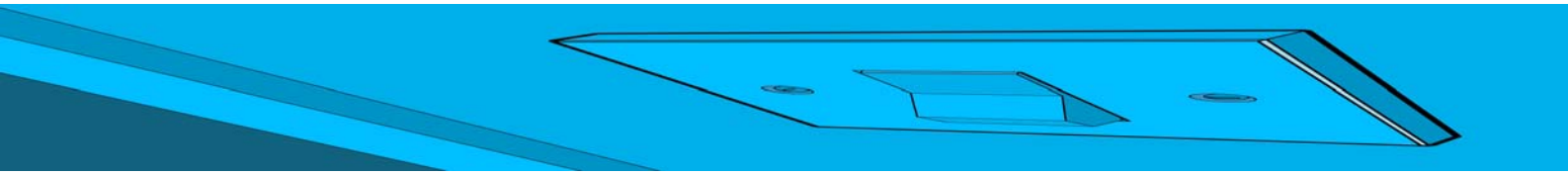

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UKERC Review of Evidence for the Rebound Effect

Technical Report 3: Elasticity of substitution studies

Working Paper

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Preface

This report has been produced by the UK Energy Research Centre's Technology and Policy Assessment (TPA) function.

The TPA was set up to address key controversies in the energy field through comprehensive assessments of the current state of knowledge. It aims to provide authoritative reports that set high standards for rigour and transparency, while explaining results in a way that is both accessible to non-technical readers and useful to policymakers.

This report forms part of the TPA's assessment of evidence for a rebound effect from improved energy efficiency. The subject of this assessment was chosen after extensive consultation with energy sector stakeholders and upon the recommendation of the TPA Advisory Group, which is comprised of independent experts from government, academia and the private sector. The assessment addresses the following question:

What is the evidence that improvements in energy efficiency will lead to economy-wide reductions in energy consumption?

The results of the project are summarised in a Main Report, supported by five in-depth Technical Reports, as follows:

Evidence from evaluation studies
Evidence from econometric studies
Evidence from elasticity of substitution studies
Evidence from CGE modeling studies
Evidence from energy, productivity and economic growth studies

A shorter Supplementary Note provides a graphical analysis of rebound effects. All these reports are available to download from the UKERC website at: www.ukerc.ac.uk/

The assessment was led by the Sussex Energy Group (SEG) at the University of Sussex, with contributions from the Surrey Energy Economics Centre (SEEC) at the University of Surrey, the Department of Economics at the University of Strathclyde and Imperial College. The assessment was overseen by a panel of experts and is extremely wide ranging, reviewing more than 500 studies and reports from around the world.

Technical Report 3 focuses upon empirical estimates of the elasticity of substitution between energy and capital. This parameter has been identified as a key determinant of the likely magnitude of the rebound effect in different sectors. The report clarifies the meaning and importance of this parameter, summarises and compares empirical estimates of this parameter, evaluates the reasons that have been proposed for the differing results, discusses whether a consensus has been reached to whether energy and capital can be considered as 'substitutes' or 'complements' and draws some implications for the rebound effect.

Executive Summary

Introduction

Statements regarding the magnitude of the elasticity of substitution between energy and other inputs appear regularly in the rebound literature. For example, Saunders (2000b) states that:

> "It appears that the ease with which fuel can substitute for other factors of production (such as capital and labour) has a strong influence on how much rebound will be experienced. Apparently, the greater this ease of substitution, the greater will be the rebound" (Saunders, 2000, p. 443).

The elasticity of substitution between energy and other inputs is also a crucial variable for Computable General Equilibrium (CGE) models of the macro-economy. The assumptions made for this variable can have a major influence on model results in general and estimates of the rebound effect in particular.

These observations suggest that a closer examination of the nature, determinants and typical values of elasticities of substitution between energy and other inputs could provide some useful insights into the likely magnitude of rebound effects in different sectors. This was the motivation for this report, which includes an in-depth examination of empirical estimates of the elasticity of substitution between energy and capital. However, the empirical literature on this subject is confusing and contradictory and more than three decades of empirical research has failed to reach a consensus on whether energy and capital can be described as either 'substitutes' or 'complements'. Moreover, the relationship between this literature and the rebound effect is more complex than it first appears.

Defining and measuring elasticities of substitution

There are at least four definitions of the elasticity of substitution in common use and several others that appear less frequently. The lack of consistency in the use of these definitions and the lack of clarity in the relationship between them, combine to make the empirical literature both confusing and contradictory.

For all definitions, substitution between two inputs is 'easier' when the magnitude of the elasticity of substitution between them is greater, while the sign of the elasticity of substitution is commonly used to classify inputs as substitutes or complements. But the appropriate classification depends upon the particular definition being used (i.e. factors may be substitutes under one measure and complements under another).

The majority of existing empirical studies use the sign of the Allen-Uzawa elasticity of substitution (AES) to make this classification. However, it can be argued that this measure has a number of drawbacks and therefore its quantitative value may lack meaning. In many cases, the Cross Price Elasticity (CPE) or Morishima Elasticity of Substitution (MES) might be more appropriate, but these have yet to gain widespread use. Furthermore, the sign of the MES is less useful as a means of classifying substitutes and complements, since in nearly all cases the MES is positive. However (unlike the AES) both the CPE and MES are arguably more representative of actual economic behaviour since they are asymmetric.

A large number of empirical studies estimate elasticities of substitution between different factors (or groups of factors) within different sectors and countries and over different time periods. These rely upon a variety of assumptions, including in particular the specific form of the production or cost function employed (e.g. CES or Translog). The estimated values may be expected to depend in part upon the assumptions made.

Standard methodological approaches rely heavily upon assumptions about the separability of different inputs or groups of inputs. These assumptions are not always tested, and even if they are found to hold, the associated estimates of the elasticity of substitution between two inputs in the same group could still be biased. Assumptions about the nature and bias of technical change may also have a substantial impact on the empirical results, but distinguishing between price-induced technical change and price-induced factor substitution is empirically challenging. The level of aggregation of the study is also important, since a sector may still exhibit factor substitution in the aggregate due to changes in product mix, even if the mix of factors required to produce a particular product is relatively fixed. This suggests that the scope for substitution may be greater at higher levels of aggregation. However, individual factors cannot always be considered as independent, notably because energy is required for the provision of labour and capital. This suggests that the scope for substitution may appear to be smaller at higher levels of aggregation.

In general, the actual scope for substitution may be expected to vary widely between different sectors, different levels of aggregation and different periods of time, while the estimated scope for substitution may depend very much upon the particular methodology and assumptions used.

Empirical estimates of the elasticity of substitution between energy and capital

From an engineering perspective, energy and capital appear to be substitutes, since a decrease in use of the first may be compensated (at least to some extent, within a particular system boundary) by an increase in use of the second. Investment in improved energy efficiency can be understood as the substitution of capital for energy. But this neglects the associated adjustments of labour and material inputs within the production process as a whole. When the full pattern of adjustments is taken into account, energy and capital may well turn out to be complements under the definition given above. This is what Berndt and Wood (1975) found in their pioneering study of factor substitution in US manufacturing and, given its potential economic importance, has since stimulated a great deal of empirical research.

A comprehensive review has been undertaken of empirical estimates of the elasticity of substitution between aggregate energy and aggregate capital. The results were analysed to see how estimates varied with factors such as the sectors covered, the functional form employed, the use of static versus dynamic estimation and the assumptions regarding separability and technical change. The most striking result from the analysis is the lack of consensus that has been achieved to date, despite three decades of empirical work. While this may be expected if the degree of substitutability depends upon the sector, level of aggregation and time period analysed, it is notable that several studies reach different conclusions for the same sector and time period, or for the same sector in different countries.

If a general conclusion can be drawn, it is that energy and capital typically appear to be either complements (AES<0) or weak substitutes (0<AES<0.5). However, very little confidence can be placed in this conclusion, given the diversity of the results and their apparent dependence upon the particular specification and assumptions used. While there appears to be some agreement on the possible causes of the different results, there is no real consensus on either the relative importance of different causes or the likely direction of influence of each individual cause (i.e. whether a particular specification/assumption is likely to make the estimate of the substitution elasticity bigger or smaller). Moreover, there is no clear agreement on the 'best' way to proceed in estimating such relationships. Given both the range of possible influencing factors that have been identified and the apparent sensitivity of the results to these factors, the empirical literature on this topic must be considered far from robust.

Arguably, a key weakness of many of the existing studies is that specific restrictions (such as, Hicks-neutral technical change) are assumed rather than statistically tested. This suggests that any future work should ensure that such assumptions are tested for and only accepted on empirical grounds. A full meta-analysis of existing studies would also be beneficial to better ascertain the effect of the different factors on the results.

Relevance to the rebound effect

There are considerable differences between the assumptions used by empirical studies of elasticities of substitution and those employed within CGE models. The same applies to the assumptions used in theoretical investigations of the rebound effect by Saunders and others. As a result, the empirical literature may be of relatively little value in either parameterising CGE models or in providing guidance on the likely magnitude of rebound effects.

With regard to the requirements of CGE models, most empirical studies differ with regard to the assumed functional form, the assumptions regarding separability, the associated nesting of production factors, the definitions of elasticity of substitution, the aggregation of individual factor inputs and the aggregation of individual sectors. Combined with the fact that the process of compiling CGE parameter values is rarely transparent and sensitivity tests are uncommon, this suggests that the results of such models should be treated with great caution - quite apart from the range of other theoretical and practical difficulties associated with the CGE approach (see Technical Report 4).

The relationship between empirical estimates of elasticities of substitution and the magnitude of rebound effects is also more complex than is generally assumed. Saunders' (2000) statement that "…the ease with which fuel can substitute for other factors of production (such as capital and labour) has a strong influence on how much rebound will be experienced" is potentially misleading. A better statement would, first, refer to 'energy services' (or 'effective energy') rather than fuel; second, clarify that the elasticity in question is the AES between energy services and a composite of other inputs; third, include the qualification that this only applies when energy services can be considered to be separable from this composite; and fourth, clarify that this conclusion derives from a particular nesting structure in a CES production function. Since the majority of empirical studies use Translog cost functions, measure energy rather than energy services, do not impose any separability restrictions and estimate the AES between energy and individual inputs, they do not provide a direct test of Saunders proposition.

In more recent work with a Translog cost function, Saunders (2006b) has shown that the magnitude of the elasticities of substitution between each pair of inputs may play an important role in determining the magnitude of any rebound effects. But not only does this describe a more complex situation than suggested by the above quote, it also suggests that a finding that energy is a weak AES substitute for another factor, or even a complement to that factor, is not necessarily inconsistent with the potential for large rebound effects from certain types of energy efficiency improvements. This is arguably consistent with Berndt and Wood's (1979) explanation of how energy and capital may be AES complements rather than substitutes. Although not previously recognised as such, this explanation effectively describes how an energy efficiency improvement stimulated by an investment credit may lead to backfire. However, Berndt and Wood's explanation of the origins of E-K complementarity is not universally accepted.

These conclusions suggest that our survey of empirical estimates of the elasticity of substitution between energy and capital may have provided relatively little insight into the likely magnitude of rebound effects. It also suggests that the discussion about elasticities of substitution in the literature may have obscured the real issue, which is the own-price elasticity of energy services in different contexts. While estimates of elasticities of substitution are relevant to this, the relationship is not straightforward. Also, the discussion regarding substitution elasticities may have obscured the important point that rebound effects are also determined by the price elasticity of output in the sector in which the energy efficiency improvement is achieved.

However, the results do arguably reinforce one of the main conclusions of Technical Report 5 - namely that the scope for substituting capital for energy may be less than is commonly assumed. This arguably suggests the possibility of a strong link between energy consumption and economic output and potentially high costs associated with reducing energy consumption. At the same time, limited scope for substitution is not necessarily incompatible with the potential for large rebound effects from certain types of energy efficiency improvements. However, such conclusions must be treated with great caution, given the numerous limitations of the evidence described above.

Contents

1 Introduction

Many types of energy efficiency improvement may be understood as the 'substitution' of capital for energy inputs. For example, insulation materials (capital) may be substituted for fuel (energy) to maintain the internal temperature of a building at a particular level. Simple engineering intuition suggests that an increase in energy prices will induce the substitution of capital for energy, since the latter has become relatively more expensive. But changes in relative prices can induce changes in the mix of all input types, with consequences that differ between sectors and between the short and long term. As a result, the relationship between energy and capital inputs may not always be a simple as engineering intuition suggests. Indeed, a large number of empirical studies suggest that energy and capital are frequently complements', which implies that an increase in energy prices will reduce the demand for capital as well as for energy. Such a result would suggest that energy and capital are closely linked in economic production and that one cannot be easily substituted for another.

Within neoclassical production theory, the scope for substitution between two inputs (i,j), or two groups of inputs, is determined by the 'elasticity of substitution' (EoS_{ij}) between those inputs. High values of the elasticity of substitution between energy and other inputs mean that a particular sector or economy is more 'flexible' and may therefore adapt relatively easily to changes in energy prices. In contrast, low values of the elasticity of substitution between energy and other inputs suggest that a particular sector or economy is 'inflexible' and that increases in energy prices may have a disproportionate impact on productivity and growth.

Statements regarding the magnitude of the elasticity of substitution between energy and a composite of other inputs ($EoS_{E,N}$) also appear regularly in the rebound literature. For example, Saunders (2000b) states that:

> "It appears that the ease with which fuel can substitute for other factors of production (such as capital and labour) has a strong influence on how much rebound will be experienced. Apparently, the greater this ease of substitution, the greater will be the rebound" (Saunders, 2000, p. 443).

This suggests a possible trade off in climate policy:

> "…If one believes $EoS_{E,N}$ is low, one worries less about rebound and should incline towards programmes aimed at creating new fuel efficient technologies. With low $EoS_{E,N}$ carbon taxes are less effective in achieving a given reduction in fuel use and would prove more costly to the economy. In contrast, if one believes $EoS_{E,N}$ is high, one worries more about rebound and should incline towards programmes aimed at reducing fuel use via taxes. With high $EoS_{E,N}$, carbon taxes have more of an effect at lower cost to the economy." (Saunders, 2000b)

The economy-wide impact of energy efficiency improvements cannot be adequately captured within a partial equilibrium framework, but may be usefully explored through Computable General Equilibrium (CGE) modelling of the macroeconomy. As described in Technical Report 4, the assumptions made by these models for the elasticities of substitution between energy

and other inputs can have a significant influence on the results - both in general and for rebound effects in particular. As an example, Grepperud and Rasmussen (2004) estimate rebound effects to be higher in the Norwegian primary metals sector than in the fisheries sector, owing largely to the greater opportunities for input substitution in the former (Grepperud and Rasmussen, 2004).

These observations suggest that a closer examination of the nature, determinants and typical values of elasticity of substitution between energy and other inputs could provide some useful insights into the likely magnitude of rebound effects in different circumstances. This is the motivation for the current report, which includes an in-depth examination of empirical estimates of the elasticity of substitution between energy and capital (EoS_{KE}). At first sight, this elasticity appears particular relevant to the rebound effect since many of the energy efficiency improvements relevant to modern economies appear to result from the substitution of capital for energy, rather than labour or materials. Moreover, while there is some consensus on the degree to which energy and labour may be considered substitutes, there is much less consensus on the degree of substitutability between energy and capital. Indeed, the latter has been the subject of controversy within energy economics for over three decades. This report therefore:

- provides a straightforward summary of the relevant production theory;
- clarifies the different definitions of the elasticity of substitution and the relationships between them;
- highlights the challenges associated with obtaining empirical estimates of elasticities of substitution;
- summarises the available estimates of the elasticity of substitution between energy and capital;
- summarises and evaluates the reasons that have been proposed for the widely differing results.
- identifies whether and to what extent a consensus on this subject has been reached;
- identifies weaknesses and gaps in the literature; and most importantly:
- examines the relevance of the above to rebound effects and identifies the lessons that may be learned

The report includes an extensive search of the literature on EoS_{KE} (see Annex 2), including papers that cited the seminal study by Berndt and Wood (1975). Almost all subsequent research on this topic stems from this paper, together with the follow up papers by Griffin and Gregory (1976) and Berndt and Wood (1979). The key contribution of Berndt and Wood to production economics was to argue that energy is a necessary factor of production which should be incorporated into empirically estimated production functions along with the more traditional inputs of capital and labour.[1] Put another way, Berndt and Wood showed that the traditional neglect of energy in empirical studies was likely to have led to biased and misleading results. Berndt and Wood (1975) were one of the first to measure the elasticity of substitution between capital (K), labour (L), energy (E), and materials (M) and came to the conclusion that capital and energy were complements. Arguably the insights offered by

[1] Although there is evidence of research prior to this, for instance Nerlove, (1963), who's data was subsequently used by Christensen and Greene (1976) discussing multi-factor production functions containing energy as a factor input.

Berndt and Wood (1975) resulted in an overhaul of production theory, leading many to question the standard assumptions of the theory, and to raise concern over the inconsistency of empirical results.[2]

Even a cursory examination of the voluminous literature on elasticities of substitution reveals it to be extremely confusing, with competing definitions of the appropriate measures to be used, persistent measurement difficulties, conflicting results and competing explanations for these results. This makes it difficult to provide an overview of the literature and to draw any general conclusions. Moreover, while the statements quoted above suggest that a review of the literature on elasticities of substitution should throw some light on the rebound effect; the situation turns out to be more complex than it first appears. Indeed, one of the unanticipated conclusions from this report is that: first, the above statements by Saunders are potentially misleading; second, the empirical basis for assumed parameter values in energy-economic models appears to be extremely weak; and third, empirical estimates of elasticities of substitution may in practice tell us relatively little about the size of the rebound effect.

While at first sight these appear to be rather negative conclusions, the investigation, clarification and interpretation of this subject has nevertheless proved to be a valuable exercise. In addition to highlighting the limitations of some key studies in the rebound literature, a potentially important conclusion is that the scope for substituting capital for energy may be less than is commonly assumed. A possible implication is that decoupling energy consumption from economic growth could be costly.

The report is structured as follows. Section 2 introduces some key concepts from neoclassical production theory, and provides an intuitive explanation as to why, in some circumstances, energy and capital may appear as complements rather than substitutes. Section 3 summarises the different definitions of the elasticity of substitution, clarifies the relationship between them and highlights a number of issues relevant to estimating elasticities of substitution which are often overlooked in the empirical literature. Section 4 summarises the results of a literature review of empirical estimates of the elasticity of substitution between energy and capital, classifies these results in a number of ways and evaluates the reasons that have been put forward to explain the wide variation in these results. Section 5 explores the relevance of elasticities of substitution to the rebound debate, considering in turn: the relationship between empirical estimates and the requirements of CGE models; the importance of 'separability' assumptions and 'nesting' structures; and the appropriate modelling of technical change. It concludes that the relationship between this parameter and the rebound effect is rather more complex than is commonly assumed and that an empirical finding of complementarity between energy and capital might in some circumstances be compatible with large rebound effects - the opposite of what some authors have suggested. Section 6 summarises and concludes.

[2] For example, Raj and Veall (1998).suggest that the Berndt and Wood (1975) data has produced 38 different elasticities ranging from -3.94 to 10.84.

