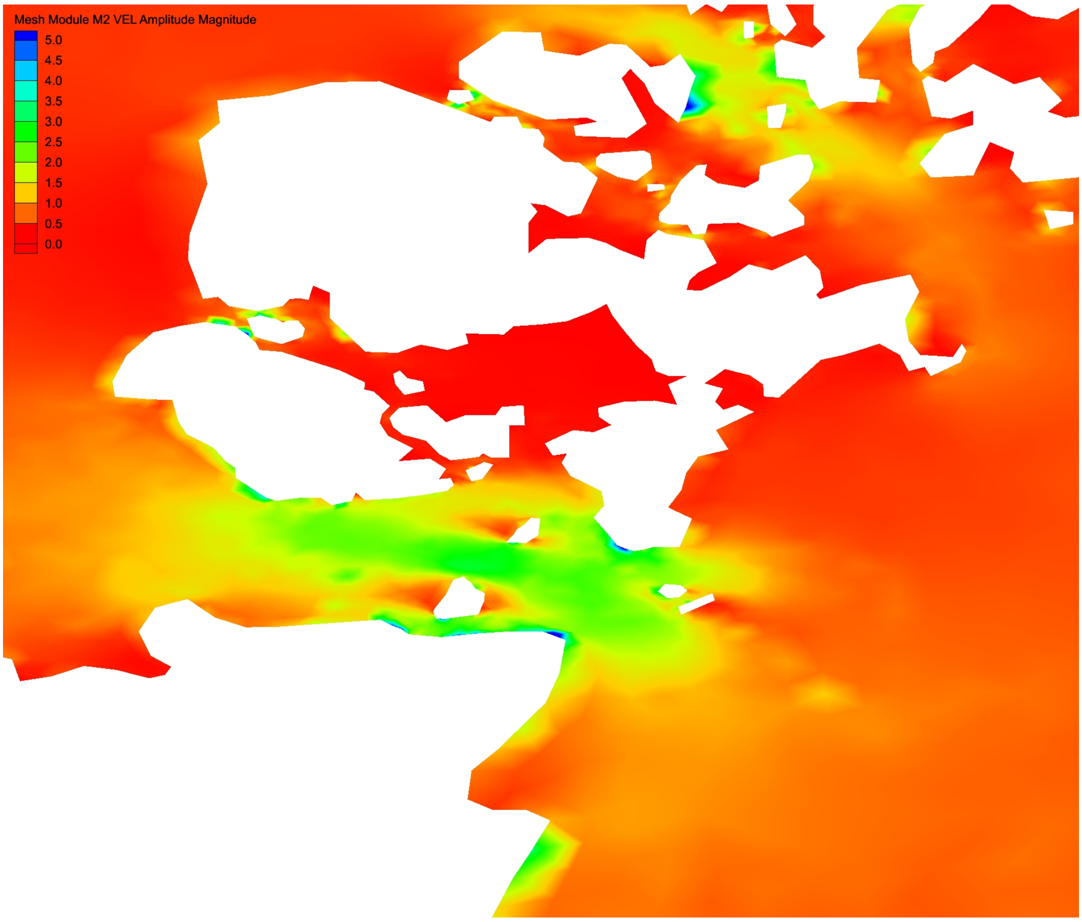

Figure 15 Predicted M2 tidal current amplitude

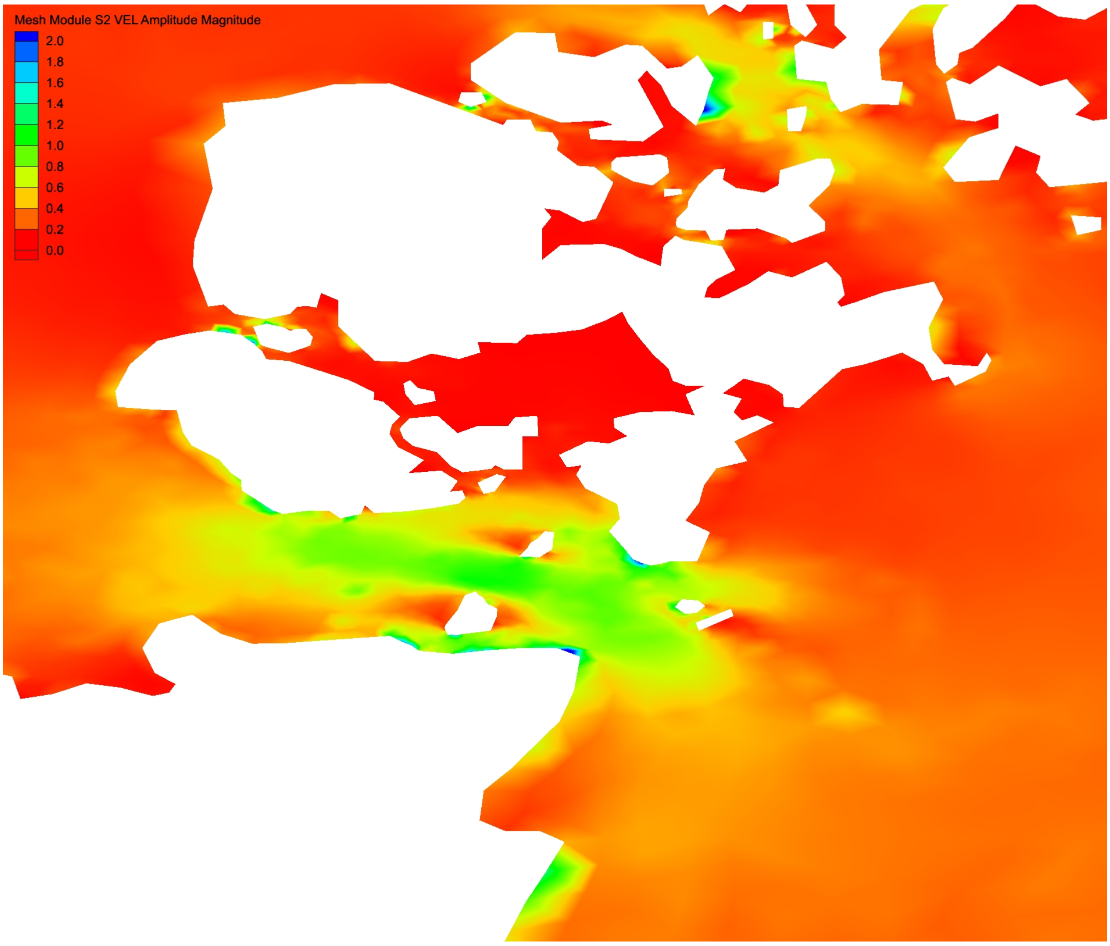

Figure 16 Predicted S2 tidal current amplitude

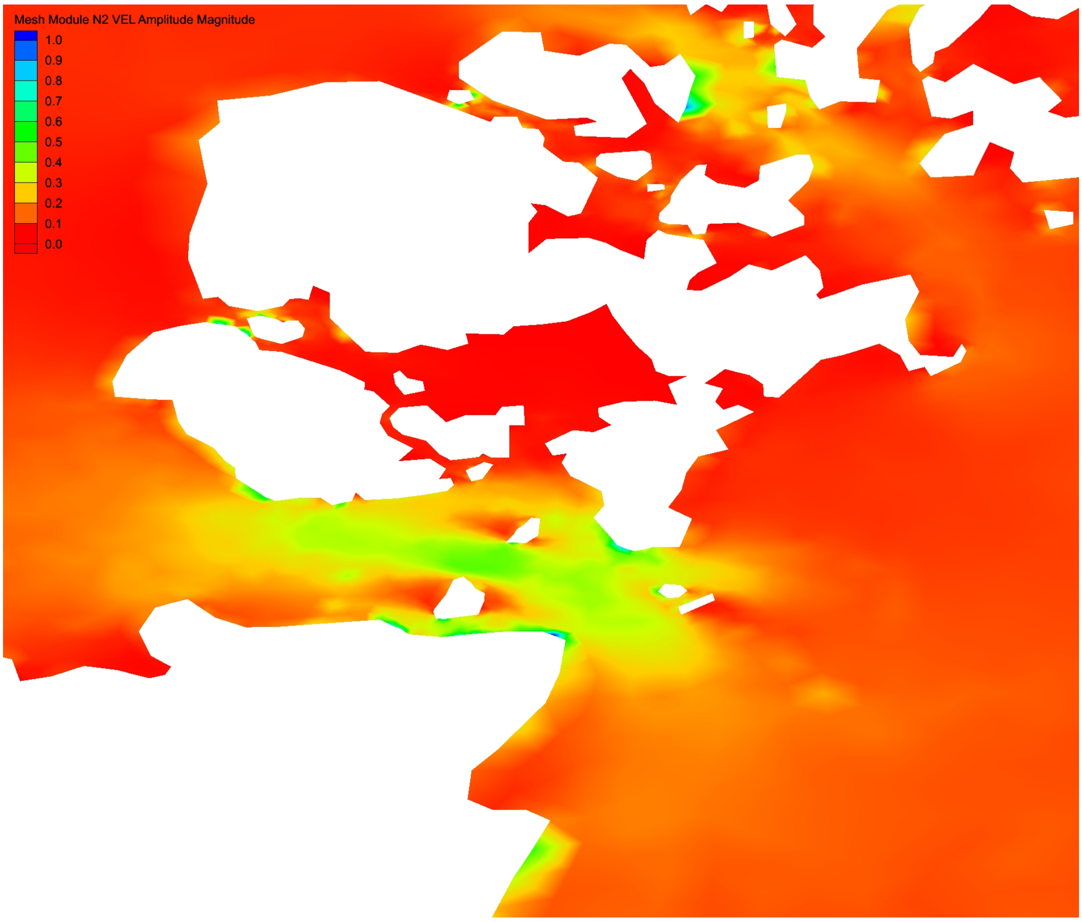

Figure 17 Predicted N2 tidal current amplitude

Validation — water levels

The model will be compared to amplitude measurements taken from the Admiralty Total Tide software. These are derived from measured data that have undergone harmonic analysis. In this report we have sufficient data coverage from the "primary ports" — ones where the predicted levels are based on in situ measurements. Data from "secondary ports" are also available, but these levels have only been inferred from other locations and are unreliable for validation of tidal models.

Within the Pentland Firth there are three "primary ports" (Scrabster, Gills Bay and Bur Wick). These are shown in Figure 18.