2 Understanding substitution and complementarity

This section introduces the notions of substitution and complementarity and provides an intuitive explanation as to why, in some circumstances (and under some measures) energy and capital may appear as 'complements' rather than 'substitutes'. The discussion is preceded by a brief introduction to some relevant concepts from neoclassical production theory.

2.1 Neoclassical production theory

Production theory is the area of economics concerned with understanding the optimal proportions of factor inputs used in obtaining any level of output within an economy, i.e. making sure that the resources available to the economy are utilised in the best possible manner. The term 'economy' here may refer to a whole nation, a sector, a subsector or even an individual firm. However, in the following description, the term 'agent' will be used interchangeably for different levels of aggregation.3 This introductory discussion will largely be confined to the three factor input case, namely capital (K), labour (L) and energy (E), although it is also fairly common practice to include materials (M); the resulting production functions being identified as KLE or KLEM functions respectively. In subsequent sections, it will be argued that the omission materials may bias empirical estimates.

A standard production function can be expressed as;

$$Y_{it} = f(K_{it}, L_{it}, E_{it}) \qquad (2.1)$$

Equation (2.1) states that agent (i)'s gross output (Y) in a given time period (t), is a function of the factor inputs (K, L, E) used in the same time period. Thus, the general functional form may be expressed for any number of agents and/or time periods. If, for instance, the production function were to be expressed purely for a cross section, the subscript (t) would be dropped from equation (2.1), leaving just the subscript (i), identifying each of the individual agents. Similarly, to represent only one agent over multiple time periods, the subscript (i)'s would be dropped.

The production function represents the maximum output obtainable from a specified set of inputs given the existing technical opportunities. This abstracts from the engineering and managerial problems associated with maximising the 'technical efficiency' of a production process, so that analysis can focus on the optimal combination of factor inputs. The agent is assumed to be making optimal choices concerning how much of each input factor to use, given the price of the inputs and the existing technological constraints. This 'optimising' assumption is central to neoclassical theory and is maintained in what follows to facilitate interpretation of the relevant literature. However, it is worth noting that, although the neo-classical paradigm remains central to mainstream economic analysis, it has not gone unchallenged (e.g. Hodgson (1988)) and there have been several attempts to supersede it (e.g. (Kahneman and Tversky, 2000).

[3] The level of aggregation used in production functions, in particular the appropriateness or otherwise of their use for the whole economy, is the cause of much debate in the literature; recent discussions include Felipe and Fisher (2003) and Temple (2006).

The relationship between the level of output and the demand for any factor input is represented by the isoquants of the production function (i.e. (Y, x) curves, where x=K, L, E). These are characterised by non-negativity (i.e. there is either a positive output or no output at all), and typically by diminishing returns to scale (i.e. as scale increases, agents use factor inputs less efficiently). However, it is standard to assume that production functions are homogenous of degree one, which means that the function exhibits constant returns to scale - implying that when each input is increased by some proportion k, output increases by exactly the same proportion.[4] Homogeneous functions are a special class of homothetic functions, where the slope of an isoquant is constant for different levels of output (i.e. along rays from the origin in (Y,x) space). For non-homothetic production functions, the implication is that:

$$\left[\frac{dY}{dx_i}\middle|\{Y = y_1\}\right] \neq \left[\frac{dY}{dx_i}\middle|\{Y \neq y_1\}\right] \qquad (2.2)$$

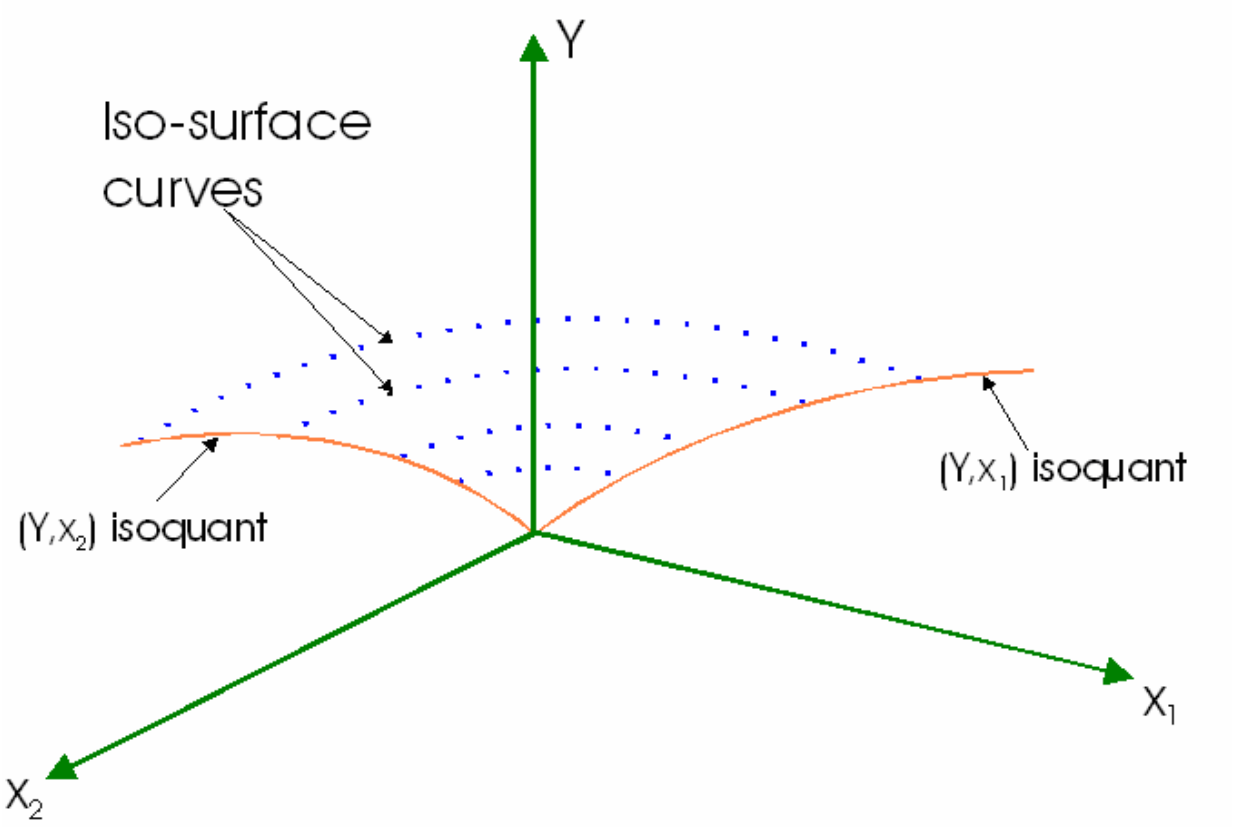
In other words, the curvature of the isoquants is not constant with respect to the scale of production. Although it is standard to assume homogeneous, or at least homothetic production functions, Spady and Friedlander (1978) have shown that this may bias the empirical estimates of various parameters – including elasticities of substitution.

The surface between any two isoquants is termed the iso-surface curve and defines the feasible interaction among factor inputs, i.e. how the demand for one factor will react when the demand for another factor increases or decreases. This iso-surface curve is normally assumed to be 'bowed outwards' between the two isoquants, indicative of gains in efficiency to be made from specialisation. Figure 2.1 provides a graphical representation of this relationship, although for illustrative purposes it considers only two factor inputs.

It is generally assumed that agents are rational and that behaviour is characterised by profit maximisation. It is further assumed that all agents are producing output at maximum efficiency with no wastage occurring in the production process. In these circumstances, movement along the iso-surface curves (between factor inputs) when the level of output is held constant, is analogous to movement along the 'production possibilities frontier' (PPF). The iso-surface curve will have as many dimensions as there are factor inputs to the production function.

[4] The production function $Y = f(X_1,X_2)$ is said to be homogeneous of degree n if, given any positive constant k, $f(kX_1,kX_2) = k^n f(X_1,X_2)$. When $n > 1$, the function exhibits increasing returns, and decreasing returns when $n < 1$. When it is homogeneous of degree 1, it exhibits constant returns (n=1).

Figure 2.1: Graphical representation of a two-input production function.



Production functions are normally represented by a pre-specified functional form, which is intended to approximate the (n-dimensional) relationship between factor inputs and observed output. That is to say, their purpose is to define a set of parameters which, given the observed data, reasonably approximate and define the n-factor iso-surface curve. This is useful in understanding the rate at which agents within an economy can feasibly (or have historically) move(d) between alternative factor input combinations, and may help understand how they may subsequently react in the future. Common functional forms which will occur regularly in what follows are the Cobb Douglas, Constant Elasticity of Substitution (CES) and Translog production functions. Each of these are briefly introduced in Annex 1.[5]

[5] See also Annex 2 of Technical Report 5

2.2 Substitution and complementarity

Factors of production are frequently described as either substitutes or complements. These terms typically, but not always, refer to the changes in the relative proportion of different input factors while output is held fixed. In different contexts two factors may be described as substitutes (complements) when:[6]

> Definition 1: the usage of one increases (decreases) when the usage of the other decreases (increases);
>
> Definition 2: the usage of one increases (decreases) when the price of the other increases (decreases);
>
> Definition 3: the usage of one relative to the other increases (decreases) when the price of the other increases (decreases);
>
> Definition 4: the usage of the first relative to the second increases (decreases) when the price of the second relative to the first increases (decreases)

The existence of different definitions of substitution and complementarity can complicate the interpretation of empirical studies. Whether two factors may be described as substitutes or complements depends upon the definition being used. It is quite possible for two factors to be described as substitutes under one definition and complements under another. Care must therefore be taken in comparing one study with another.

The first two of these definitions are the most commonly used. For example, under Definition 1, energy and capital would be described as substitutes if a decrease in energy usage was associated with an increase in capital usage, holding output constant. Alternatively, energy and capital would be described as complements if a decrease in energy usage was associated with a decrease in capital usage, holding output constant. The meaning of capital usage depends upon how capital (or capital services) is measured, but should depend upon both new investment and the estimated obsolescence of existing capital stock.

Definition 2 is particularly relevant for exploring the economic impact of changes in relative prices. For example, energy and capital would be described as substitutes under this definition if an increase in energy prices increased the demand for capital inputs, holding output constant. Alternatively, energy and capital would be described as complements if an increase in energy prices decreased the demand for capital inputs, holding output constant. If energy and capital are found to be complements under this definition, the economic impact of an increase in energy prices could be significant:

> "A reduction in the use of energy by itself will have a relatively small economic impact, determined to first order by energy's small value share. But if the reduced use of energy also produces a reduction in the use of capital, the larger value share of capital applies and the economic impact is magnified. This indirect effect through capital can be the largest component of the economic impact of reduced energy use... but this effect is often ignored in economic impact analyses of energy policy" (Hogan, 1979)

[6] This list is not exhaustive. For example, in some cases economists are interested in the effect of changes in quantities on changes in prices (e.g. the effect of immigration on relative wages). This is not simply the inverse of the effect of changes in prices on changes in quantities, and may lead to a different classification of substitutes and complements.

Hence, in the context of the oil price shocks of the 1970s, Berndt and Wood's (1975) finding of energy-capital complementarity under this definition was clearly significant. Furthermore, the feasibility of increasing economic output with rising energy prices (e.g. due to resource depletion) may depend in part upon the scope for substitution between capital and energy. But despite the substantial empirical literature on this subject, there still appears to be little consensus on whether energy and capital should be considered as substitutes or complements.

The process of improving the energy efficiency of a subsystem normally involves 'substituting' energy for capital inputs. Hence, energy and capital would normally be considered as substitutes under Definition 1. For example, insulation materials can be substituted for fuel inputs to maintain the internal temperature of a building at a particular level. Similarly, waste heat recovery equipment can be substituted for fuel inputs to maintain a given level of steam production. At first sight, therefore, the notion that energy and capital may be complements appears somewhat odd from an engineering perspective. Nevertheless, beginning with Berndt and Wood (1975) a number of econometric studies at different levels of aggregation have come to precisely that conclusion. Some clues to the puzzle may be obtained by recognising that there are number of competing definitions of substitution/complementarity that necessarily applied to multifactor production functions. In a two-factor production function, the inputs must be substitutes under Definition 1, since, if one input is reduced, output can only be maintained by increasing the other input. However, the interpretation is different in a multifactor production function, since the behaviour of one input in response to a change in the level or price of another input will also depend upon the behaviour of all the other inputs. Berndt and Wood (1979) provide a helpful account of this process which provides one possible explanation of how energy and capital can be complements under either Definition 1 or Definition 2. This is reproduced below.

2.2.1 Graphical exposition

Berndt and Wood (1979) begin with a four input production function:[7]

$$Y = F(K, L, E, M) \tag{2.3}$$

Under certain assumptions, there will be a corresponding cost function, which gives the minimum cost of producing a particular output level Y given input prices PK, PL, PE, and PM:

$$C = G(P_K, P_L, P_E, P_M, Y) \tag{2.4}$$

Berndt and Wood's exposition depends upon the notion of 'separability' between K and E on the one hand and L and E on the other. Separability is a standard (although arguably problematic) assumption in empirical work and may be defined in two ways, which are commonly taken as equivalent (Frondel and Schmidt, 2004):[8]

Primal: The optimum ratio of two factors is unaffected by the level of other inputs.
Dual: The optimum ratio of two factors is unaffected by the prices of other inputs.

[7] Here and elsewhere, production and cost functions are assumed to meet certain standard conditions such as linearly homogeneous, twice differentiable and quasi concave.

[8] A distinction is commonly made between 'weak' and 'strong' separability - for definitions, see Berndt and Christensen (1973). In this report, all references are to 'weak' separability.

If energy and capital are separable from labour and materials, a composite input of 'utilised capital' (K*) can be defined which is derived from capital and energy alone. The production and cost functions for this input are as follows:

$$K^* = f(K, E) \quad (2.5)$$

$$P_{K^*} = g^*(P_K, P_E, K^*) \quad (2.6)$$

Similarly, a separate composite input of labour/materials (L*) can be defined which is derived from labour and materials alone. The production and cost functions for this input are as follows:

$$L^* = f(L, M) \quad (2.7)$$

$$P_{L^*} = h^*(P_L, P_M, L^*) \quad (2.8)$$

Then, if separability holds, the 'master' production and cost functions for output Y can be written as a function of these composite inputs:

$$Y = F(K^*, L^*) \quad (2.9)$$

$$C = G(P_{K^*}, P_{L^*}, Y) \quad (2.10)$$

Figure 2.2 provides a graphical representation of the master production function for a competitive, cost minimising firm producing output level Y. The original input prices for 'utilised capital' (P_{K^*}) and the labour/materials composite (P_{L^*}) are represented by the iso-cost line AA'. The firm minimises the costs of producing Y* by using K*1 of utilised capital and L1* of the labour/materials composite.

Figure 2.2 Master production function – initial input prices

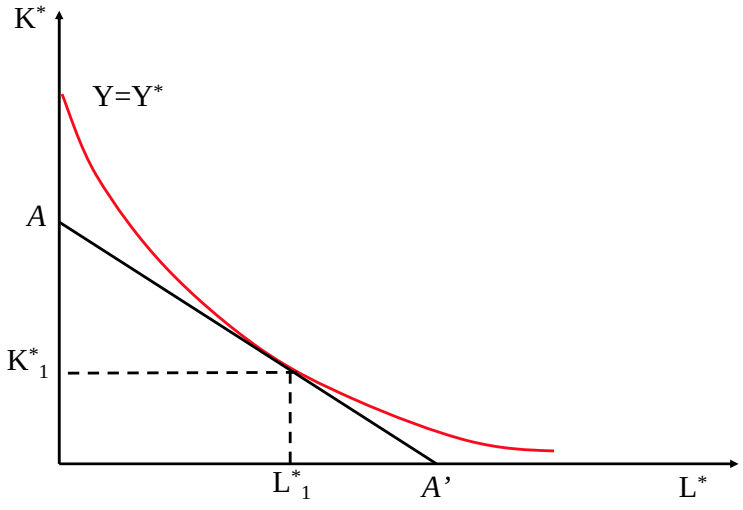
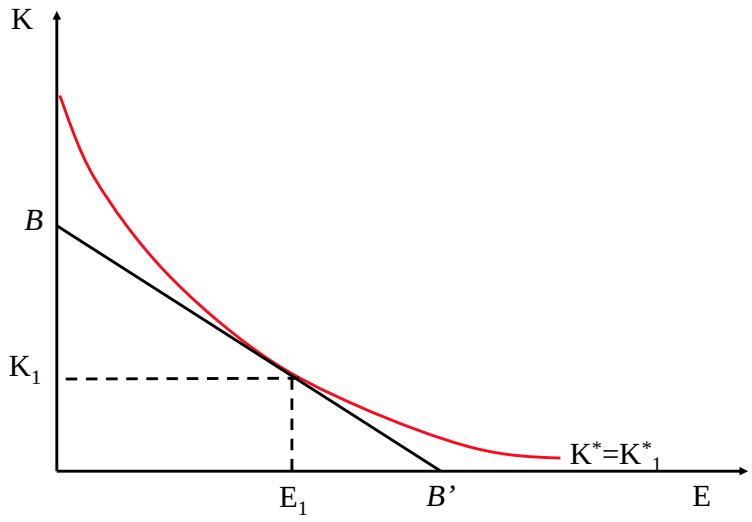


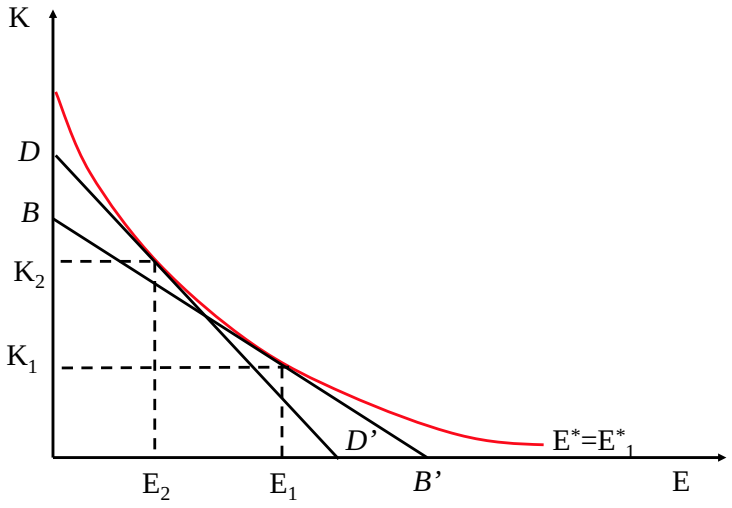
Figure 2.3 provides a graph of the production sub-function for a competitive, cost minimising firm producing utilised capital K*1 from capital and energy inputs. Given the original prices PK and PE reflected in the iso-cost line BB', the firm produces K*1 using K1 units of capital and E1 units of energy.