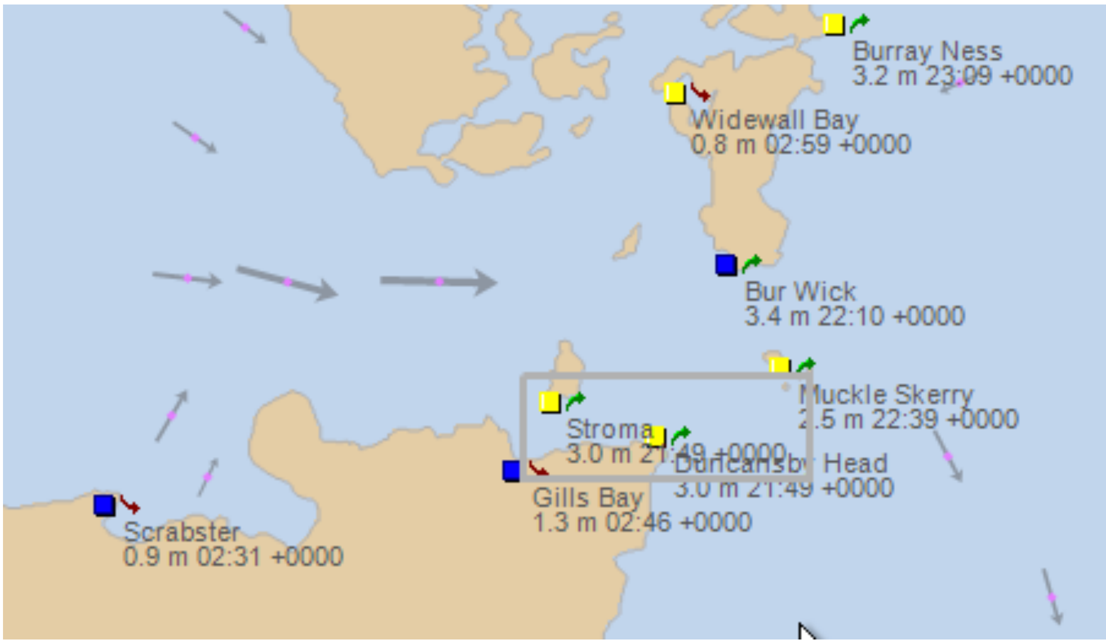

Figure 18 Data available from the Admiralty for the Pentland Firth. Primary ports are shown with blue dots. Secondary ports are shown with yellow squares. Tidal stream data are shown by arrows.

As the TotalTide data have been processed to extract tidal harmonics, comparison may be made for any time period. In the subsequent figures we start the simulations on 31 December 2012. There is a short period of "spin up" as the tidal wave propagates from the boundary of the domain to the sites being studied, and this transient effect is removed from the comparisons presented in this report. Thus the comparisons start 12 hours after the analysis start time. The time on the graphs is in hours after 00:00 31 December 2012.

A comparison between the model and the processed measurements is given in Figure 19, Figure 20 and Figure 21. The agreement is generally excellent. All sets of amplitudes and phases are very close.

A quantitative comparison can be found by harmonic analysis of the signals. The field data are processed using the method suggested by Pawlowicz et al. (2002) with some minor modifications. Numerical results are analysed using ADCIRC's input harmonic analysis tool. Table 2 provides a comparison between the results.