Figure 2.3 Production sub-function for utilised capital - initial prices



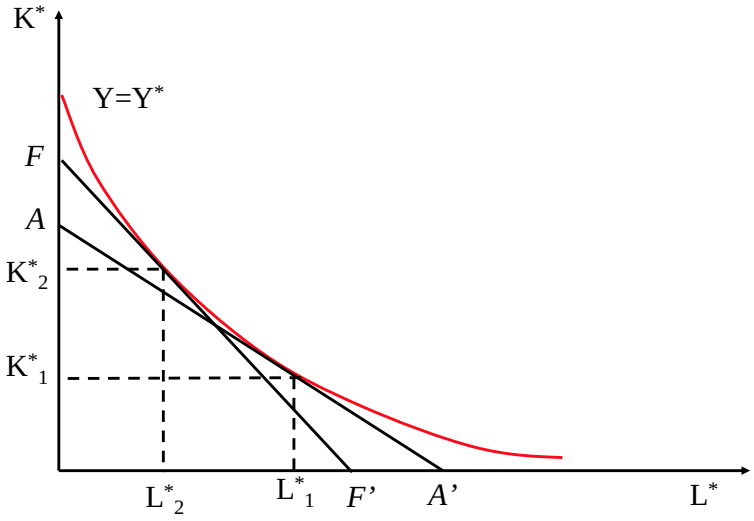
Suppose now that the cost of capital falls relative to the price of energy (say, for example, by a government subsidy to encourage investment in capital equipment). Then firstly, holding the production of utilised capital constant at K*1, the steeper iso-cost line DD' in Figure 2.4 indicates that the demand for capital would increase from K1 to K2 while the demand for energy would fall from E1 to E2. At the level of the subfunction, capital has substituted for energy inputs to produce the same level of utilised capital (K*1). Hence, at the level of the subfunction, capital and energy are substitutes (under this definition) as may be expected from simple engineering intuition.

Figure 2.4 Production sub-function for utilised capital - new prices, constant utilised capital



But since the cost of capital has fallen, so has the price of 'utilised capital' (P_{K^*}). As illustrated in Figure 2.5, this changes the slope of the iso-cost line for the master cost function from AA' to FF' and results in a new cost minimising optimum of L*2 for the labour/materials composite and K*2 for utilised capital. Hence, at the level of the master production function, the capital subsidy has led to utilised capital substituting for labour/materials. The increased demand for utilised capital will correspondingly increase the demand for energy inputs, thereby offsetting some (or all) of the original reduction in energy consumption.

Figure 2.5 Master production function – new input prices

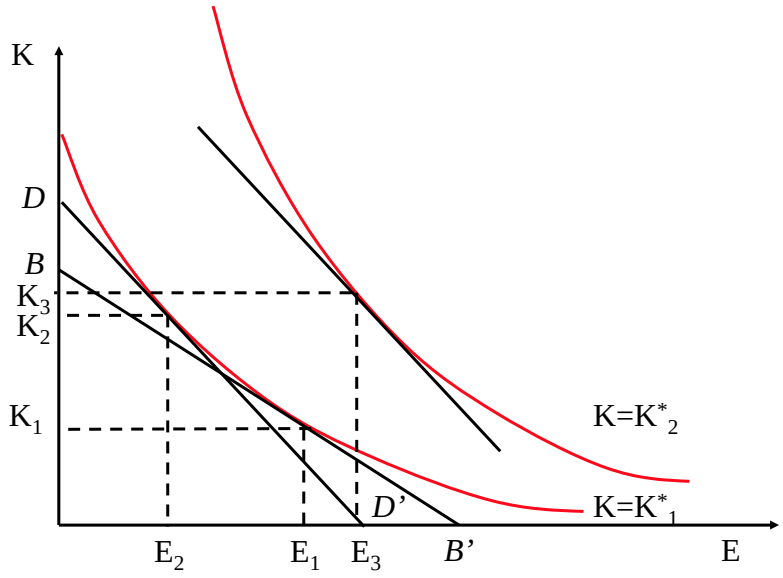


The increased demand for utilised capital (K*2) may be represented by a shift of the K* isoquant in the sub-function for utilised capital from K*1 to K*2 – as illustrated in Figure 2.6. The optimum input combinations for producing this new level of utilised capital are given by K3 and E3.

Hence, holding output constant, the net result of the subsidy on capital investment is that:

the demand for 'utilised capital' has increased from K^*_1 to K^*_2;
the demand for capital has increased from K_1 to K_3; and
the demand for energy has <u>also</u> increased from E_1 to E_3.

Figure 2.6 Production sub-function for utilised capital - new prices, new level of utilised capital



This outcome might be considered counterintuitive, given that the reduction in the price of capital has increased, not decreased the demand for energy, holding output constant. This appears inconsistent with the 'engineering' interpretation of a subsidy on capital investment leading to the substitution of capital equipment for energy purchases. This 'engineering' substitution does take place at the level of the sub-function for utilised capital and is represented by Figure 2.4. But one consequence of this E-K substitution is that the unit cost of utilised capital is reduced, leading to an increased demand for utilised capital (from K*1 to K*2) and a consequent reduction in demand for the labour/materials composite to hold output constant. In the example shown here, the degree of substitution between utilised capital and the labour/materials composite is high, so there is a substantial increase in utilised capital demand. This leads to an increase in energy demand (from E2 to E3) which is more than sufficient to offset the original reduction in energy demand (from E1 to E2) brought about by the investment in capital equipment. It should be clear that the final outcome depends upon the shape of the isoquants and in practice, it is possible for the final energy demand (E3) to be less than, equal to or greater than the original demand (E1).

Berndt and Wood (1979) termed the movement along the K-E partial isoquant (Figure 2.4) the gross substitution effect and the movement between K-E partial isoquants (Figure 2.6) the expansion effect; with the overall change being termed the net substitution effect.[9] The gross effect represents the change in demand for capital and energy holding the demand for utilised capital fixed, while the expansion effect represents the change in demand for capital and energy resulting from the increased demand for the (cheaper) utilised capital holding output fixed. This decomposition is analogous to the decomposition of changes in factor inputs into a substitution effect (holding output constant) and an output effect (resulting

[9] In their original paper, Berndt and Wood (1975) used the term 'scale effect' rather than expansion effect, but the terminology was subsequently modified to avoid confusion with 'returns to scale'.

from an increase in output) (see Supplementary Report). However, in this example, the overall output of the producer (Y) is held constant in the definition of net substitution effects, but utilised capital inputs are allowed to vary. Put another way, Berndt and Wood's expansion effect may be interpreted as the output effect for the production (sub) function for utilised capital.

The final demand for capital and energy will be the net result of the gross substitution and expansion effects – and in principle may be greater or less than the original demand. Berndt and Wood term capital and energy as net complements where the expansion effect is larger than the gross effect (demand for both increases following the reduction in the cost of capital), and capital and energy as net substitutes where the expansion effect is smaller than the gross substitution effect (demand for energy falls following an reduction in the cost of capital, while that for capital increases). As discussed below, this is consistent with the most common interpretation of these terms but is not the only way in which substitutes and complements may be defined.

While the independent variable for Berndt and Wood's example is a reduction in the cost of capital, analogous conclusions apply when the independent variable is an increase in the price of energy. In this case, the price of utilised capital will increase, so the labour/materials composite will substitute for utilised capital to keep output constant. This substitution will reinforce the reduction in energy consumption caused by the substitution of capital for energy and will offset the corresponding increase in capital use. Again, if the 'expansion' (contraction in this case) effect is larger than the gross substitution effect, capital and energy will appear as net complements and overall capital use will fall. The two effects need not be symmetric, however. For example, energy and capital may appear as net complements following a change in capital prices, but net substitutes following a change in energy prices.

Berndt and Wood's explanation of energy-capital complementarity hinges upon the assumption that utilised capital is separable from labour and material inputs. As discussed in Section 3, this is a restrictive assumption that may not be supported by empirical evidence. However, Berndt and Wood (1979) demonstrate that the assumption of seperability is not essential to reconcile a finding of gross substitutability between energy and capital with a finding of net complementarity.

It is important to note that Berndt and Wood's interpretation of the origins of energy-capital complementarity is not the only one that has been provided, nor is it accepted by all commentators (Griffin, 1981). A particular weakness is that Berndt and Wood assume a single homogeneous output, whereas in practice changes in the price of energy or capital may also lead to shifts in the product mix (Solow, 1987). The manner in which capital inputs are measured could also be relevant, together with the type of data on which the estimate is based. For example, Miller (1986) argues that many time-series studies are likely to underestimate the scope for substituting capital for energy because they only capture short-run responses such as retrofitting, while significant improvements in energy intensity generally require the replacement of long-lived capital equipment. In the short run, energy price increases would increase the price of energy intensive products, reduce demand for those products and reduce investment in the relevant industries, which may appear as an apparent complementarity between energy and capital. Energy and capital may be long-term substitutes in the relevant sector, but this would not be apparent in studies that use time-series data if these only cover relatively short periods.

These alternative explanations of energy-capital complementarity need to be given serious consideration, not least because it is difficult to think of real-world examples of the behaviour described above (i.e. a reduction in energy use being accompanied by reduction in capital use as energy is replaced by labour or material inputs). These alternative explanations are therefore examined further in Section 4.2. Nevertheless, Berndt and Wood have provided an internally consistent and illuminating account of the type of mechanisms that may be at work.

2.2.2 Gross and net price elasticities

As indicated, Berndt and Wood (1979) defined the gross price elasticity between capital and energy (CPE^{*}_{EK}) as that which is conditional upon a fixed level of utilised capital (K*):

$$CPE^{*}_{EK} = \left.\frac{\partial \ln E}{\partial \ln P_K}\right|_{K^* \text{ and } Y \text{ constant}} \tag{2.11}$$

The net price elasticity between capital and energy (CPE_{EK}) is defined as having no such conditions, thus permitting the level of utilised capital (K*) to change in response to changes in input prices. However, overall output (Y) is held constant.

$$CPE_{EK} = \left.\frac{\partial \ln E}{\partial \ln P_K}\right|_{Y \text{ constant}} \tag{2.12}$$

A more common name for the net price elasticity is the Cross Price Elasticity, which explains the use of CPE in the above equations. The CPE is one of the measures of the elasticity of substitution considered further in Section 3.

The net (or cross) price elasticity can be expressed as follows:

$$CPE_{EK} = \left.\frac{\partial \ln E}{\partial \ln P_K}\right|_{Y \text{ constant}} = \left.\frac{\partial \ln E}{\partial \ln P_K}\right|_{K^* \text{ constant}} + \left.\frac{\partial \ln E}{\partial \ln K^*}\frac{\partial \ln K^*}{\partial \ln P_{K^*}}\frac{\partial \ln P_{K^*}}{\partial \ln P_K}\right|_{Y \text{ constant}} \tag{2.13}$$

This reduces to:[10]

$$CPE_{EK} = CPE^{*}_{EK} + s_{K,K^*} CPE_{K^*K^*} \tag{2.14}$$

Where:

s_{K,K^*} = the share of capital costs in the total cost of utilised capital

$CPE_{K^*K^*}$ = own price elasticity of utilised capital along a Y_0 isoquant

[10] Since the production function is assumed to be linearly homogeneous $\partial \ln E / \partial \ln K^* = 1$. Also, using Shepard's Lemma: $\partial \ln P_{K^*} / \partial \ln P_K = (P_K / P_{K^*})(\partial P_{K^*} / \partial P_K) = (P_K / P_{K^*})(K / K^*) = s_{K,K^*}$

Berndt and Wood denote the second term on the right hand side of this equation ($s_{K,K^*}CPE_{K^*K^*}$) the expansion elasticity, so that:

Net price elasticity = gross price elasticity + expansion elasticity

Since $CPE_{K^*K^*}$ is negative and s_{K,K^*} is positive, it follows that the expansion elasticity is negative. Therefore: $CPE_{EK} < CPE^*_{EK}$ and the net elasticity is less than the gross elasticity. While the gross elasticity is positive, the net elasticity may be negative if the expansion elasticity is greater than the gross elasticity ($s_{K,K^*}CPE_{K^*K^*} > CPE^*_{EK}$) as in the graphical example above. Thus, it is possible that any two factors can be both gross substitutes and net complements under Definition 2.

Note that the greater the cost share of capital in the total cost of utilised capital, the more likely it is that the net elasticity will be negative. The expansion elasticity is likely to be smaller following a change in energy prices, since energy is likely to account for a smaller share of the total cost of utilised capital ($s_{E,K^*} < s_{K,K^*}$). Hence, the net (or cross) price elasticity is asymmetric.

In the example given here, the net price elasticity is really a measure of the ease of substitution between capital and energy compared with the ease of substitution between 'utilised capital' (K*) and the labour/materials composite (L*). More generally, it is a measure of the relative substitution between two inputs compared with the substitution effects of other inputs (Hogan, 1979). If the substitution between capital and energy in the sub-function is large compared to the substitution between utilised capital and the labour/materials composite in the master function, then capital and energy will be net substitutes under this definition. When the substitution between capital and energy in the sub-function is relatively small compared to the substitution between utilised capital and the labour/materials composite in the master production function, then capital and energy will be net complements under this definition. Put another way, capital and energy will be substitutes (complements) when the gross price elasticity is large (small) compared to the expansion elasticity.

Hence, while capital and energy are substitutes when viewed in isolation from the rest of the inputs to the production process, they may well be complements in the context of the overall production function. This is precisely what Berndt and Wood (1979) found in their empirical investigations of the (aggregate) manufacturing sectors in the US and Canada.

2.3 Summary

This section has introduced some key concepts from neoclassical production theory and provided a plausible (although far from universally accepted) account of how two inputs may appear as complements. It has shown how the identification of two inputs as substitutes or complements depends upon the particular definition being used. The most common definition describes two factors as substitutes (complements) if an increase in the price of the first is associated with an increase (decrease) in the usage of the second, holding output constant. This definition is useful when assessing the potential impact of an increase in input prices.

From an engineering perspective, energy and capital appear to be substitutes, since a decrease in use of the first may be compensated (at least to some extent, within a particular system boundary) by an increase in use of the second. Investment in improved energy efficiency can be understood as the substitution of capital for energy. But this neglects the associated adjustments of labour and material inputs within the production process as a whole. When the full pattern of adjustments is taken into account, energy and capital may well turn out to be complements under the definition given above. This is what Berndt and Wood (1975) found in their pioneering study of factor substitution in US manufacturing period 1947-71 (during which energy prices were falling in relative terms). This finding, together with its potential economic importance, has stimulated a great deal of empirical research. But it appears that a consensus has yet to be reached on whether energy and capital can be described as substitutes or complements.

3 Defining and measuring elasticities of substitution

The elasticity of substitution is intended to measure the ease with which one factor of production can be substituted for another. The sign of this measure is commonly used to define whether factors may be considered substitutes or complements. However, there are a number of definitions of this parameter, which makes interpretation of the empirical literature difficult. This section summarises the different definitions of the elasticity of substitution and clarifies the relationship between them. It then highlights a number of issues relevant to estimating elasticities of substitution, which are frequently overlooked in the empirical literature.

3.1 Defining the elasiticity of substitution

The elasticity of substitution (EoSij) is intended to measures the ease with which one varying factor of production (i) can be substituted for another (j). Definitions of the elasticity of substitution always refer to a situation where output is held fixed. However, there are several competing definitions, which incorporate different assumptions about whether:

the independent variable refers to a change in the usage of a input or a change in the price of that input;

the independent variable refers to an absolute change in the usage or price of an input or a change relative to another input;

the dependent variable refers to an absolute change in the usage or price of an input or a change relative to another input;

other input quantities are held fixed; and

other input prices are held fixed.

The value and sign of the estimated elasticity of substitution depends upon which definition is used. Each definition can provide useful, but different information about the scope of substituting capital for energy, or vice versa. But the lack of clarity in definitions, together with inconsistency in terminology can make the empirical literature in this area difficult to interpret (Stern, 2004). This also partly explains why the substitutability between capital and energy remains a topic of controversy.

The following sections introduce five alternative measures used to define the relationship between factor inputs in terms of substitutability/complementarity. These measures, discussed further below, are:

Marginal rate of technical substitution (r)
Hicks/Direct Elasticity of Substitution (HESij)
Cross Price Elasticity (CPEij)
Allen-Uzawa Elasticity of Substitution (AESij)
Morishima Elasticity of Substitution (MESij)

In the empirical literature, by far the most common measure of the elasticity of substitution is the Allen-Uzawa. However, this has been criticised by a number of recent authors and the use of the Morishima measure of the elasticity of substitution is becoming more widespread.

Both of these can usefully be related to the Cross Price Elasticity between two factors of production which in many respects is a more useful measure. Both the Allen-Uzawa and Morishima elasticities can be related to the Direct elasticity of substitution, which in turn is a generalisation of the Hicks definition to multi-factor production functions (Hicks, 1932). The original Hicks definition, in turn, is based upon the marginal rate of technical substitution. The following sections define each of these measures, clarify the relationships between them and comment on their relative suitability for empirical work. In each case the assumptions behind the definition are summarised; the relevant formula(e) are presented; it is shown whether and how the measure may be used to classify factors as substitutes or complements; some key issues in interpretation are highlighted; and possible extensions identified. The text is largely based upon the excellent surveys by Frondel (2004) and Stern (2004).

3.1.1 Marginal rate of technical substitution

The logical starting point is the marginal rate of technical substitution (r). This measures the rate at which one factor of production can be substituted for another factor while holding output constant. r is a measure of the slope of an isoquant on the production surface.

3.1.1.1 Assumptions:

Two factors vary (xi and xj)
Other inputs fixed.
Output is fixed

3.1.1.2 Definitions

$$r = -\left.\frac{\partial x_i}{\partial x_j}\right|_{Y \text{ and } x_k \text{ constant for } k \neq i,j} \qquad (3.1)$$

An alternative definition is:[11]

$$r = -\left.\frac{f_j}{f_i}\right|_{Y \text{ and } x_k \text{ constant for } k \neq i,j} \qquad (3.2)$$

The marginal rate of technical substitution (r) between i and j is therefore given by the ratio of the marginal productivity of j to the marginal productivity of i. The marginal productivity, in turn, represents the increase in output for a unit increase in the input of a particular factor ($f_i = \partial f / \partial x_i$).

3.1.1.3 Substitutes and complements

The marginal rate of technical substitution provides a direct measure of how much of one factor is required to substitute for another – with other inputs and output fixed. The terms substitutes and complements are not used directly in relation to this measure, but r does provide a basis for the Hicks elasticity of substitution, discussed next.

[11] This can be derived by differentiating the production function (f) with respect to x_j : $\frac{\partial f}{\partial x_i}\frac{\partial x_i}{\partial x_j} + \frac{\partial f}{\partial x_j} = 0$

3.1.1.4 Issues and extensions

In the two-input case, r is positive. To hold output constant, xj must increase when xi decreases. The value of r will depend upon both the level of output and the relative proportion of each factor (i.e. the point on the isoquant map). Typically (with convex isoquants) r is diminishing for increasing inputs of xi.12

One drawback of r is that it depends upon the units in which factors are measured. Frondel (2004) has proposed an alternative, non-dimensional measure of relative changes in factor inputs, termed the 'total elasticity of substitution' (TES):

$$TES_{ij} = -\frac{\partial x_i / x_i}{\partial x_j / x_j} = \frac{\partial x_i}{\partial x_j}\frac{x_j}{x_i} \qquad (3.3)$$

However, this elasticity measure is not in widespread use and none of the papers summarised in the empirical review section apply this measure.

3.1.2 The Hicks / Direct elasticity of substitution

Hicks (1932) introduced this as a measure of the ease with which a decrease in one input can be compensated by an increase in another while output is held constant. The Hicks Elasticity of Substitution (HESij) measures the ratio of the relative change in factor proportions to the relative change in the marginal rate of technical substitution. The definition refers to movement along a partial isoquant on the production surface and is a scale-free measure of the curvature of this isoquant. The original Hicks definition applies to a production function with only two inputs. The subsequent generalisation to multi-input production functions is sometimes termed the Direct Elasticity of Substitution (Chambers, 1988). In what follows, both of these will be referred to as the Hicks Elasticity of Substitution (HES).[13]

3.1.2.1 Assumptions

Two factors vary (xi and xj)
Other inputs fixed.
Output is fixed

[12] The definition can be extended to the case where three inputs vary to give: $r = -\frac{\partial x_i}{\partial x_j} = \frac{f_j}{f_i} + \frac{f_j}{f_i}\frac{\partial x_k}{\partial x_j}\bigg|_{Y \text{ and } x_k \text{ constant for } k \neq i,j}$.

In this case, *r* is no longer unambiguously positive. If $r<0$, x_j could decline when x_j declines. The output is held constant by an increase in x_k.

[13] Terminology does not seem to be consistent in this area, which can be a major source of confusion. For example, Berndt and Wood (1979) refer to the *HES* (as defined here) as the Direct elasticity of substitution, and refer to the *AES* as the Hicks-Allen elasticity of substitution.

3.1.2.2 Definition

For a two-input production function that satisfies certain conditions,[14] the HESij is defined as:[15]

$$HES_{ij} = \frac{\partial \ln(x_i / x_j)}{\partial \ln r} = -\frac{\partial \ln(x_i / x_j)}{\partial \ln(f_j / f_i)}\bigg|_{Y \text{ and } x_k \text{ constant for } k \neq i,j} \quad (3.4)$$

The HES may be generalised to a multi-input production function by holding other inputs fixed. Under the assumption of perfect competition and profit maximisation, the marginal rates of technical substitution between two factors (-fj/fi) should be equal to the ratio of their prices (pj/pi). This leads to a more convenient definition as follows:

$$HES_{ij} = \frac{\partial \ln(x_i / x_j)}{\partial \ln(p_j / p_i)}\bigg|_{Y \text{ and } x_k \text{ constant for all } k \neq 1,j} \quad (3.5)$$

This generalisation of the basic Hicks definition may therefore be termed a two-factor, two-price elasticity.

3.1.2.3 Substitutes and complements

If only two inputs are allowed to vary, output can only be held constant if a decrease in one input (i) is compensated by an increase in a second (j) - in other words, one factor must 'substitute' for another. Therefore, the HES classifies all inputs as substitutes according to Definition 1 of Section 2.

3.1.2.4 Factor shares

The HES provides information on the effect of a change in usage of an input on the share of that input in the value of output ($s_i = x_i p_i / P_Y$).[16] It may be shown that (Sato and Koizumi, 1973):

$$\frac{\partial(s_i / s_j)}{\partial(x_i / x_j)} \gtreqless 0 \text{ according to } HES_{ij} \gtreqless 1 \quad (3.6)$$

Hence, if HESij>1 (<1,) the share of input i in the value of output becomes larger (smaller) relative to j as the usage of i becomes larger (smaller) relative to j.

---

[14] A continuous function with positive first order partial derivatives and continuous second-order partial derivatives that is quasi concave.

[15] An alternative formulation is: $HES_{ij} = -\left\{\left[\frac{1}{x_i f_i} + \frac{1}{x_j f_j}\right] / \left[\frac{f_{ii}}{f_i^2} + 2\frac{f_{ij}}{f_i f_j} - \frac{f_{jj}}{f_j^2}\right]\right\}\Bigg|_{Y \text{ and } x_k \text{ constant for } k \neq i,j}$, where f_i represents the marginal productivity of input *i*, or the first derivative of the production function with respect to *i*. .

[16] Under competitive market conditions, the latter is equal to the share of a factor in total input costs ($s_i = x_i p_i / C$).

3.1.2.5 Issues and extensions

The HES measures the curvature of the surface of the production function in a particular direction. It follows from the concavity of the production function that HESij>0.

From Equation 3.4, if r does not change at all with changes in the ratio xi/xj, it indicates that substitution is easy, because the ratio of marginal productivities of the two inputs does not change as the input mix changes. Alternatively, if r changes rapidly for small changes in the ratio xi/xj, it indicates that substitution is difficult because minor variations in the input mix will have a substantial effect on the relative productivities of the two inputs.

Taking the two factor case, if HESij is large, r will not change much relative to the input ratio, xi/xj, and the isoquant will be relatively flat. On the other hand, if HESij is small, the isoquant will be sharply curved. The extremes are: a linear production function, where $HES_{ij} = \infty$, and a 'Leontief' (fixed proportions) production function, where $HES_{ij} = 0$. For a 'Cobb Douglas' production function HESij=1, while for a 'Constant Elasticity of Substitution' (CES) production function, HESij is constant (as the name suggests) between 0 and infinity. The CES may be generalised to the multifactor case, but this places restrictive conditions on the elasticity values (McFadden, 1963). The 'Translog' production function allows for multiple substitution possibilities between pairs of factors, so HESij can vary. More information on these different types of production function is given in Annex 1.[17]

Since the extension of the Hicks definition to multi factor production functions requires the assumption that other factor inputs are fixed, the practical value of this definition is limited. As shown in Section 2, it is likely that in practice any change in the ratio of two inputs will also be accompanied by changes in the levels of other inputs. Some of these inputs may be complementary with the ones being changed, whereas others may be substitutes, and to hold them constant creates a rather artificial restriction.

3.1.3 The Cross Price Elasticity

The cross price elasticity (CPEij) forms the basis of Berndt and Wood's net and gross elasticities, described in Section 2. It also forms the basis of the Allen Uzawa and Morishima measures of the elasticity of substitution described below. Frondel (2004) is one of a number of authors who have argued that the CPEij provides the best and most intuitive measure of factor substitutability, despite the fact that most empirical studies do not state this measure explicitly.

3.1.3.1 Assumptions

One input price varies
Other input prices (not quantities) are fixed
Output is fixed

3.1.3.2 Definition

$$CPE_{ij} = \left.\frac{\partial \ln x_i}{\partial \ln p_j}\right|_{Y \text{ and } p_k \text{ constant for } k \neq j} \qquad (3.7)$$

[17] See also Annex 2 of Technical Report 5.

3.1.3.3 Substitutes and complements

The CPE classifies a pair of inputs as substitutes (complements) if an increase in the price of one causes the usage of the other to decrease (increase). Thus i and j are substitutes (complements) if CPEij >0 (<0).

3.1.3.4 Issues and extensions

The CPE focuses solely on the relative change in use of factor i due to a change in the price of factor j, with output and other factor prices held fixed. So this is a one-factor-one-price elasticity of substitution. The CPE is asymmetric (i.e. $CPE_{ij} \neq CPE_{ji}$) and is equivalent to the net price elasticity as defined by Berndt and Wood (i.e. the sum of the gross price and expansion elasticities). Hence, given that the gross price elasticity is necessarily positive, the CPE is only negative if the expansion elasticity is greater than the gross price elasticity.

3.1.4 The Allen-Uzawa Elasticity of Substitution

This is by far the most common measure of the EoS in empirical work and stems from the original definition by Allen (1938). As with the CPE, the AES is a one-factor-one-price elasticity.

3.1.4.1 Assumptions

One input price varies (pi)
Other input prices (not quantities) are fixed
Output is fixed

3.1.4.2 Definition

Allen derived a formula for the AES based upon the production function as follows:

$$AES_{ij} = \frac{\sum_{k=1}^{n} x_k f_k}{x_i x_j} \frac{|F_{ij}|}{|F|} \Bigg|_{Y \text{ and } p_k \text{ constant for } k \neq j} \qquad (3.8)$$

Where xk is an input factor, fk is the marginal productivity of that factor, F is the bordered Hessian of the production function and Fij is the cofactor of fij.

Uzawa (1962) derived an alternative definition using the cost function (C), and it is this form which is most used in the empirical literature (hence the term Allen-Uzawa elasticity of substitution):

$$AES_{ij} = \frac{cc_{ij}}{c_i c_j} \Bigg|_{Y \text{ and } p_k \text{ constant for } k \neq j} \qquad (3.9)$$

Where ci is the first directive of the cost function with respect to input i. This definition is useful empirically, but is difficult to interpret. However, Blackorby and Russell (1981; 1989) showed how this form is related to the more familiar cross price elasticity, as follows:[18]

$$AES_{ij} = \frac{CPE_{ij}}{s_j}\Bigg|_{Y \text{ and } p_k \text{ constant for } k \neq j} \tag{3.10}$$

Where sj represents the share of input j in total costs ($s_j = x_j p_j / c$). This formulation is much more useful for an intuitive understanding of the meaning of the AES.

3.1.4.3 Substitutes and complements

As with the CPE, the AES classifies a pair of inputs as substitutes (complements) if an increase in the price of one causes the usage of the other to decrease (increase). Therefore i and j are substitutes (complements) if AESij >0 (<0).

3.1.4.4 Factor shares

The AES provides information on the effect of a change in the price of input j on the share of input i in the value of output ($s_i = x_i p_i / P_Y$):

$$\frac{\partial \ln s_i}{\partial \ln p_j} \gtreqless 0 \text{ according to } AES_{ij} \gtreqless 1 \tag{3.11}$$

$$\frac{\partial \ln s_i}{\partial \ln p_j} = s_j (AES_{ij} - 1) \tag{3.12}$$

So, an increase in the price of input j causes the cost share of the input i to increase (decrease) if the magnitude of the AES is greater than (less than) unity. While the sign of the AES provides information on whether si increases or decreases following an increase in pj, to quantify this change information is also required on si.

3.1.4.5 Issues and extensions

The AES is popular in empirical studies, because it may be calculated easily from econometrically estimated cost functions using Equation 3.9. However, Frondel (2004) argues that it has a number of drawbacks.

First, AES always has the same sign as the CPE, hence, AES classifies inputs as complements or substitutes in the same way as the CPE. In essence, the AES adds no more information to that contained in the CPE, so that arguably, the CPE could be used without complicating the issue by dividing by the cost share. Moreover, weighting of the CPE by the cost share can result in small variations in the cost share of a particular factor leading to large variations in the magnitude of the AES. This is particularly the case for energy, whose share of total cost typically varies between 1% and 10%. As a result, the quantitative value of the AES may have little meaning. As Chambers (1988) notes, Equation 3.10 is the:

18 Their derivation uses Shephard's Lemma – namely that the contingent demand function for any input is given by the partial derivative of the cost function (c) with respect to that input's price: $\partial c / \partial p_i = c_i = x_i$. Using this: $CPE_{ij} = \frac{\partial \ln x_i}{\partial \ln p_j} = \frac{p_j}{x_i}\frac{\partial x_i}{\partial p_j} = \frac{p_j}{x_i}\frac{\partial C_i}{\partial p_j} = \frac{p_j}{x_i}\frac{\partial^2 C_{ij}}{\partial p_i \partial p_j} = \frac{p_j}{x_i} C_{ij}$. Then multiply top and bottom by $x_j c$: $CPE_{ij} = \frac{p_j x_j c}{x_i x_j c} c_{ij} = \frac{p_j x_j}{c}\frac{c}{c_i c_j} c_{ij} = \frac{CPE_{ij}}{s_j}$

> "……most compelling argument for ignoring the Allen measure in applied analysis … The interesting measure is CPE_{ij} - why disguise it by dividing by a cost share? This question becomes all the more pointed when the best reason for doing so is that it yields a measure that can only be interpreted intuitively in terms of CPE_{ij}" (Chambers, 1988)

Second, the AES does not define the curvature of the isoquant in the same manner as the HES (Blackorby and Russell, 1975). The only circumstances in which it does is when the production function takes a non-nested Constant Elasticity of Substitution (CES) form (including Cobb-Douglas), or when there are only two factor inputs.

Third, the AES is symmetric (AESij = AESji.), which implies that that movement in one direction across the iso-surface is equally as easy as movement in the exact opposite direction. However, this symmetry is only achieved by dividing the CPE by the cost share. As indicated above, the CPE is not symmetric: in other words, the change of factor i due to a change in the price of factor j (with output and other factor prices held fixed) is not the same as the change in factor j due to a change in the price of factor i. This asymmetry may be important in practice, but is disguised by the AES. For example:

> "Reducing energy use due to energy-price shocks might be compensated optimally by an additional use of a third factor, say labor, while capital remains constant. Yet, conversely, a further expansion in capital use due to lower capital prices may necessitate more energy for an economically optimal way of production" (Frondel, 2004, p. 993)

3.1.5 The Morshima Elasticity of Substitution

The Morishima elasticity of substitution (MESij), due originally to Morishima (1967)), has been used instead of the AES in some more recent empirical work. In contrast to the AES and CPE, the MES measures the change in a ratio of inputs, rather than a single input – and hence is closer to the HES definition. However, in contrast to the HES, the measure is defined with respect to a change in a single price rather than a ratio of prices. The MES is thus a two-factor-one-price elasticity.

3.1.5.1 Assumptions

One input price varies (pi)
Other input prices (not quantities) are fixed
Output is fixed

3.1.5.2 Definition

Taking the general HES, but assuming that the change in pj/pi is solely due to a change in pj gives:[19]

$$MES_{ij} = \left.\frac{\partial \ln(x_i / x_j)}{\partial \ln(p_j)}\right|_{Y \text{ and } p_k \text{ constant for } k \neq j} \qquad (3.13)$$

[19] $\frac{\partial \ln(x_i / x_j)}{\partial \ln(p_j / p_i)} = \frac{\partial \ln(x_i / x_j)}{(p_i / p_j) / p_i \partial (p_j)} = \frac{\partial \ln(x_i / x_j)}{\partial \ln(p_j)}$

Blackorby and Russell (1975) use the cost function to define the MES as follows:

$$MES_{ij} = \frac{p_j c_{ij}}{c_i} - \frac{p_j c_{jj}}{c_j}\Bigg|_{Y \text{ and } p_k \text{ constant for } k \neq j} \quad (3.14)$$

Equation 3.14 is useful for empirical work with cost functions, but only applies when the function is continuous, linearly homogeneous and concave. Use of a cost function in this way also implies perfect competition and cost minimising behaviour. Hence, Equation 3.14 is a more restrictive definition than Equation 3.13.[20]

3.1.5.3 Substitutes and complements

The MES classifies a pair of inputs as substitutes (complements) if an increase in the price of one leads to an increase (decrease) in the usage of the other relative to the usage of the input whose price has changed. So i and j are substitutes (complements) if MESij >0 (<0). In practice, the MES classifies the great majority of inputs as substitutes.

3.1.5.4 Factor shares

The MES provides information on the effect of a change in the price of input j on the share of input i in the value of output relative to the share of input j:

$$\frac{\partial \ln(s_i / s_j)}{\partial \ln p_j} \gtreqless 0 \text{ according to } MES_{ij} \gtreqless 1 \quad (3.15)$$

$$\frac{\partial \ln(s_i / s_j)}{\partial \ln p_j} = (MES_{ij} - 1) \quad (3.16)$$

So, an increase in the price of input j causes the cost share of input i relative to that of input i to increase (decrease) if the magnitude of the MES is greater than (less than) unity.

Note that if two inputs are MES substitutes (MESij>0), it does not necessarily follow that the cost share of input i will decrease relative to that of input j following an increase in the price of j. This will occur only if MESij>1. In this sense, the magnitude of the MES relative to unity may provide a more useful indication of 'ease of substitution' than its magnitude relative to zero – although the latter is conventionally used to distinguish MES substitutes and complements. Substitution is clearly easier if producers can reduce their relative expenditure on the factor whose price have risen than if they are forced to increase their relative expenditure on this factor: i.e. substitution is easier if MESij>1.

3.1.5.5 Issues and extensions

Relationship between the MES and the CPE

Blackorby and Russell (1975) show that:

$$MES_{ij} = CPE_{ij} - CPE_{jj} \quad (3.17)$$

[20] From Shephard's Lemma $c_i = x_i$ Substituting this into the cost function definition of the *MES* gives: $MES_{ij} = \frac{p_j}{x_i}\frac{\partial x_i}{\partial p_j} - \frac{p_j}{x_j}\frac{\partial x_j}{\partial p_j} = \frac{\partial \ln x_i}{\partial \ln p_j} - \frac{\partial \ln x_j}{\partial \ln p_j} = \frac{\partial \ln(x_i / x_j)}{\partial \ln p_j}$. This is equivalent to the first definition derived directly from the production function.

Where CPEjj= the own price elasticity of input j. This shows that the effect of varying pj on the quantity ratio xi/xj (holding output and other prices constant) is divided into two constituent parts:

the proportional effect on xi, given by the cross price elasticity (CPEij) and
the proportional effect on xj itself, given by the own price elasticity (CPEjj)

Inversely, the effect of varying pi on the quantity ratio xj/xi - holding output and other prices constant - is given by:

$$MES_{ji} = CPE_{ji} - CPE_{ii} \quad (3.18)$$

This shows that the MES (like the CPE but unlike the AES) is asymmetric. By implication, two factors i and j may be MES complements with respect to changes in pi and MES substitutes with respect to changes in pj. However, the MES is closer to the original (Hicks) notion of a substitution elasticity in that it measures the percentage change in a factor ratio. In contrast, the CPE measures the percentage change in factor demand.

In the simple case of only two inputs, the MES cannot be negative (this would imply that a decline in the availability of one input could be made up through a decline in the availability of the second input). However, with more than two inputs it is theoretically possible for the MES to be negative. It would require the cross price elasticity between two inputs to be negative (indicating that they are complements under the CPE/AES definition) and to be greater in absolute value than the own price elasticity. For example, it would imply that a 1% increase in the price of energy would reduce inputs of capital by a greater proportion than it decreased inputs of energy. This would imply a substitution away from capital despite the fact that the relative price of capital has decreased. Hence, the MES is likely to be positive in the majority of cases. If substitutability is defined as MESij >0, this means that factors will be found to be MES substitutes in practically all cases. As a result, the sign of the MES may not be useful means of distinguishing substitutes and complements.

Relationship between the MES and the AES

Blackorby and Russell (1981) prove that AES and MES are identical if and only if the production technology has a non-nested CES or Cobb Douglas structure, or if there are only two inputs. Further, by combining Equation 3.17 and Equation 3.10, it can be seen that:

$$MES_{ij} = s_j(AES_{ij} - AES_{jj}) \quad (3.19)$$

The AES and the CPE classify a pair of inputs as substitutes (complements) if an increase in the price of one causes an increase (decrease) in the quantity demanded of the other. In contrast, the MES classifies a pair of inputs as substitutes (complements) if an increase in the price of one causes the quantity of the other to increase (decrease) relative to the quantity of the input whose price has changed. If two inputs are AES substitutes, then they must be MES substitutes. But if two inputs are AES complements, either they may be MES substitutes or (less likely) complements. Because AES_{jj} is always negative, two inputs being AES substitutes (AESij>0) are also inevitably MES substitutes. However, AES complements (AESij<0) are also likely to be MES substitutes. The effective inability to distinguish between substitutes and complements is arguably a drawback of the MES.