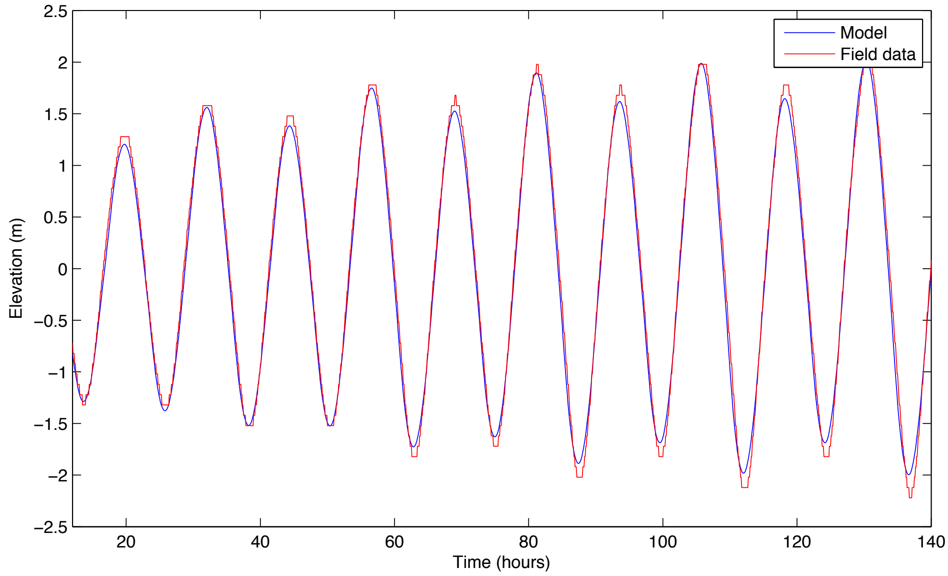

Figure 19 Comparison of water level at Scrabster

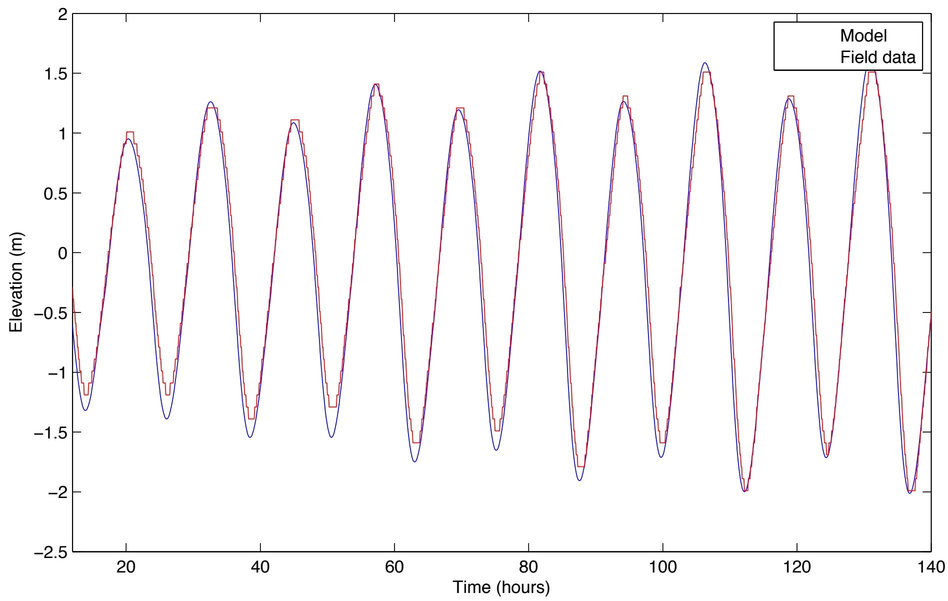

Figure 20 Comparison of water level at Gills Bay

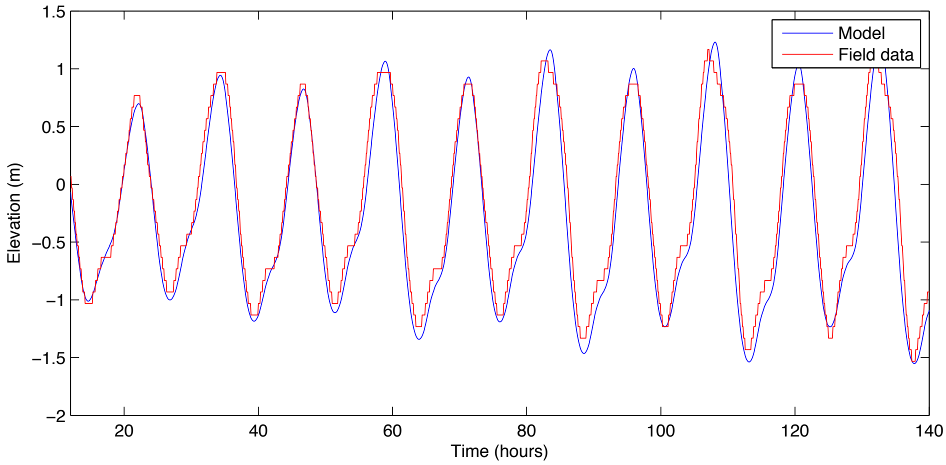

Figure 21 Comparison of water level at Bur Wick

Component	M2		S2		N2	
	Field data	Model	Field data	Model	Field data	Model
Scrabster	1.37	1.32	0.64	0.53	0.28	0.21
Gills Bay	1.12	1.18	0.41	0.41	0.19	0.15
Bur Wick	0.87	0.87	0.35	0.32	0.12	0.10

Table 3 Comparison of water level tidal harmonics. Levels are in m.

There is generally good agreement between the water levels. The error is smallest for the M2 tides. The model was tuned for the M2 tides and, as pointed out by le Provost (1991), this tends to lead to a small under-prediction in the other tidal components. We choose to accept these results because the M2 tide will be by far the most important in terms of energy extraction. The power dissipated by a tide with multiple harmonics relative to one with only the single dominant harmonic is given by

$$\frac{P}{P_{ref}} = 1 + \frac{9}{16}(r_1^2 + r_2^2 + \cdots) + \cdots$$

where r_n is the ratio of the smaller harmonics to the dominant harmonic. Thus, for resource assessment, improving the accuracy of the dominant harmonic at the cost of other components is justified. Considering the current measurements in Table 4 this implies that for the Pentland Firth very approximately 90% of the power will be extracted from the M2 tide.

Validation — currents

Introduction

It is vital to validate tidal models against currents as well as against amplitudes. There is no unique set of currents which satisfy a given set of water levels and so it is possible to predict correct water levels with incorrect currents or *vice versa*. In particular, for a given difference in elevation along a channel such as the Pentland Firth, the current will depend strongly on the bed friction coefficient.

Part of the difficulty in validating currents is that the data available are limited. Current measurements are typically of short duration and only for one location in the water column. As discussed in WG3 WP6 D3 this is particularly true for the Pentland Firth where data coverage is extremely poor.

It is often difficult to match amplitudes of tidal currents as the model produces depth-averaged values, whilst measurements are typically only available for one elevation in the water column. Where measurements are available near the top of the water column amplitude comparisons are possible — standard metocean practice is to assume a $1/7^{\text{th}}$ power law profile[2]. When the measurements are close to the sea-floor comparison of amplitudes becomes almost impossible as the measured data are highly sensitive to the exact nature of the boundary layer, and extrapolating such data to a depth-averaged value is not robust.

To make sure the model is correctly tuned it is also important to consider the phase of the current. Consider the 1d equation for the flow in a channel that is given by

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{d\zeta}{dx} + c_d\frac{u|u|}{h} = 0,$$

where u is the velocity ζ the free surface elevation, *x,* the distance along the channel, c_d the friction coefficient and h the water depth. The third term equates

[2] Unless water depths are greater than 100 m — i.e. the effective boundary layer is thinner than the water depth.

to the difference in amplitude across the channel. Let us assume that the driving amplitude is sinusoidal equivalent to an M2 tide. Given this forcing, the first term will give velocity components at the M2 frequency with a 90 degree phase shift. The second term is dependent on the geometry of the channel and will give terms which occur at double the frequency of u, thus creating M4 components. The fourth term can be expanded using Fourier series (le Provost, 1991) which gives only odd order harmonics. Thus this will produce components at M2, M6, etc. Thus the phase of the M2 current is dependent on the first and last terms and so is dependent on the friction coefficient used. Thus the comparison of phase is an important test of whether the friction in the channel is correct.

It might also be thought useful to consider higher order components as a test of both friction and channel geometry (in a similar way to the approach of Adcock & Taylor (2009) where linear information is deduced from the non-linear harmonics). Unfortunately, investigating this novel approach using the Pentland Firth model has not proved successful. This is probably due to the dependence of the M2 term on friction and the 1D model being an over-simplification for the prediction of the non-linear terms.

Admiralty data

Only one measurement is available within the Pentland Firth itself. The location is shown in Figure 22.

Figure 22 Location of Admiralty TotalTide measurement point in Pentland Firth

This measurement is assumed to be a surface velocity measurement. This assumption is based on passed experience of using TotalTide and because the database is primarily for shipping who need surface measurements. Thus the record depth-average velocity has been calculated from $U_{mean} = U_{surface} \int_0^h \left(\frac{z}{h}\right)^{1/7} \mathrm{d}z/h = \frac{7}{8} U_{surface}$ to give an approximation for the depth-averaged velocity. A comparison between these data and the model is shown in Figure 23 and in more detail in Figure 24.

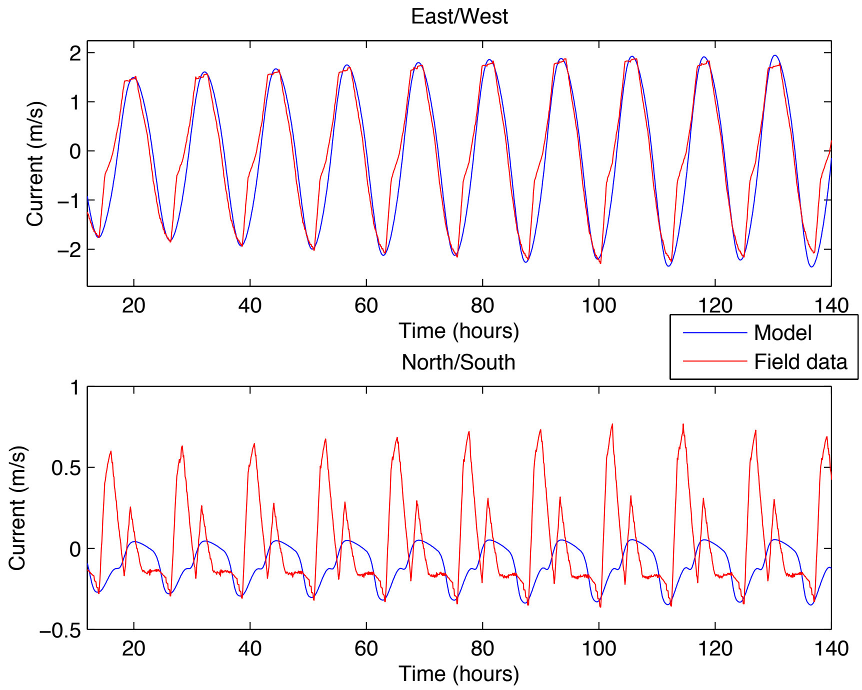

Figure 23 Comparison of current measurements in Pentland Firth

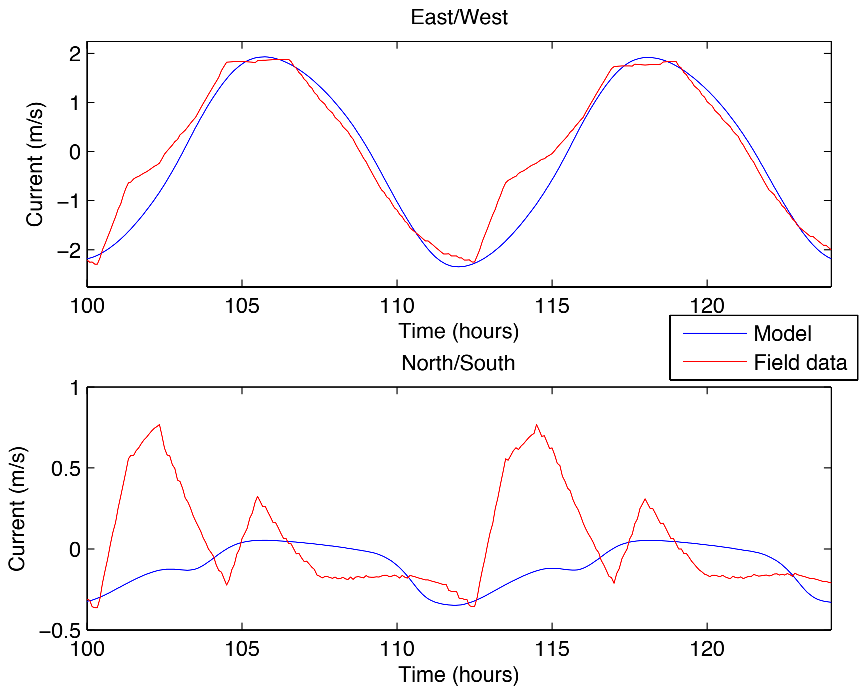

Figure 24 Comparison of current measurements in Pentland Firth

There is clearly good agreement in the east/west direction. Both amplitude and phase are close. In the cross-channel, north/south direction there is a discrepancy, although as a fraction of the total current this is small. This can also be seen in Figure 25 where the magnitude and direction of the currents are shown. The field data suggest there is a flow towards the north a few hours after the ebb tide, rather than towards the east as the model is predicting. It is not obvious what physical mechanism could be driving such a current. There is, of course, the possibility that the measurement is in error rather than the analysis.