3.1.6 Summary of definitions

Table 3.1 summarises the key features of each of the definitions of the elasticity of substitution discussed above. Given this range of definitions, care must be taken when referring to 'the' elasticity of substitution, or to defining two factors as either substitutes or complements.

While the HES has an important role both theoretically and in energy-economic modelling, empirical studies tend to estimate the CPE, AES or MES. It is only with respect to these three measures that factors can meaningfully be classified as substitutes and complements. In all cases, substitution between two inputs is 'easier' when the elasticity of substitution between them is greater.

Most empirical studies use the AES and classify inputs as either substitutes or complements on the basis of the sign of the AES. This reflects the most common understanding of these terms, which is the effect of a change in the price of one factor on the demand for another (i.e. Definition 2 in Section 2). However, exactly the same information is provided by the CPE – the AES just scales that parameter by one of the cost shares, which is not necessarily very helpful since it implies that the quantitative value of the AES lacks meaning. Hence, in many circumstances, it would be simpler to just estimate the CPE. The sign of the MES is less useful as a definition of substitutes or complements, since in nearly all cases the MES is positive. However, the MES (like the CPE but unlike the AES) is asymmetric, which is more representative of actual economic behaviour.

The magnitude of each measure relative to unity provides an indication of how the cost share of each factor will change in either absolute or relative terms. The sign of the AES indicates whether an increase in the price of input j will increase or decrease the share of input i in the value of output, while the MES provides immediate quantitative information on the effect of a change in the price of input j on the share of input i relative to the share of input j. This is potentially valuable information.

Table 3.1 Comparing different definitions of the elasticity of substitution

Measure	Output	Other factor inputs	Other factor prices	Definition	Type	Substitutes (complements)	Factor shares $\text{EoS} \gtreqless 1$	Symmetrical
Hicks/Direct	Fixed	Fixed	-	$HES_{ij} = -\frac{\partial \ln(x_i / x_j)}{\partial \ln(f_{x_j} / f_{x_i})}$ $HES_{ij} = \frac{\partial \ln(x_i / x_j)}{\partial \ln(p_j / p_i)}$	Two factor, two price	All inputs are substitutes	$\frac{\partial(s_i / s_j)}{\partial(x_i / x_j)} \gtreqless 0$	No
Cross price	Fixed	Variable	Fixed	$CPE_{ij} = \frac{\partial \ln x_i}{\partial \ln p_j}$	One factor, one price	CPE>0 (<0)	$\frac{\partial \ln s_i}{\partial \ln p_j} \gtreqless 0$	No
Allen	Fixed	Variable	Fixed	$AES_{ij} = \frac{1}{s_j}\frac{\partial \ln x_i}{\partial \ln p_j}$	One factor, one price	AES>0 (<0)	$\frac{\partial \ln s_i}{\partial \ln p_j} \gtreqless 0$	Yes
Morishima	Fixed	Variable	Fixed	$MES_{ij} = \frac{\partial \ln(x_i / x_j)}{\partial \ln(p_j)}$	Two factor, one price	MES>0 (<0) (most inputs are substitutes)	$\frac{\partial \ln(s_i / s_j)}{\partial \ln p_j} \gtreqless 0$	No

Relationships:

$MES_{ji} = CPE_{ji} - CPE_{ii}$; $\quad MES_{ij} = s_j(AES_{ij} - AES_{jj})$

Factor shares:

$\frac{\partial \ln s_i}{\partial \ln p_j} = s_j(AES_{ij} - 1)$; $\quad \frac{\partial \ln(s_i / s_j)}{\partial \ln p_j} = (MES_{ij} - 1)$

3.2 Issues in estimating elasticities of substitution

Empirical studies of production relationships assume a particular functional form for the production or cost function and estimate the parameters of that function econometrically. Elasticities of substitution may then be calculated from those estimated parameters. The most common functional forms are the Cobb Douglas, the Constant Elasticity of Substitution (CES) and the Translog. Each of these is briefly introduced in Annex 1. By implication, the empirical results may depend very much on the particular form that is chosen.

Empirical studies may also be prone to bias due to omitted variables and other factors. Four particularly important issues relevant to estimating elasticities of substitution are:

separability of input factors;
aggregation and changes in product mix;
the importance of cost shares when using static Translog cost functions; and
the treatment of technical change
Each of these is discussed in turn below.

3.2.1 Separability

Assumptions regarding the separability of production factors are common in empirical work. Separability is defined in one of two ways:

> Primal: The marginal rate of technical substitution between two inputs is unaffected by the level of other inputs:

$$\frac{\partial}{\partial x_k}\left[\frac{\partial x_j}{\partial x_i}\right] = 0 \qquad (3.20)$$

> Dual: The marginal rate of technical substitution between two inputs (xi/xj) is unaffected by the prices of other inputs:

$$\frac{\partial}{\partial p_k}\left[\frac{x_j}{x_i}\right] = 0 \qquad (3.21)$$

If two inputs (i,j) are separable from a third input (k), then the ease of substitution between i and k (as measured by the CPE, AES or MES) is equal to that between j and k (e.g. AESik=AESjk).

The separability of inputs in production functions is commonly used within production theory to justify both the omission of inputs (notably materials) for which data is unavailable and the grouping, or nesting, of different inputs. With nesting, the assumption is that producers engage in a two-stage decision process: first optimising the combination of inputs within each nest, and then optimising the combination of nests required to produce the final output. Two factors may only be legitimately grouped within a nest if they are separable from factors outside of the nest. For example, a (KL)E nesting structure requires that capital and labour are separable from energy under the definitions given above. One of the contributions of Berndt and Wood (1975) was to show that a nest of capital and labour inputs (often referred to as 'value-added') was not separable from either energy or materials within their dataset. The only nesting structure that was supported by Berndt and Wood's data was $Y = f(g(KE), h(L, M))$, which corresponds to the example given in Section 2.

However, defined in this way, the assumption of separability does not mean that measures of the CPE, AES or MES between inputs within a nest (e.g. i and j) are unaffected by the level (price) of inputs outside the nest (e.g. k) (Frondel and Schmidt, 2004). For example, even if capital and labour inputs were separable from energy inputs according to the standard definition, this would not necessarily imply that the ease of substitution between capital and labour (as measured by the CPE, AES or MES) was unaffected by the level (price) of energy inputs. Therefore, omitting measures of any input from empirical work could lead to error. This is particularly important for empirical estimates based upon KLE production/cost functions, as opposed to those which include materials (KLEM). The omission of materials could lead to biased estimates, even if materials are separable from other production factors according to the definitions above.

Frondel and Schmidt (2002) show that for AESij and CPEij to be unaffected by changes in the price of k, both the separability condition defined above must hold and changes in the price of k must not affect the cost shares of inputs i and j. Similar comments apply to MESij. Since this appears unlikely in practice, it seems reasonable to assume that measures of CPEij and AESij will generally depend upon the price of other factors, even when i and j are separable from those factors. This suggests that empirical studies that use assumptions of separability to justify the omission of other factors - notably materials - are likely to be biased.

Empirical work based upon Translog cost functions should be less vulnerable to this problem, since these functions do not rely upon nesting structures and do not impose separability restrictions (although such restrictions can be tested). However, Translog studies that omit particular inputs due to lack of data could still be prone to bias.

3.2.2 Aggregation

Measures of the elasticity of substitution are defined with respect to constant level of output and (implicitly) assume that this output consists of a single homogeneous product. Measures of the elasticity of substitution are then interpreted as relating to changes in the mix of factor inputs required to produce this single product; but in practice, this might be misleading. Individual firms/subsectors/sectors may produce a range of different products with each requiring a different input mix. Hence, changes in factor prices could lead to a change in the mix of products, as well as changes in the mix of factors used to produce an individual product (Miller, 1986; Solow, 1987). More generally, changes in factor prices could lead to a change in the relative contribution of individual subsectors to the output of a sector, or the relative contribution of individual sectors to the output of an economy. The scope for such changes is likely to increase with the level of aggregation for which the elasticity of substitution is estimated. For example, a product mix for a sector defined at the two digit level is greater than that for a sector defined at the three digit level, which in turn is greater than that for an individual firm. In practice, sector definitions at the two or three digit level frequently include both the energy intensive production of basic materials and the less energy intensive manufacturing of products from that basic material.

Miller (1986) argues that such effects are likely to lead the elasticity of substitution between capital and energy to be overestimated in studies using a static functional form with cross-sectional data. This is because the errors due to the excluded product mix variable are likely to be systematically correlated with factor prices. For example, suppose the data set includes two regions or countries with very different levels of energy prices. One may expect

the region with higher energy prices to specialise in less energy-intensive products, and vice versa. But these differences in product mix may be disguised by the chosen level of sectoral aggregation. If the sector is wrongly assumed to produce a homogeneous product, the data set will suggest a considerable scope for substituting capital for energy in the manufacture of that product and hence lead to a high estimate for the elasticity of substitution. This bias may be moderated, but not necessarily eliminated, by any correlation between the energy and capital intensity of the relevant sub-sectors (Miller, 1986). The resulting estimates of the elasticity of substitution could overstate the potential for substituting capital for energy over time within that sector. A single nation may not be not be able to shift its production in the same manner as a single region, and the world as a whole will not be able to shift its production in the same manner as an individual country - especially if the energy intensive products are intermediate inputs into the production of non-energy intensive final goods. The results of such studies could therefore be misleading.[21]

In a similar manner, Solow (1987) has illustrated how a sector or economy may still exhibit factor substitution in the aggregate due to changes in product mix, even when the production of individual products is governed by fixed factor ratios. Moreover, if the scope for substitution within a particular production process is relatively limited, changes in relative factor prices may induce changes in the composition of production at higher levels of aggregation (Koetse, et al., 2007).

A second aggregation issue is highlighted by ecological economists such as Stern and Cleveland (2004). Measures of the elasticity of substitution between energy and capital refer to a particular level of aggregation and implicitly assume that the relevant factors are independent of each other. However, such estimates may overestimate the possibility for substitution between energy and capital at a higher level of aggregation, such as the economy as a whole. This is because they do not include the indirect energy consumption that is required to produce and maintain the relevant capital, or the indirect capital and labour that is required to produce and deliver energy commodities. For example, energy is required to produce and install home insulation materials and energy efficient motors. This suggests that the scope for substitution between energy and capital at the level of the macro-economy may be less than indicated by an analysis of an individual sector:[22]

> "From an ecological perspective, substituting capital and/or labour for energy shifts energy use from the sector in which it is used to sectors of the economy that produce and support capital and/or labour. In other words, substituting capital and/or labour for energy increases energy use elsewhere in the economy" (Kaufmann and Azary-Lee, 1990)

However, the above two observations are potentially contradictory. Consideration of potential changes in product/sector mix suggests that empirical studies at higher levels of aggregation may indicate a greater scope for substitution between energy and capital than studies at a lower level of aggregation, while the consideration of indirect energy consumption suggests the opposite. The balance between the two will depend upon the

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[21] Miller (1986) also argues that studies using time-series data may be biased towards a finding of complementarity between energy and capital, since they typically reflect only short-term adjustments. This echoes the earlier arguments of Griffin (1981) and others, although Miller highlights other factors such as differences in the measurement of capital that may also contribute to this result. Hence, Miller provides a number of explanations why energy and capital may be found to be substitutes in cross-sectional studies and complements in time-series studies.

[22] The implications of this, and their relevance to the rebound effect, are discussed further in *Technical Report 5*.

particular sectors and level of aggregation being studied. For example, the indirect energy consumption associated with the manufacture of insulation will only be relevant to estimates of the elasticity of substitution if the sector that manufactures insulation is included in the estimates. Generally, changes in product mix should be more relevant to estimates of the elasticity of substitution between energy and capital within a particular sector, while indirect energy consumption should be more relevant to economy-wide estimates. Also, empirical estimates at lower levels of aggregation are more likely to isolate 'pure' substitution effects.

A final aggregation issue relates to the potentially important distinctions between different types of capital (e.g. machinery and structures), different types of labour (e.g. skilled and unskilled) and different types of energy input (e.g. electricity and fuel). There will be scope for substitution within these subcategories (e.g. between electricity and fuel), as well as between subcategories (e.g. machinery and fuel) and studies that use more aggregate measures will not reveal these individual patterns. As an example, there may be less scope for substitution between capital and skilled labour than between capital and unskilled labour. Similarly, aggregate energy and capital may be estimated to be substitutes, but capital may be a complement for electricity inputs but a substitute for fuel oil. The estimated elasticity of substitution involving an aggregate is not necessarily a weighted average of the elasticities of substitution for the disaggregate inputs. However, this does not necessarily mean that estimates of substitution possibilities between more aggregate measures of inputs will be biased.

In sum, aggregation issues may be a potentially significant source of bias within empirical studies and may best be minimised by using the highest level of sectoral disaggregation that the data permits and also by distinguishing between different types of fuel, capital equipment and labour (Apostolakis, 1990).

3.2.3 Cost shares

The most common approach to measuring elasticities of substitution is to estimate a static cost function, in which inputs are implicitly assumed to adjust their long run desired levels within one time period. Typically, studies employ a homothetic, Translog cost function (see Annex 1) of the form:

$$\ln C(p_1,.....p_I,Y) = \beta_0 + \beta_Y \ln Y + \sum_{i=1}^{I} \beta_i \ln p_i + \frac{1}{2}\sum_{i,j=1}^{I,J} \beta_{ij} \ln p_i \ln p_j \qquad (3.22)$$

With the following restrictions:

$$\sum_{i=1}^{I} \beta_i = 0 \qquad (3.23)$$

$$\sum_{i=1}^{I} \beta_{ij} = 0 \qquad (3.24)$$

In this case (using Shephard's lemma), the cost share (si) for each factor is given by:

$$s_i = \frac{x_i p_i}{C} = \frac{\partial C}{\partial p_i}\frac{p_i}{C} = \frac{\partial \ln C}{\partial \ln p_i} = \beta_i + \sum_{i=1}^{I} \beta_{ij} \ln p_j \qquad (3.25)$$

Frondel and Schmidt (2002) show that, in this instance the cross price elasticity is given by:

$$CPE_{ij} = \frac{\beta_{ij}}{s_i} + s_j \qquad (3.26)$$

This shows that estimates of the CPE (and hence also the AES and MES) from a static Translog cost function will depend upon the cost shares of the relevant factors. In particular CPEij will be close to the cost share of factor j if the cost share of factor i is large relative to the second order coefficient βij (since the first term will be small). Since, in practice, βij is often small, the cost share of factor j is likely to strongly influence the magnitude of the estimated cross price elasticity. According to Frondel and Schmidt (2002), the economic intuition behind this is that the larger the cost share of factor i (si), the harder it is to substitute factor i for a factor j whose price is increasing.

In the case of capital and energy, the cost share of energy is likely to have a strong role in determining the magnitude of CPEKE (and hence of AESKE and MESKE). Costs shares, in turn, depend upon which factors are measured. For example, the inclusion or otherwise of materials may be expected to have a significant influence on the estimated cost shares. Frondel and Schmidt (2002) review the empirical literature on energy-capital substitution and find that evidence of complementarity only occurs in cases where the cost share of energy is small. When material inputs are included in the specification, the cost shares of capital and energy necessarily become smaller and the finding of complementarity becomes more likely. Frondel and Schmidt (2002) argue that this may explain the differences between time-series and cross-sectional results noted by Apostolakis (1990) and others. The standard interpretation is that time-series results reflect short-run substitution possibilities while cross-sectional results represent long-run (equilibrium) substitution possibilities (Griffin and Gregory, 1976; Apostolakis, 1990; Frondel and Schmidt, 2002). Since time-series studies appear more likely to find energy and capital to be complements, while cross-sectional studies appear more likely to find them to be substitutes, this suggests that there may be more scope for substitution between capital and energy in the long-run (when capital can be replaced). However, Frondel and Schmidt (2002) explain these results by noting that the time series studies were more likely to include data on materials.

Prior to Frondel and Schmidt (2002), this linkage between costs shares and elasticity estimates (with the static, Translog functional form) does not appear to have been recognised. It may explain why the estimates of CPEKE (i.e. with PE varying) are commonly small – since the cost share of energy is typically small. Frondel and Schmidt (2004) argue that '… inferences obtained from previous empirical analyses appear likely to be an artefact of cost shares and have little to do with statistical inference about technology relationships' (p. 72). However, Stern (2004) argues that KLE studies are likely to be biased owing to the omission of materials and that cost shares are not arbitrary but are instead a function of the underlying technology. For example, energy intensive industries have a stronger incentive to substitute capital for energy when energy prices increase, suggesting that the finding of substitution may be more common when the cost share of energy is large. The implication is that KLEM studies are likely to reveal the 'true' relationship, while KLE studies will not. In either case, these observations suggest that dynamic functional forms may be preferable, although in practice these appear to be less commonly used.[23]

[23] Static means that the production function is formed only of 'current period' variables, whereas a dynamic model includes information on all or some of the other variables in the past.

3.2.4 Technical change

Technical change occurs when technological improvements in an industry act as a precursor to increased levels of output associated with any given level of factor inputs, for example through engineering efficiency improvements. While substitution may be represented as a movement along an isoquant of a production function, 'technical change' refers to the development of new technologies and methods of organisation that shift the isoquant to the left, allowing the same level of output to be produced from a lower level of inputs.

The inclusion of technology into production functions can be done in a number of ways so as to reflect different assumptions about the role of technical change, the most common being the assumption of Hicks-neutral technical change or non-neutral (factor augmenting) technical change.[24]

Hicks neutral technical change, is shown representatively as:

$$Y = \upsilon(t)\, f(K, L, E) \tag{3.27}$$

Where the multiplier $\upsilon(t)$ denotes technology and is assumed to be greater than unity ($\upsilon(t) \geq 1$). The assumption that technical change can be separated in this way from the rest of the factors of production ensures that it can be uniquely identified. Moreover, the parameters of the estimated production function reflect only the decision to use a certain combination of the available factor inputs and not the change in input use owing to technological advancements. The effects of Hicks-Neutral technical change are indiscriminate of factor input, e.g. the technical change benefits the use of labour equally as much as it benefits the use of capital. More specifically,

> "Hicks neutral technical change shifts the iso-surface parallel toward the origin, and hence decreases the input level of all production factors in the same proportion" (Kako, 1980)

It appears more likely, however, that the effects of technical change over time will serve to be more beneficial to one factor input than another and should thus be disaggregated to allow for differentiation across the factor inputs. For instance, it may be found that technical change has improved capital productivity to a greater extent than labour productivity. Such representations of technical change are known as 'factor-augmenting (or non-neutral)' and may be represented as follows:[25]

$$Y = f(\upsilon_K(t)K, L, E) \tag{3.28}$$

$$Y = f(K, \upsilon_L(t)L, E) \tag{3.29}$$

$$Y = f(K, L, \upsilon_E(t)E) \tag{3.30}$$

Where Equations (3.28), (3.29) and (3.30) represent capital augmenting (Solow-Neutral), labour augmenting (Harrod-Neutral) and energy augmenting technical change respectively. Many empirical models allow the incorporation of multiple factor augmenting technical change, as follows:

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[24] In the empirical work discussed in Section 4, it is not always clear exactly what assumptions are being made. Moreover, when a Translog cost function is estimated using derived cost share equations, the omission of any statement about technical change often implies the assumption of Hicks neutral technical change.

[25] In other words, factor augmenting technical change implies that proportional savings on factor *X* inputs are greater than the average proportional savings on all inputs.

$$Y = f(\upsilon_K(t)K, \upsilon_L(t)L, \upsilon_E(t)E) \quad (3.31)$$

Alternatively, empirical models may omit technical change altogether. This may be justified if technical change can be assumed to be separable from the other factors in the production function; which means that it omission from the functional form should not lead to any bias in the estimation of the relevant substitution parameters. However, such assumptions are not always tested and the particular approach taken in an empirical study may have a strong influence on the results obtained. In general, there appears to be no preferred approach, since the role technology plays may be expected to vary widely between different agents and over different time periods.

A standard assumption in empirical studies is that the rate and bias of technical change is fixed. But in a long-run relationship, it seems reasonable to expect that changes in relative prices will stimulate technical change in particular directions, as well as encouraging substitution - i.e. movements along an existing production function. Induced technical change has become a central feature of empirical research in this area, but distinguishing between price-induced factor substitution and price induced technical change is empirically challenging.

3.3 Summary

This section has summarised the different definitions of the elasticity of substitution, clarified the relationship between them and highlighted a number of important issues relevant to estimating these elasticities empirically. Taken together, the analysis suggests that considerable caution is required when interpreting the literature in this area. In particular, the different definitions of the elasticity of substitution, the lack of consistency in the use of these definitions and the lack of clarity in the relationship between them, all contribute to making the empirical literature both confusing and contradictory.

For all definitions, substitution between two inputs is 'easier' when the elasticity of substitution between them is greater. But the appropriate classification of factors as substitute or complements depends upon the particular definition of the elasticity of substitution being used (i.e. factors may be substitutes under one measure and complements under another). The majority of existing studies use the sign of the Allen-Uzawa elasticity of substitution (AES) to make this classification. However, according to some authors, this measure has a number of drawbacks and its quantitative value lacks meaning. Furthermore, in many cases, the Cross Price Elasticity (CPE) or Morishima Elasticity of Substitution (MES) measures may be more appropriate, but these have yet to gain widespread use. Furthermore, the sign of the MES is less useful as a means of classifying substitutes and complements, since in nearly all cases the MES is positive. However, the MES is seen by some authors as more representative of actual economic behaviour since (unlike the AES) it is asymmetric.

For all definitions, the magnitude of the elasticity of substitution relative to unity can provide an indication of how cost shares will change following a change in the independent variable. While the precise interpretation depends upon the definition, the magnitude relative to unity may provide an appropriate basis for classifying two factors as either 'weak substitutes' (EoS<1) or 'strong substitutes' (EoS>1).

A large number of empirical studies estimate elasticities of substitution between different factors (or groups of factors) within different sectors and countries and over different time periods. These rely upon a variety of assumptions, including in particular the specific form of the production or cost function employed (e.g. CES or Translog). The estimated values may be expected to depend in part upon the assumptions made.

Standard methodological approaches rely heavily upon assumptions about the separability of different inputs or groups of inputs. These assumptions are not always tested, and even if they are found to hold, the associated estimates of the elasticity of substitution could still be biased. Assumptions about the nature and bias of technical change may also have a substantial impact on the empirical results, but distinguishing between price-induced technical change and price-induced factor substitution is empirically challenging. The level of aggregation of the study is also important, since a sector may still exhibit factor substitution in the aggregate due to changes in product mix, even if the mix of factors required to produce a particular product is relatively fixed. This suggests that the scope for substitution between energy and capital may be greater at higher levels of aggregation. However, individual factors cannot always be considered as independent, notably because energy is required for the provision of labour and capital. This suggests that the scope for substitution may appear to be smaller at higher levels of aggregation.

In general, the actual scope for substitution may be expected to vary widely between different sectors, different levels of aggregation and different periods of time, while the estimated scope for substitution may depend very much upon the particular methodology and assumptions used.

4 Empirical estimates of the elasticity of substitution between energy and capital

In Berndt and Wood's (1975) study of US manufacturing, factors were classified as either substitutes or complements on the basis of the sign of the AES. The results were:

Capital and Labour:	AES substitutes
Capital and Materials:	AES substitutes
Labour and Energy:	AES substitutes
Labour and Materials:	AES substitutes
Energy and Materials:	AES substitutes
Capital and Energy:	AES complements

Subsequent empirical studies have found broadly comparable results for all of the above factor pairs except for capital and energy (Chung, 1987). In this case, no consensus appears to have been reached.

This section summarises the results of a literature search (see Annex 2) on empirical estimates of the elasticity of substitution between energy and capital (EoSKE). The search was confined to estimates of the elasticity of substitution between aggregate measures of capital and energy, so the substitution within and between different types of capital and different types of energy carrier were not considered.[26] This type of substitution may nevertheless be important in determining economic behaviour in general and rebound effects in particular (Grepperud and Rasmussen, 2004).

The search identified around 100 relevant papers, that included over 200 empirical estimates of EoSKE. Of these, 7 papers covered or included the UK, including 25 estimates of EoSKE for the UK. Throughout what follows, energy and capital will be classified as substitutes or complements according to the sign of the estimated AES or MES in these studies. Since the overwhelming majority of studies estimate the AES (Section 4.1.6), these dominate the overall results. However, since studies that used the MES are less likely to classify factors as complements (Section 3.1.6), this creates a slight bias towards substitutability in the aggregate results.

Section 4.1 summarises and classifies these results by firstly considering 'papers' to identify the different approaches taken, then by considering the 'results'; that is classifying the individual EoSKE estimates. Section 4.2 summarises and evaluates the different reasons that have been put forward to explain the large variation in the empirical results. The most striking result from the analysis is the lack of consensus that has been achieved to date, despite the numerous studies that have been undertaken.

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4.1 Summary of results from existing studies

4.1.1 Classification according to papers

In order to demonstrate the key approaches adopted, the following classifies the identified 'papers' according to:

Form of production or cost function utilised
Form of separability (KLE vs non-KLE)
Form of technical change adopted (Hicks neutral vs non-nuetral)
Measure of EoSKE adopted
Static vs dynamic model
Type of data (time series vs cross section vs panel)
Countries covered.

Figure 4.1 shows that the overwhelming majority of studies assumed a Translog cost function and estimated the AES using a static[27] approach with time series data. About half only included capital, labour and energy as factors of production (i.e. they omitted materials); about a half assumed Hicks neutral technical change and just under a half used data from the US.

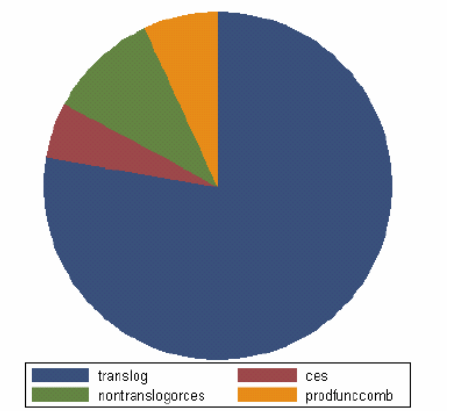
4.1.2 Classification according to estimated results

The above classification gives an indication of the predominant methodologies employed in the studies reviewed, but in order to try and ascertain the effect of the different approaches the 'results' from the estimated EoSKE's are classified. Before that, however, it is useful to consider the overall results of estimating EoSKE. Figure 4.2 therefore summarises the estimated EoSKE's for the overall results ('world'), as well as for the US and the UK individually. This shows that the average estimate for the world as a whole for EoSKE is close to zero suggesting that K and E are either weak complements or weak substitutes given a mean of -0.07 and a median of 0.08, although there is some dispersion around this which is a little skewed. For the US the average estimated EoSKE is -0.04 suggesting weak complementarity. However, for the UK the average estimated EoSKE is about 0.4 to 0.9 suggesting that K and E are substitutes; but this is based on a small number of papers and the results are dominated by those given in Harris et al (1993).

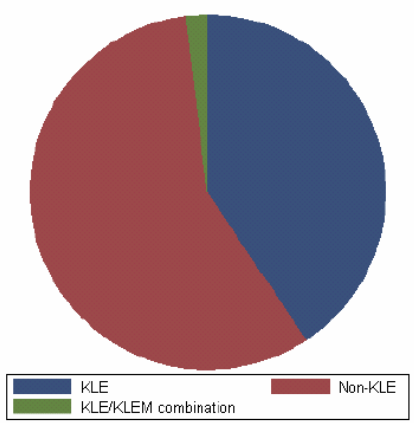
[27] Static means that the production function is formed only of 'current period' variables, whereas a dynamic model would include information on all or some of the other variables in the past.

Figure 4.1 Classification according to papers

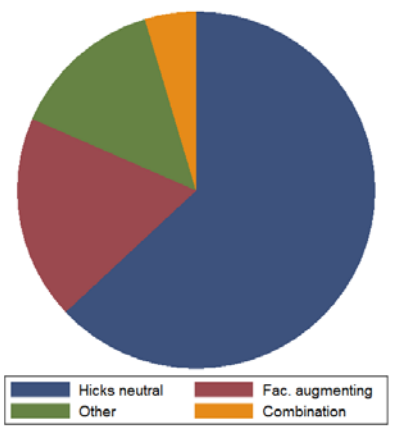
Form of function utilised



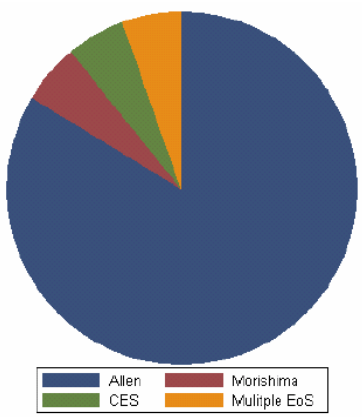
Form of separability



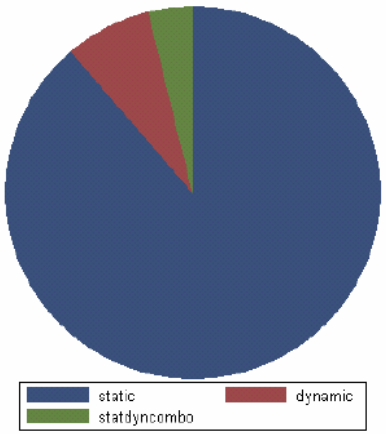
Form of technical change



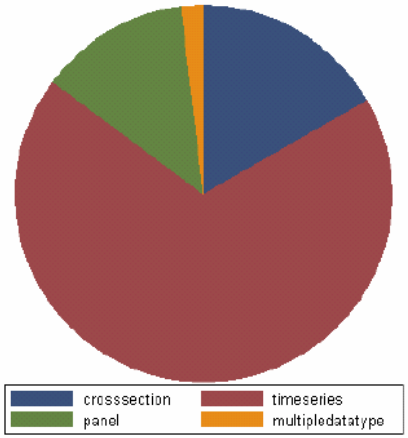
Measure of σEK



Static vs dynamic model



Type of data



Countries

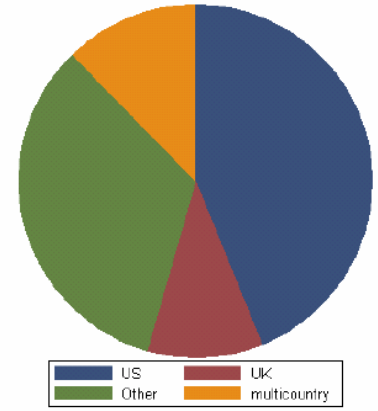
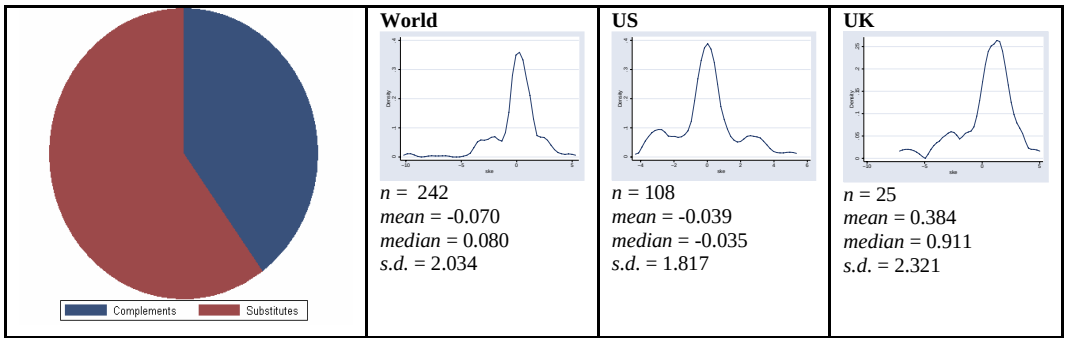


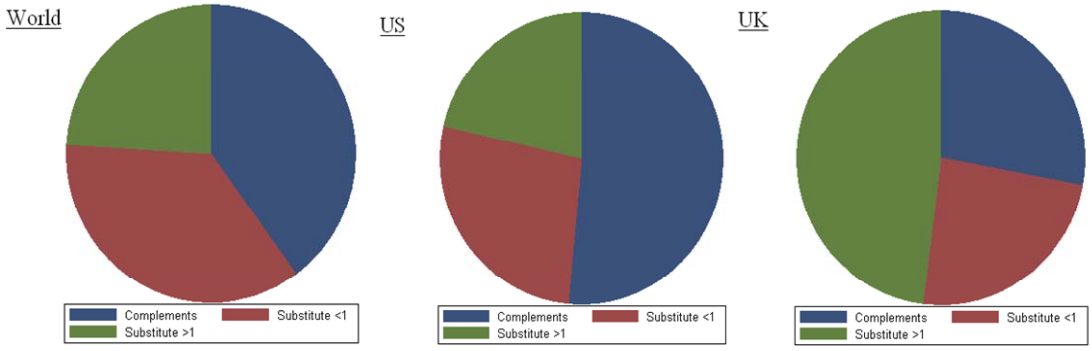
Figure 4.2 Summary of overall EoS_{KE} Results[28]



Another way of presenting these results is given in Figure 4.3. Overall, just over 40% of the estimates suggest that energy and capital are complements and 60% suggest they are substitutes. Of the latter, around 60% (representing about 35% of the total) are less than unity. Hence, 75% of the estimates suggest that energy and capital are either complements or weak substitutes.

Not surprisingly, there is a similar pattern for the USA. For the UK, however, only a quarter of the results suggest that energy and capital are complements and nearly a half suggest that energy and capital are strong substitutes (AES>1).It should be emphasised, however that these results come from only 7 papers and that about three-quarters of the estimates derive from Harris et al (1993).

Figure 4.3 Summary of EoS_{KE} results



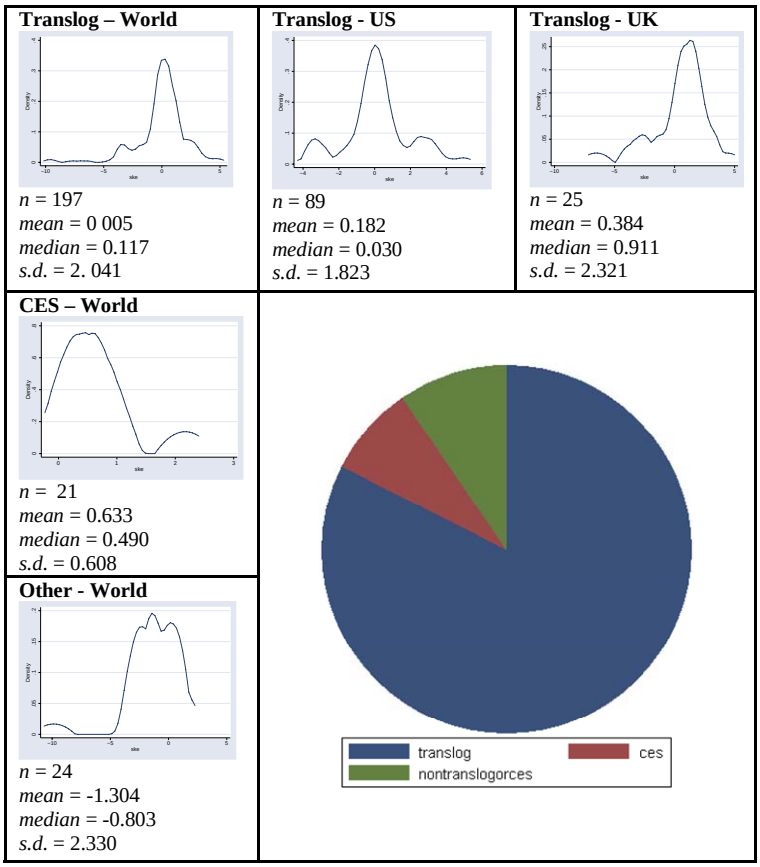
Behind these aggregate results, however, there are a range of different approaches, specifications, measurements and so on. Therefore, in order to try and understand the effect of these, the following sections classify the 'results' according to:
Form of production or cost function utilised
Form of separability (KLE vs non-KLE)
Form of technical change adopted (Hicks neutral vs non-nuetral)
Measure of EoSKE adopted
Static vs dynamic model
Type of data (time series vs cross section vs panel)
Sector analysed.

[28] 'Outliers', where EoS_{KE} exceeds plus or minus 10 are excluded from the results.

For each classification the pie charts (similar to above) are presented plus the means, medians, and distributions of the estimated EoSKE's for the overall results and (where applicable) similar information for the US and the UK. This is followed by a short discussion.

4.1.3 Functional form

Figure 4.4 Functional form (results)

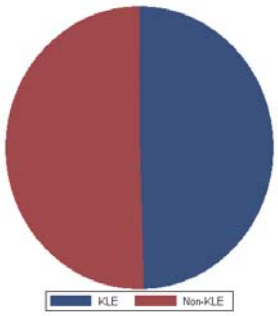


This shows that, as with the papers, the Translog specification dominates with just over 80% of the overall results derived from this specification. This is also true for the US results and for the UK all the estimates are derived from the Translog specification. The overall mean from the Translog models is about 0.005 (with a median of 0.117) suggesting that on average K and E are (very weak) substitutes although as can be seen there is a wide distribution around the average. For the US, the average estimated EoSKE from using the Translog specification is positive but quite low with a mean of about 0.18 and median 0.03, whereas for the small number of UK estimates the distribution is wider with the mean of the estimates being 0.4 but a much larger median of 0.9 – but again suggesting that on average K and E are substitutes for the UK. However, these averages here (and below) must be treated with caution and are only indicative given that the other factors below are not 'controlled for'.

For the small number of CES estimates the average overall estimated EoSKE is about 0.5 to 0.6. This suggests that K and E are substitutes, but this is only to be expected given the construction of the CES function (Annex 1). However, the estimates from the 'other' functions suggest that K and E are complements overall with the average estimated EoSKE between -0.8 and -1.3.