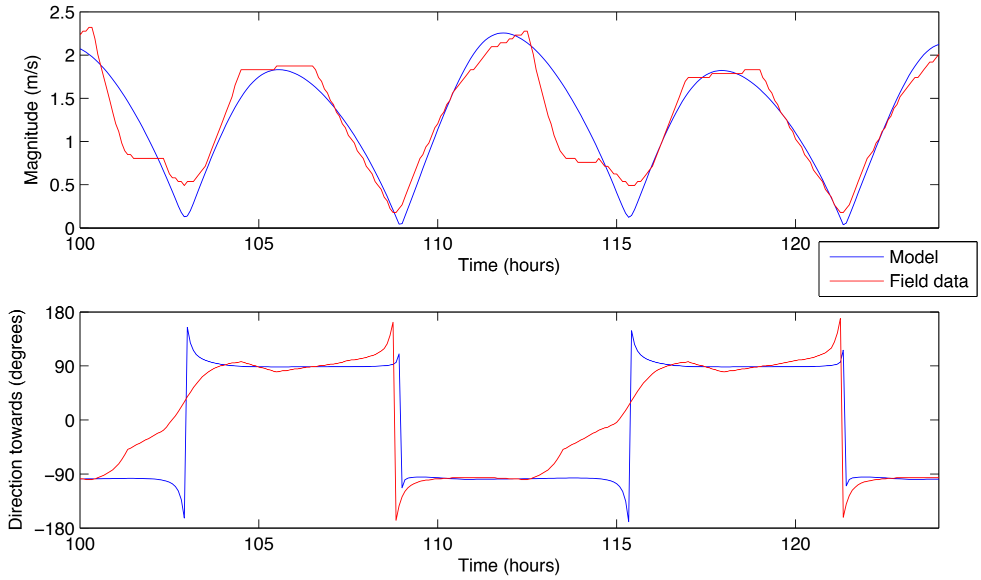

Figure 25 Comparison of magnitude (top) and direction (lower) of currents between model and measured data

A comparison of the main tidal harmonics is given in Table 4.

	M2	S2	N2
Field data	1.55	0.597	0.371
Model	1.62	0.63	0.29

Table 4 Comparison of tidal current harmonic amplitudes. Values in m/s

Admiralty tidal atlas

The admiralty publishes a tidal atlas (Admiralty, 1986) which contains maps of tidal currents. An example of such a map is given in Figure 26. The source of these data is unknown and it should therefore be treated with caution. It does, however, allow some comparison to be made for the flow through the three main channels which are candidate sites for tidal turbines. The three channels are the “Inner Sound” between mainland Scotland and Stroma, the “Outer Sound” between Stroma and Swona, and the “North Channel” between Swona and South Ronaldsay. This comparison is shown in Table 5.

Component	Flood, Spring		Flood, Neap		Ebb, Spring		Ebb, Neap	
	Atlas	Model	Atlas	Model	Atlas	Model	Atlas	Model
Inner Sound	2.2	2.8	0.9	0.9	2.2	2.2	0.9	0.9
Main channel	4	3.9	1.8	1.9	3.6	3.3	1.6	1.9
North Channel	3.6	2.8	1.6	1.0	2.7	2.4	1.1	0.9

Table 5 Comparison of tidal stream atlas with model. Values in m.

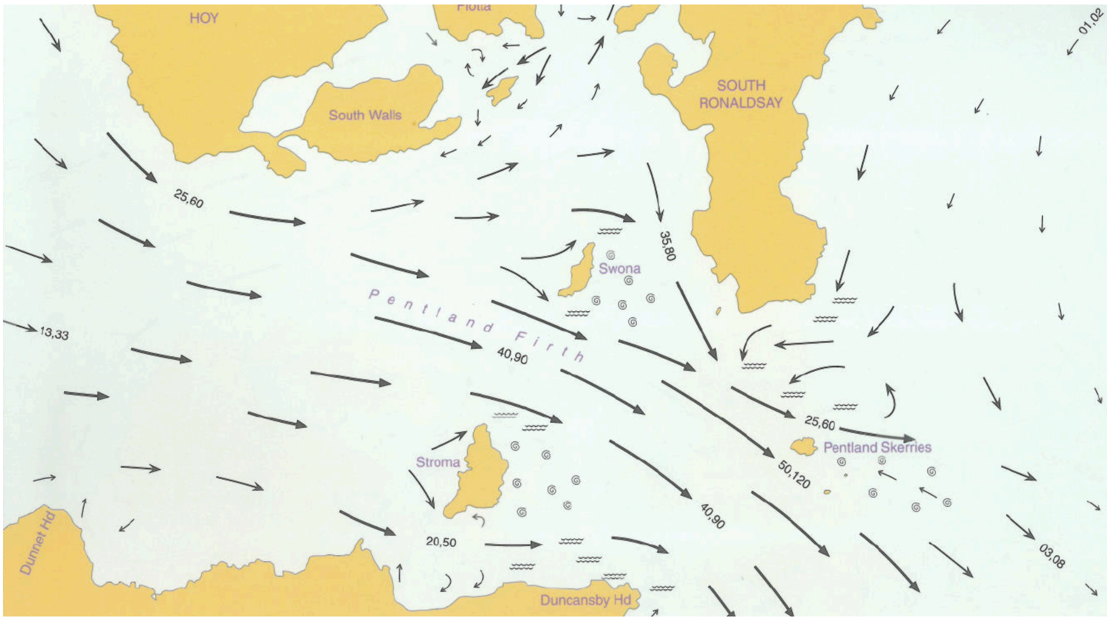

Figure 26 Example of figure from Admiralty Tidal Stream Atlas. Numbers are the current in knots×10 at neap,spring tide

It would be wrong to read too much into this comparison, given the limitations of the available data. However, generally agreement is satisfactory except for the northern channel between Swona and South Ronaldsay where the current appears to be under-predicted. The reasons for this are not clear, although it is evident from Figure 26 that the dynamics of this channel are complex, as eddies form in this channel off both Swona and South Ronaldsay. It would be possible to reduce locally the model's shear stress and eddy viscosity in this region, which would result in higher velocities through this channel. However, given the limited information we have chosen to keep the model as simple as possible. It may be that the details of this channel simply cannot be captured in a depth-averaged model.