4.1.4 Separability assumptions

Figure 4.5 Form of separability (results)



KLE – World	KLE - US	KLE - UK
n = 115 *mean* = 0.403 *median* = 0.480 *s.d.* = 1.204	*n* = 36 *mean* = 0.172 *median* = 0.080 *s.d.* = 0.831	*n* = 23 *mean* = 0.849 *median* = 1.026 *s.d.* = 1.683
Non-KLE – World	**Non-KLE - US**	**Non-KLE - UK**
n = 127 *mean* = -0.499 *median* = -0.114 *s.d.* = 2.492	*n* = 72 *mean* = -0.145 *median* = -0.128 *s.d.* = 2.145	*n* = 2 *mean* = -4.961 *median* = -4.961 *s.d.* = 2. 205

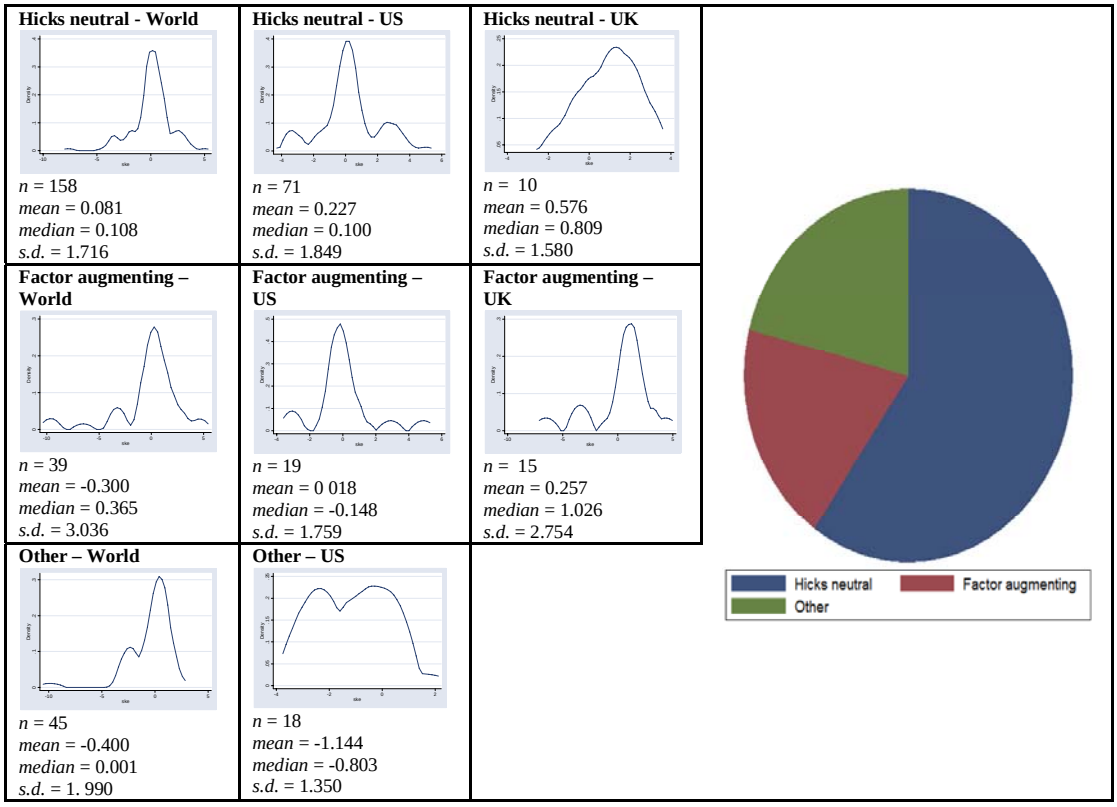
This shows that about half of the estimates use only KLE in the specification and about half use KLEM (or similar). This would appear to have an effect on the results since the average estimated EoSKE for KLE specifications is about 0.4 to 0.5 suggesting K and E are substitutes whereas the average estimated EoSKE for non-KLE specifications gives a mean of -0.5 and a median of about -0.1 suggesting that K and E are complements; although the dispersion in both is quite wide. These results are consistent with the predictions of Frondel and Schmidt (2002), in that the inclusion of materials appears to make the estimates lean more towards complementarity, possibly because the cost share for energy is reduced. The KLE formulations may therefore be vulnerable to omitted variable bias.

This distinction also looks to hold for the US and UK. The average estimated EoSKE for results from specifications including KLE only are about 0.1 and 0.8 to 1.0 for the US and UK respectively; suggesting K and E are substitutes. Whereas the average estimated EoSKE for results from non-KLE specifications are centred around -0.1 and -5.0 for the US and UK respectively; suggesting K and E are complements (although the UK figure is based on only two estimates).

In summary it would appear that the separability assumption does have an impact on the estimated elasticities. Specifically, when materials are included (i.e a particular source of omitted variable bias is avoided) there appears to be a greater tendency for energy and capital to be found to be complements. However, this is again a tentative conclusion since a range of other factors are not 'controlled for'.

4.1.5 Assumptions about technical change

Figure 4.6 Form of technical change (results)[29]



Classification of the treatment of technical change is difficult given the various explanations given by some authors. For example some authors make no mention of technical progress and hence exclude a technical progress term from their specified Translog cost function; whereas others specify that they are making the assumption Hicks neutral technical progress. However, the estimated cost shares are observationally equivalent, so for Figure 4.6 above they have all been classified together as 'Hicks neutral' as opposed to where a Translog specification is used with the explicit assumption of non-neutral technical progress. The other category therefore includes all non-Translog estimation.

It can be seen that the majority of results are based on the assumption of Hicks neutral technical change although a fair proportion assumed non-neutral (or factor augmenting) technical change. Moreover the distributions illustrate that, similar to the issue of separability above, the assumption about the form of the technical change appears to have an effect on the estimated results, although the distinction is not so clear. The avrage of the estimated EoSKE's for the Hicks-neutral specification is about 0.1 suggesting K and E are (weak) substitutes whereas for the non-neutral (factor augmenting) specification the mean estimated EoSKE is -0.3suggesting that K and E are complements whereas the median is 0.4 suggesting they are substitutes. For the US Hicks neutral specification the average estimated EoSKE suggests that K and E are substitutes with a mean 0.2 and a median of 0.1. However, for the non-neutral (factor augmenting) specification the mean of the estimated EoSKE is 0.02 suggesting that K and E are weak substitutes whereas the median

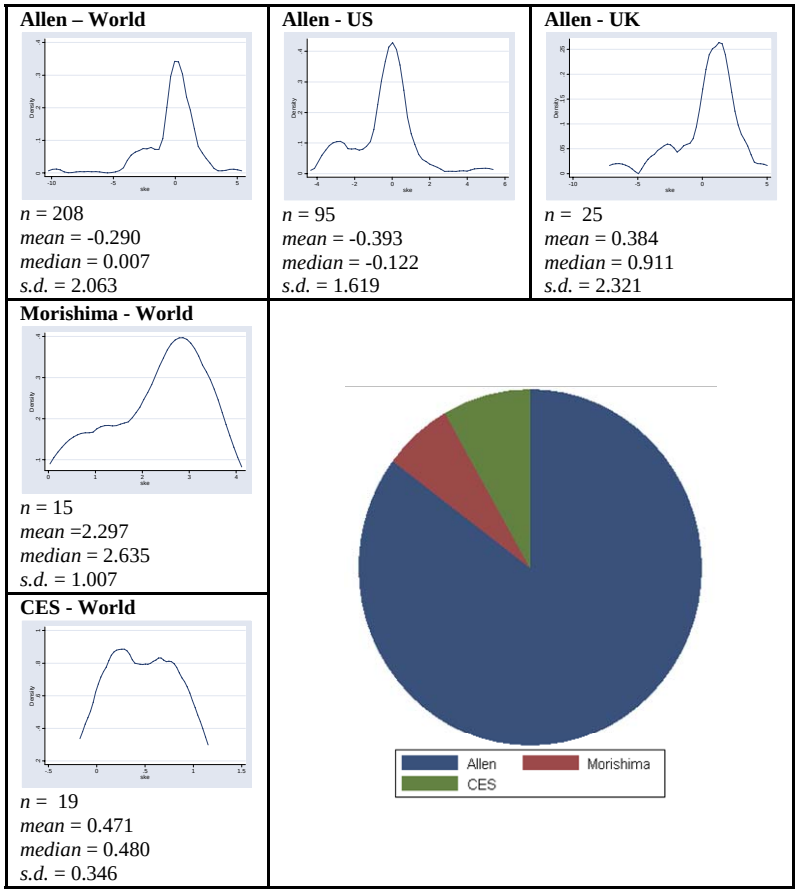
[29] For this classification, studies that employed a Translog cost function but without any statements about technical progress are grouped together with those that explicitly state the assumption of Hicks neutral technical progress, since the estimated share equations are observationally equivalent.

is -0.1 suggesting that K and E are complements. However, for the UK there is no distinction between the two specifications in that they both suggest that K and E are substitutes with the average of the estimated EoSKE's being about 0.6 to 0.8 for the Hicks neutral specification and from about 0.3 to 1.0 for the non-neutral (factor augmenting) specification.

In summary, although the different assumptions about technical change appear to have an effect on the estimated EoSKE's it is difficult to ascertain a clear pattern and would appear to depend upon the underlying data generating process.

4.1.6 Definitions of the elasticity of substitution

Figure 4.7 Measure of EoS_{KE} (results)

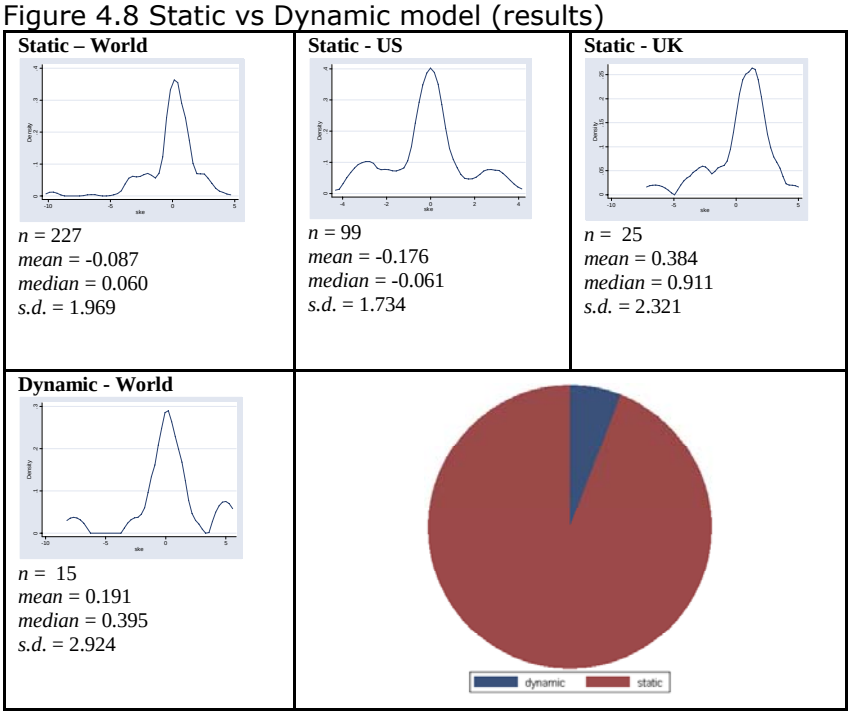


This shows that the vast majority of results are based on the Allen measure of EoSKE with about 6% based on the Morishima definition (and about 8% derived from a CES function). Given the relative small number of results not based on the AES, it is not possible to ascertain any particular strong pattern in the results. The mean of the estimated Allen EoSKE's for the overall results is -0.3, suggesting E-K complementarity, whereas the median is 0.02 suggesting (weak) E-K substitutability; but the distribution is skewed somewhat so the median is probably a better guide suggesting E-K substitutability. Interestingly within this there is a clear difference between the US and the UK results; the average estimate of the Allen EoSKE for the US is about -0.1 to -0.4, suggesting that K and E are complements, whereas the average estimate of the Allen EoSKE for the UK is 0.4 to 0.9, suggesting K and E are substitutes.

Although based on a relatively small number of results, the average estimated Morishima EoSKE is 2.3 to 2.6, suggesting strong substitutability. But this measure is expected to give higher numerical results than the Allen EoSKE for the reasons described in Section 4. Finally, the results based on the CES function (which are constrained to be positive and hence give E-K substitutability) give an average estimated EoSKE of 0.5.[30]

4.1.7 Static versus dynamic estimation

Figure 4.8 Static vs Dynamic model (results)

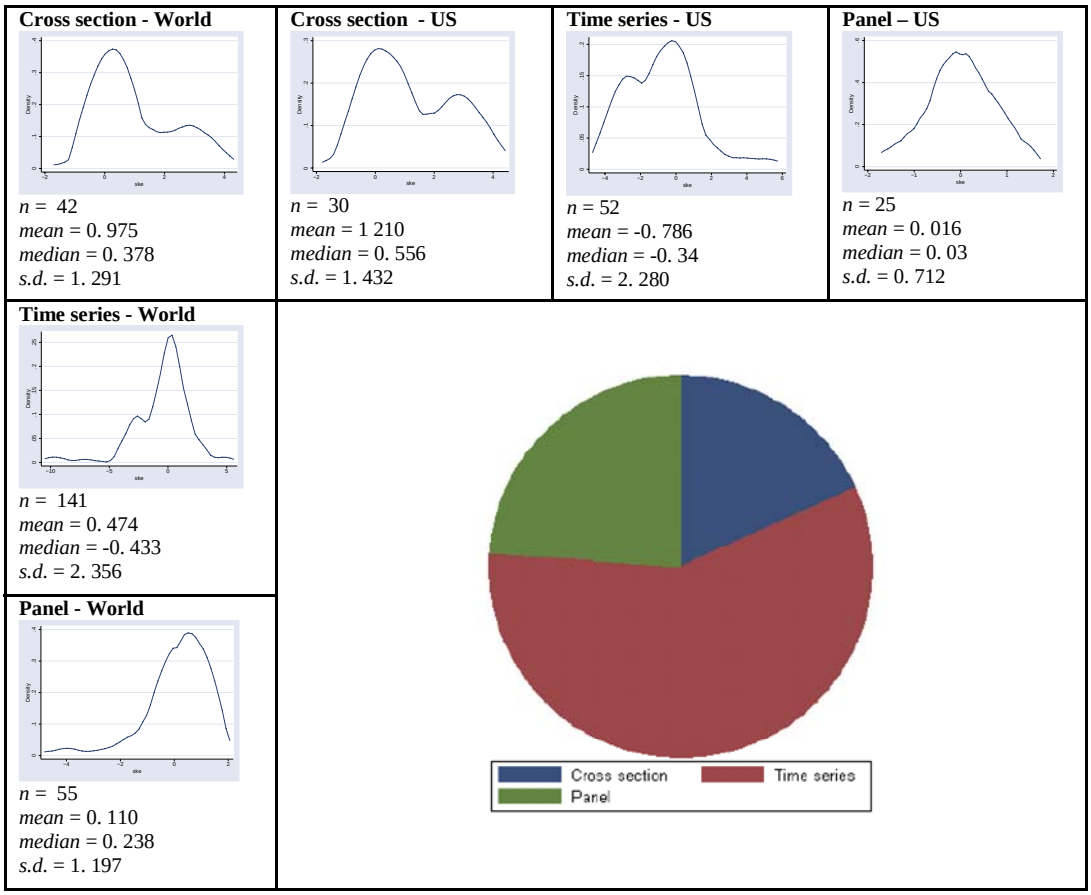


This shows that the results are based overwhelmingly on static models, hence it is not possible to ascertain any particular pattern given the small number of results based upon a dynamic specification. The dominance of static models is unfortunate, given both the advances in time series econometrics and the potential difficulties with static models highlighted by Frondel and Schmidt (2002).

[30] It is worth noting that Thompson and Taylor (1995) re-calculate a large number of MES_{KE} estimates for previous studies that had used the AES_{KE} and find that "out of 148 estimates of the MES only 4 are negative" (p. 566). These results are not included in the above given they use data from previous studies.

4.1.8 Type of data

Figure 4.9 Type of data (results)

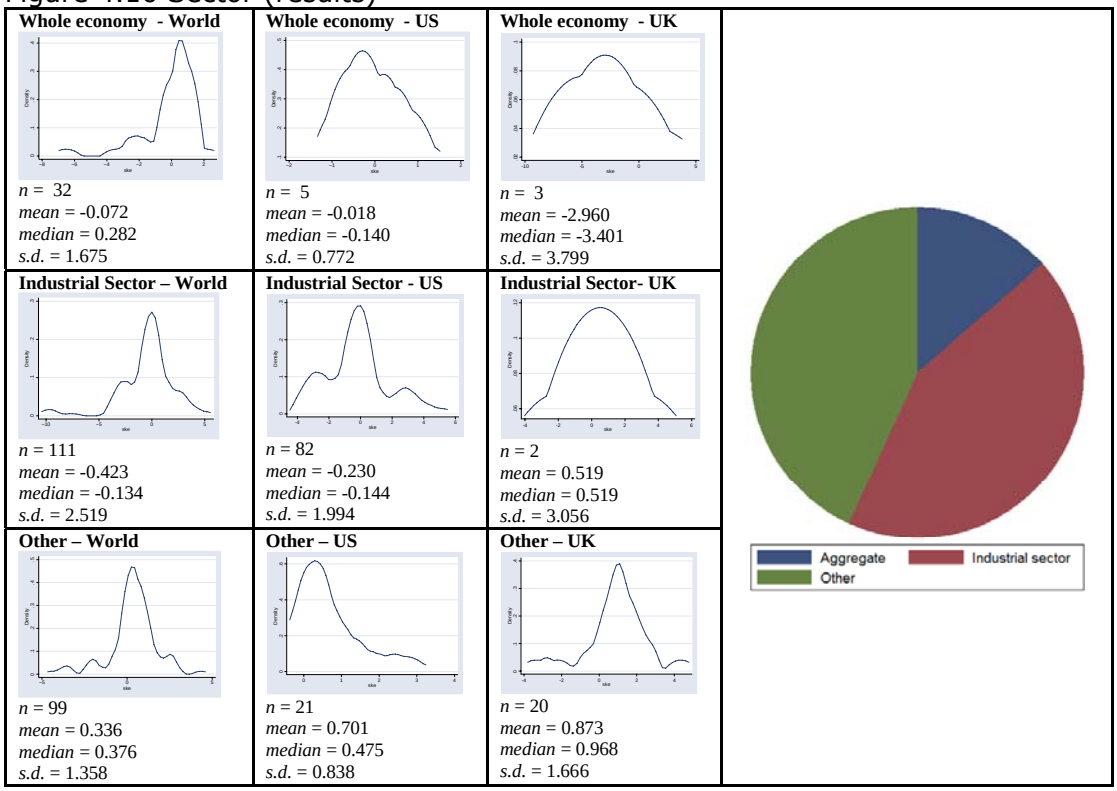


This shows that the majority of results derive from single country estimates based upon time series data. However, about 18% are based upon cross section multi country studies and about 24% based upon panel multi country studies. When considering the overall results it would appear that the choice of data is important. The average for the estimated EoSKE from time series results is about -0.1 to -0.4 whereas for the results from cross-section studies it is 0.4 to 1.0 and for the results from panel studies it is 0.1 to 0.2. Hence, time series studies on the whole suggest E-K complementarity whereas the multi country cross section and panel studies suggest E-K substitutability. While this could be suggestive of greater substitutability in the long-run (as represented by cross-section studies) compared to the short run (as represented by time series studies), there are several difficulties with this interpretation and the result may equally well reflect the more common inclusion of materials in time series studies (Section 3.2.3).