Direct field measurements

As set out in WG3 WP6 D3, field measurements are available from a number of sites for the British Oceanographic Data Centre. These are shown in Figure 27. Unfortunately, these measurements were taken just 3 m above the sea-bed making it impossible to extrapolate to obtain a depth-averaged velocity. We therefore plot the measurements (after filtering to remove non-tidal components) on different axes to the model data. For the location $58^{0}43.67'$ $3^{0}28.31'$ comparison is shown in Figure 28 and Figure 29. This is a location in the middle of the channel.

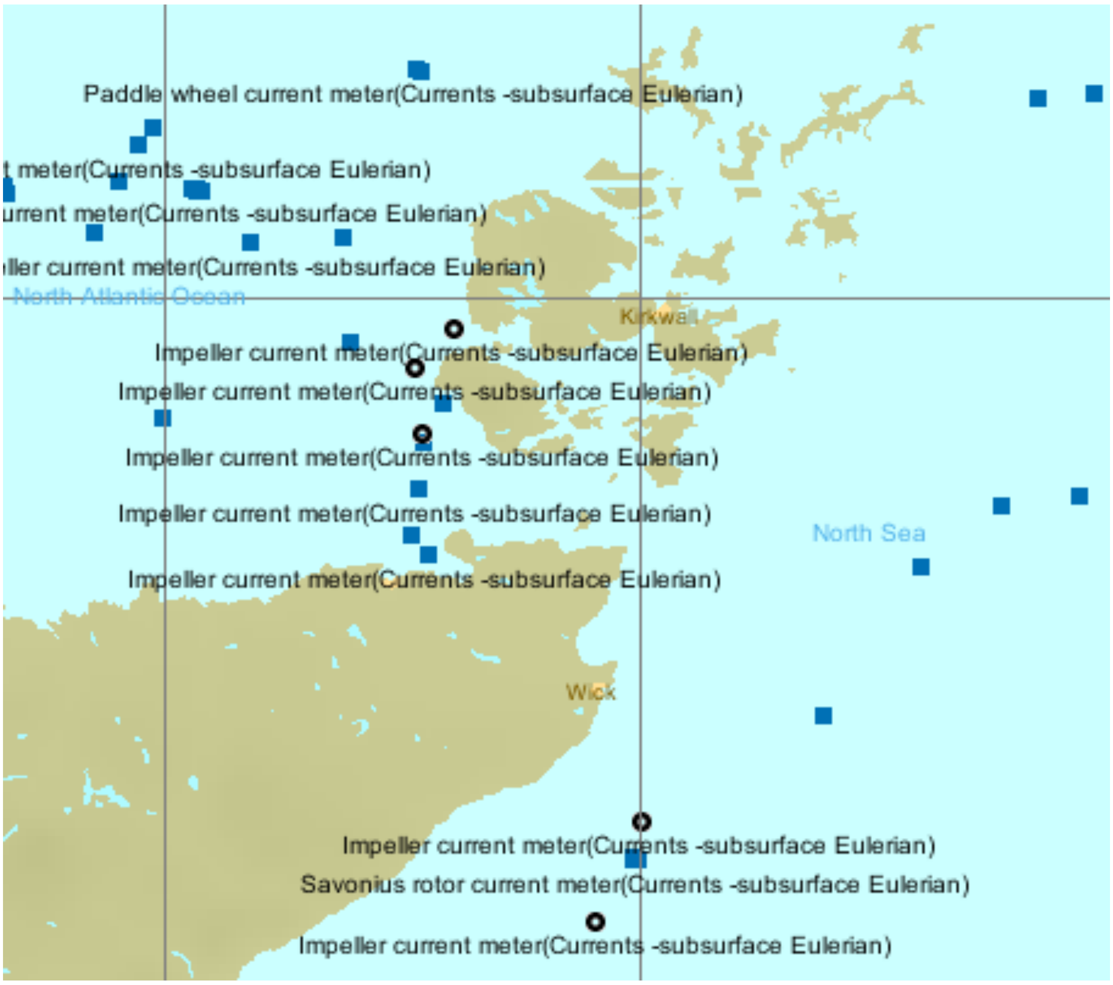

Figure 27 Current measurements for the Pentland Firth available from BODC

The measurements and the model show good agreement in terms of phase in both directions. In the east/west direction the measured current is rather smaller than the predicted current by the depth averaged model — this is to be expected as the measured data were sampled close to the seabed. In the north/south direction the current is larger in the measured data than in the model predictions. This is surprising and no explanation is offered. However, the amplitudes in the north/south direction are much smaller than in the east/west direction.

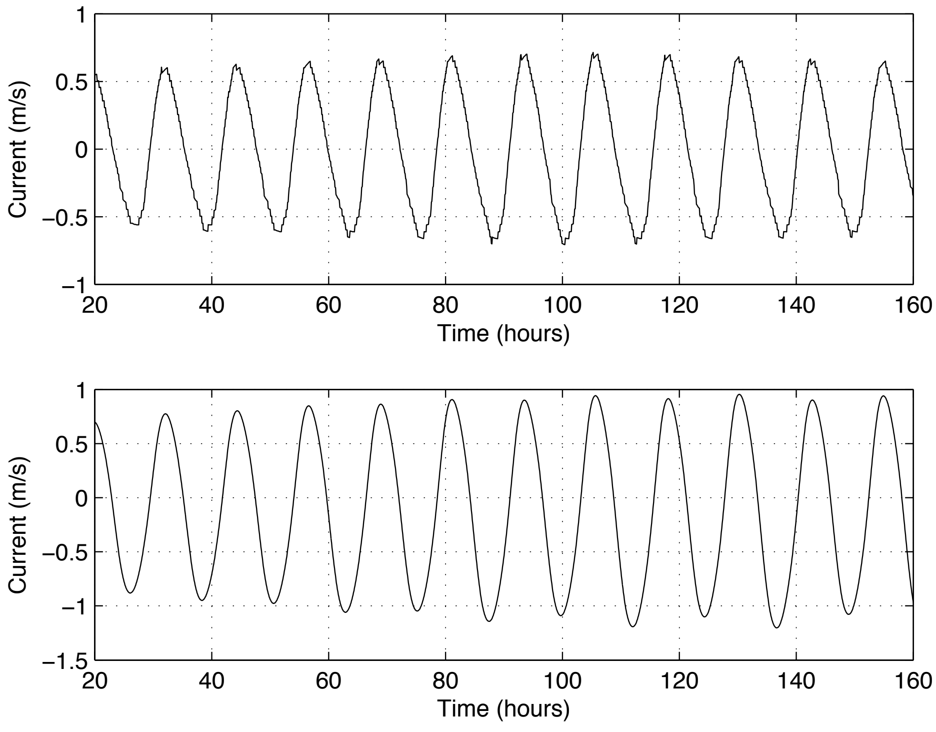

Figure 28 Comparison between filtered measurement (top) and model predictions (below) of current to west of Pentland Firth and model (in East/West direction).

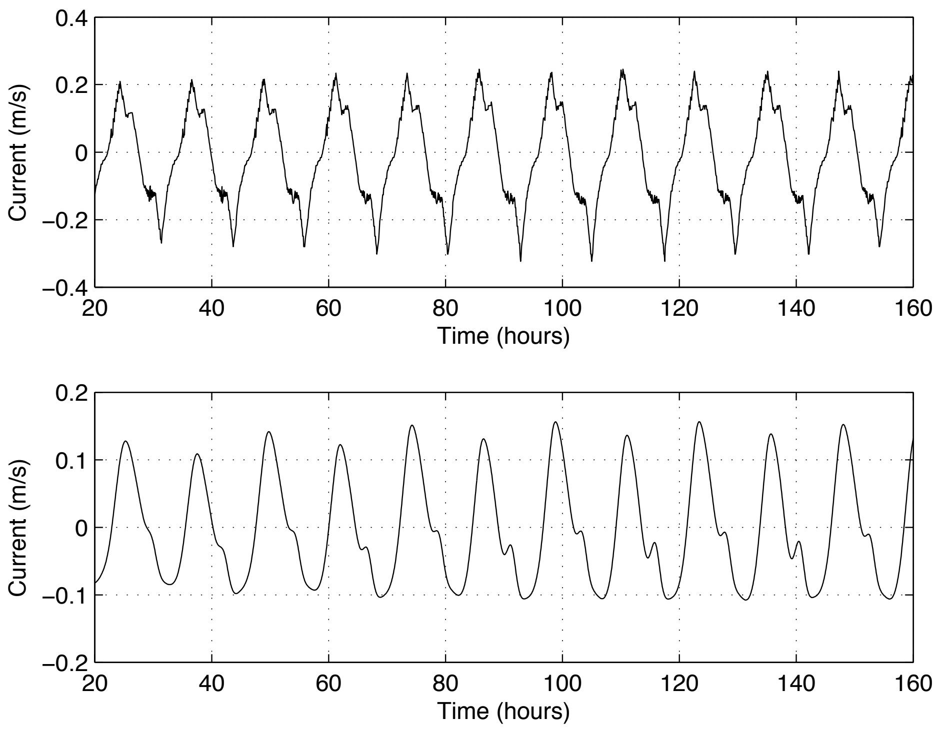

Figure 29 Comparison between filtered measurement (top) and model predictions (below) of current to west of Pentland Firth and model (in North/South direction).

Conclusions

A numerical model of the Pentland Firth has been developed. The model shows good agreement with field measurements in the east/west direction which dominates the flow in the channel and is the most important for calculating the resource of the channel. There are some minor unexplained anomalies between field data and the model in the north/south direction but these will only have a small effect on the estimated tidal resource. The fact that the model simultaneously predicts values of both elevations and velocities that agree well with field observations indicates that the bed friction coefficient (the most important tunable parameter) has been selected at an appropriate value. The next step is to include tidal turbines in the model.

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Appendix 1

Bathymetry data

The Bathymetry data used was derived from Seazone (http://www.seazone.com). These come in various formats. To us these were supplied in ascii format. A MATLAB script for converting these to x, y, z format ready for importing to the meshing software is included with this deliverable. The areas needed for this simulation are

Survey gridded bathymetry:

NW55850030, NW55850035, NW55850040, NW55900030, NW55900035, NW55900040

Chartered gridded bathymetry

NW25600020, NW25600040, NW25600060, NW25600080, NW25600100, NW25800020, NW25800040, NW25800060, NW25800080, NW25800100, NW26000020, NW26000040, NW26000060, NW26000080, NW26000100