Similar results are found for the US, with an average estimated EoSKE of -0.3 to -0.8 and 0.6 to 1.2 for time series and cross section studies respectively. For the panel study results from the US, both the mean and median are close to zero. No UK distinction is given since all bar one of the UK estimates for EoSKE come from single country time series studies.

4.1.9 Analysis by sector

Figure 4.10 Sector (results)



Classification of results by sector needs to be viewed with caution since the interpretation of the distributions and data are not straightforward. The definitions of sectors are not homogenous with respect to different countries and cannot necessarily be directly compared on such a basis. As a result, the information presented should be seen as 'indicative' compared to the distributions given in previous figures where it is clearer how to classify the different groupings. With these caveats in mind the data is categorised into the two predominant categories analysed, 'whole economy' and 'industrial sector'[31], plus 'other'[32].

The UK whole economy figures suggest a strong complementary relationship between K and E, whereas for the US a weak complementary relationship is suggested, but for the whole world the distribution is concentrated around zero with a negative mean (suggesting complements) but with a positive median (substitutes); although the tail of the probability goes much deeper into the negative (complements) zone.[33] For the industrial sector, the average estimate suggests substitutability (but is based on only two estimates) however the averages for the US and the world industrial sectors suggest complementarity, with both the means and the medians being in the negative part of the distribution. The 'other' sector however, shows far more consistent evidence of substitutability between K and E, but again it must be emphasised that, given the way this category is formed by the bundling together of everything not specifically whole economy or the industrial sector, this should be treated with caution.

[31] Note the 'Industrial Sector' includes those studies that have stipulated that they have estimated the EOS_{KE} for the industrial sector as a whole and/or the manufacturing sector.

[32] These other sector cover a range of industries. With too few observations to draw meaningful conclusions on their own, they are bundled together.

[33] However, it must be stressed that these distributions are derived from small numbers of observations.

4.1.10 Summary

Estimates of the elasticity of substitution between energy and capital appear to be highly variable between sectors, countries and time periods. The results demonstrate no clear consensus regarding whether energy and capital may be regarded as either substitutes or complements. Different studies have analysed different sectors using different model specifications, data sets and methods of estimation and have often come to quite different conclusions. While this may be expected if the degree of substitutability depends upon the sector, level of aggregation and time period analysed, it is notable that several studies reach different conclusions for the same sector and time period, or for the same sector in different countries. Perhaps the most telling example of this is that according to (Raj and Veall, 1998) studies using the original Berndt and Wood (1975) data have produced 38 different estimates of EoSKE, ranging from -3.94 to 10.84.

If a general conclusion can be drawn, it is that energy and capital appear to be either complements (AESKE<0) or weak substitutes (0<AESKE<0.5). If capital and energy are AES complements, then increases in energy prices are likely to lead to reductions in capital use as well as energy use, thereby increasing the economic impact of the price increase. If capital and energy are weak substitutes (i.e. with a small AES), then a large amount of capital may need to be substituted to maintain a constant flow of services from capital and energy combined.

However, it is clear that the assumptions made by the modellers can have a considerable influence on the results obtained. Although there are some indications that the assumptions about separability and technical change may be particularly significant, it is not possible to come to a definitive conclusion since it is not possible to control for all the relevant influences.

The following section therefore attempts to classify the reasons suggested in the literature for the widely different results.

4.2 Possible reasons for the different results

Not surprisingly, various authors have proposed different reasons for the wide variation in empirical results for EoSKE. The first notable attempt was that by Berndt and Wood (1979) who attempted to reconcile their 1975 results of E-K complementarity for the US using time series data with that of Griffin and Gregory (1976) who found E-K substitutability in a multi-country cross section study. According to Griffin and Gregory, the time series results were estimating the short run EoSKE whereas the cross section results were estimating the long-run EoSKE. In contrast, Berndt and Wood (1979) argued that there was an important distinction between 'gross' and 'net' substitutes/complements, as described in Section 2 and that Griffin and Gregory's omission of materials had biased their results. Griffin (1981) contested this explanation, pointing out (amongst other things) that omitting materials from Berndt and Wood's data still led to a finding of E-K complementarity. Berndt and Wood (1981) replied that Griffin's use of the AES rather than the CPE was confusing matters and introduced an alternative explanation, namely conceptual differences in the measurement of capital. In the intervening 25 years, this debate has not been satisfactorily concluded.

The divergence in results has also been suggested as deriving from:

differences in substitution possibilities between different sectors;

differences in substitution possibilities between different levels of aggregation (Section 3.2.2);

inappropriate separability assumptions and omitted variable bias (Section 3.2.1);

differences in cost shares when static Translog cost functions are estimated (Section 3.2.3)

the time period covered by the data, including the level of energy prices and the asymmetrical response to price changes (Kuper and Soest, 2002);

the estimation method chosen; and

differences in the treatment of technical change.

As an example of the latter, Hunt (1986) and Harris et al (1993) argue that if non-neutral technical change is accepted by the data then the results of E-K complementarity and substitutability can be reversed if Hicks neutral technical change is assumed. In a different context, Ochsen (2002) shows that the usual finding of complementarity between capital and high-skilled labour no longer holds once non-neutral technological progress is allowed for. However, nearly two thirds of the studies reviewed above simply assume Hicks-neutral technical change.

Another explanation is provided by both Field and Grebenstein (1980) and Miller (1986), who suggest that variations in the definition and aggregation of capital inputs may account for the different results. The manner in which different types of energy input are aggregated could also influence the empirical estimates, although this is rarely discussed. Technical Report 5 demonstrates that the standard method of aggregating energy inputs on a kWh basis can lead to misleading results and argues instead for the use of the Divisia index, in which different energy carriers are weighted according to their relative marginal productivities (Berndt, 1978).

Since the calculation of the AES depends upon the factor cost shares at every data point, this may be another reason for divergence given that most studies average the partial elasticity over time. However, Anderson and Thursby (1986) suggest that this is the best approach, since the means of factor cost shares are likely to be distributed normally hence allowing for standard inference. But on the other hand the distribution of point estimates may not be normal, thereby making inference problematic.

Table 2 attempts to 'quantify' the reasons put forward in the reviewed literature for explaining the different results. This distinguishes what might generally be called 'data issues' from 'technical issues'. This shows that data issues are a commonly cited reason for differences in estimated results. Furthermore, within the technical issues it is clear that homogeneity, separability, technical change and functional form are commonly cited.

Table 4.1 Summary of reasons offered in empirical studies for divergence in model results.

Reason	Times cited
Data Issues;	
Relative Factor Shares/Prices	42
Data Type	11
Level of Aggregation	11
Technical issues;	
Homogeneity	21
Separability	18
Technical change	17
Functional Form	10
Dynamic Specification	10
Elasticity Measure Used	4
Other34	24
Total	168

It is our conclusion that there is a degree of agreement to the possible causes of the different estimated elasticities. However, there is no real consensus on either the relative importance of different causes, or (at least in most cases) on the likely direction of influence of each individual cause (i.e. whether a particular specification/assumption is likely to make the estimate of the substitution elasticity bigger or smaller). Moreover, there is no clear agreement in the literature to what is the 'best' way to proceed in estimating such relationships. Given both the range of possible influencing factors that have been identified and the apparent sensitivity of the results to those factors, the empirical literature on this topic appears to be far from robust.

To improve upon this situation, it is suggested that a 'general to specific' approach be utilised that 'lets the data speak'. This means that specific restrictions (such as homogeneity, Hicks-neutral technical change, separability, etc.) should only be imposed if they are statistically accepted by the data. This means that issues such as homogeneity vs heterogeneity, separability vs non- separability, neutral vs non-neutral technical change and so on need to be tested for and only accepted on empirical grounds. Similarly, in a time series context, a dynamic model should be utilised unless a static model (implying that the short-run elasticities are equal to the long run elasticities) is shown to be statistically accepted by the data. It is to be expected that different economies and sub-sectors will have different production structures and it is correspondingly wrong to assume that a particular theoretical structure will be appropriate in all cases.

Unfortunately, the great majority of the existing empirical studies do not follow these recommendations. As a consequence many of the estimates reviewed above may well be biased.

Before leaving this overview, it is worth considering two other papers that review different estimates of EoSKE and address the issue of Allen (AESKE) versus Morishima (MESKE) elasticities of substitution.

Thompson and Taylor (1995) re-calculate a large number of MESKE estimates for previous studies that had used the AESKE and find that out of 148 estimates of the MES only 4 are negative. They conclude from this that capital and energy are MES substitutes, but this is only to be expected given the definition and behaviour of the MES, as discussed in Section 3.1.6. Since the MES means something different from the AES (and CPE) it is arguably inappropriate to use the sign of the MES to distinguish substitutes and complements in this

[34] 'Other' represents categories that received recognition by less than five papers (with the exception of the specification of capital (cited 9 times) and elasticity measure to enable consistency of this table with the previous pie charts and density plots). Reasons offered in this category include: estimation method, business cycle effects, developing countries effect and risk aversion among others.

way. Of perhaps greater interest is Thompson and Taylor's finding that for MESKE there is no dichotomy between cross-section and time series studies and there is no excessive variability in the quantitative estimates. The latter may result from the fact that the MES is less sensitive to the small and variable share of energy in total costs. Thompson and Taylor also find that the mean estimate of MES is larger when the price of energy adjusts (MESEK=1.01) than when the price of capital adjusts ((MESKE=0.76), which suggests which suggests the interesting policy conclusion that an increase in energy prices may induce a larger improvement in energy efficiency than policies such as investment tax credits.

Koetse et al. (2007)[35] used over 30 studies and (re)calculated the MESKE and CPEKE from the estimates provided in the original papers and employed these in a meta-analysis. Echoing the arguments of Griffin and Gregory (1976), they argue that time series data reflects short run substitution possibilities and cross-sectional data reflects long-run possibilities. On that basis, the CPE results suggest that energy and capital are weak substitutes in the short run (0.17) and slightly greater substitutes in the long-run (0.52). Their meta-analysis also demonstrates that model assumptions regarding returns to scale, technological change, separability, aggregation of variables, type of data used and so on, can strongly influence the estimated results; thus reinforcing the above conclusions.

4.3 Summary

This section has reviewed empirical estimates of the elasticity of substitution between aggregate energy and aggregate capital and has explored how these estimates vary with factors such as the functional form and assumptions regarding technical change. The most striking result from the analysis is the lack of consensus that has been achieved to date, despite three decades of empirical work. While this may be expected if the degree of substitutability depends upon the sector, level of aggregation and time period analysed, it is notable that several studies reach different conclusions for the same sector and time period, or for the same sector in different countries.

If a general conclusion can be drawn, it is that energy and capital typically appear to be either complements (AES<0) or weak substitutes (0<AES<0.5). However, little confidence can be placed in this conclusion, given the diversity of the results and their apparent dependence upon the particular specification and assumptions used. While there appears to be some agreement on the possible causes of the different results, there is no real consensus on either the relative importance of different causes, or the likely direction of influence of each individual cause (i.e. whether a particular specification/assumption is likely to make the estimate of the substitution elasticity bigger or smaller).

A key weakness of many of the existing studies is that specific restrictions (such as homogeneity, Hicks-neutral technical change, separability) are assumed rather than statistically tested. This suggests that any future work should ensure that such assumptions are tested for and only accepted on empirical grounds.

[35] This became available when the present report was nearing completion.

5 Elasticities of substitution and the rebound effect

This section explores the relevance of empirical estimates of elasticities of substitution to the rebound debate. It first summarises the manner in which the elasticity of substitution has appeared within the rebound literature and then explores its relationship to the rebound effect in more detail, considering in turn: a) the relationship between empirical estimates and the requirements of energy-economic models; b) the importance of separability assumptions and nesting structures; c) and the distinction between real and 'effective' energy and the appropriate modelling of technical change.

It is argued that the relationship of this parameter to the rebound effect is more subtle than is commonly assumed. In particular a finding of limited substitution or even complementarity between aggregate energy and capital may be compatible with backfire – the opposite to what some authors have suggested (Jaccard and Bataille, 2000). Moreover, existing empirical estimates of elasticities of substitution appear to be of relatively little value for either quantifying rebound effects or calibrating energy-economic models. Nevertheless, they do suggest that energy price increases may have greater economic effects than is suggested by the small share of energy in total costs.

5.1 Elasticities of substitution in the rebound literature

Statements regarding the magnitude of the elasticity of substitution between energy and other inputs appear regularly in the rebound literature. For example:

> "It appears that the ease with which fuel can substitute for other factors of production (such as capital and labour) has a strong influence on how much rebound will be experienced. Apparently, the greater this ease of substitution, the greater will be the rebound" (Saunders, 2000, p. 443).

However, the relationship between this parameter and the magnitude of rebound effects may be more subtle than it first appears.

The primary reason for the prominence of elasticities of substitution in the rebound debate is the attention paid to this parameter by Saunders (1992; 2000b; 2000a; 2007). As discussed in Technical Report 5, Saunders has made a significant contribution to the rebound literature by using neoclassical production theory and neoclassical growth theory to explore the consequences of 'energy augmenting technical change' (i.e. a form of technical change that solely improves the productivity of energy inputs). For example, Saunders (1992) uses an aggregate CES production function within a neoclassical growth model to show that both neutral and energy augmenting technical change will lead to backfire if the HES between energy and a composite of non-energy inputs is greater than unity (i.e. if energy is a strong substitute for other inputs). In more recent work with a range of functional forms, Saunders (2007) again shows that elasticities of substitution appear to be a key determinant of rebound effects.

Saunder's results suggest that a high HES between energy and a composite of other inputs could lead to large rebound effects, while a small HES could lead to small rebound effects. Following this lead, Jaccard and and Bataille (2000) use a technology simulation model to estimate an economy-wide value for the HES between energy and capital for the Canadian

economy (i.e. not necessarily the same thing). Finding a fairly low value of 0.24, they conclude that rebound effects should be relatively small. However, as argued below, this conclusion is possibly oversimplified and possibly incorrect.

A second reason for the prominence of elasticities of substitution in the rebound debate is the role they play in determining the results of Computable General Equilibrium (CGE) models of the macroeconomy. The economy-wide impact of energy efficiency improvements cannot be adequately captured within a partial equilibrium framework, but might be usefully explored through a CGE approach. However, the realism and policy relevance of CGE models is the subject of fierce debate, with different models producing widely varying results for very similar policy questions (Greenaway, et al., 1992; Barker, 2005).

The CGE literature on rebound effects is reviewed in detail in Technical Report 4. A key conclusion from this review is that the assumptions made for the elasticities of substitution between energy and other factors of production, as well as between different types of energy carrier, can have a significant influence on the model results. As an illustration, Grepperud and Rasmussen (2004) estimate rebound effects to be higher in the Norwegian primary metals sector than in the fisheries sector, owing largely to the greater opportunities for factor substitution in the former (Grepperud and Rasmussen, 2004).

However, elasticities of substitution are normally specified exogenously and can vary significantly from one CGE model to another. In principle, these parameters are established through careful surveys of relevant empirical literature, but in practice, the time periods, regions and sectors on which this literature is based may not be appropriate to the model application and there may also be substantial differences between empirical studies and model requirements in areas such as the aggregation of different inputs and the functional form of production functions. Furthermore the process of compiling parameter values is rarely transparent, sensitivity tests are uncommon and the modelling usually rests on the implicit assumption that such parameters will remain stable over time. All these features suggest that the results of such models should be interpreted extremely cautiously.

The CGE literature also suggests that the scope for substitution between energy and other inputs has a major influence on the cost of reducing carbon emissions.[36] This makes the elasticity of substitution between energy and non-energy inputs (EoSEN) a critical parameter in such models and suggests a potential policy trade-off:

> "…If one believes EoS_{EN} is low, one worries less about rebound and should incline towards programmes aimed at creating new fuel efficient technologies. With low EoS_{EN} carbon taxes are less effective in achieving a given reduction in fuel use and would prove more costly to the economy. In contrast, if one believes EoS_{EN} is high, one worries more about rebound and should incline towards programmes aimed at reducing fuel use via taxes. With high EoS_{EN}, carbon taxes have more of an effect at lower cost to the economy." (Saunders, 2000b)

However, Allan et al (2006) argue that this conclusion may be oversimplified:

> " ….the conclusion that there is consequently a key policy trade-off, so if this elasticity of substitution is low one worries less about rebound…. is strictly incorrect. The problem here is that rebound... does not depend only on the elasticity of substitution of energy for other

[36] For an insightful discussion of this point, see Hogan and Manne (1970).

> inputs. Indeed, even if this elasticity is precisely zero, as with Leontief technology, rebound and backfire remain perfectly feasible conditions, if less likely. There appears to be a widespread, but mistaken, belief in the literature that low elasticities of substitution between energy and other inputs imply that rebound must be small and backfire impossible." (Allan, et al., 2006)

The point here is that substitution effects are isolated by holding output constant. But in practice, output will not be fixed and the net effect of an energy efficiency improvement on the demand for factor inputs (including energy) will depend upon the overall reduction in production costs and the price elasticity of output (at least at the level of individual sectors). This will tend to increase energy demand further than is indicated by the substitution effects alone, so the rebound effect will be larger. The overall rebound effect will therefore also depend upon the cost share of energy and the price elasticity of the relevant product. Similar observations are made by Grepperud and Rasmussen (2004):

> "Our analysis has shown that significant rebound effects are at work in industries with limited substitution possibilities (metals) and that rebound effects can be weak in spite of a flexible technology (chemical and mineral products). The degree of substitutability among inputs (or aggregation level) is important but not essential for the results arrived at. An equally important factor is the degree to which the activity level in a sector is affected by efficiency improvements. The rebound that originates from this activity effect is not given sufficient attention in the literature." (Grepperud and Rasmussen, 2004)

Nevertheless, both Saunders' work and the CGE literature suggests that a better understanding of elasticities of substitution could offer some insight into the likely magnitude of rebound effects in different circumstances. Unfortunately, the parameters estimated in the empirical literature and reviewed in Section 4 may not be as useful for this purpose as may be expected.

5.2 Empirical estimates and modelling requirements

The first problem is that empirical studies frequently differ from theoretical/modelling studies along a number of dimensions, thereby creating problems in using empirical estimates to infer appropriate parameter values for CGE models. In particular modelling studies:

- frequently differ from empirical studies in the manner in which individual inputs are aggregated and in the level of sectoral aggregation.
- frequently assume separability between different groups of inputs, while most empirical studies do not.
- normally use the nested CES functional form, while most empirical studies use less restrictive functional forms such as the Translog.
- frequently require estimates of the elasticity of substitution between nests of inputs, while the parameters estimated by most empirical studies relate to individual pairs of inputs.
- define production functions by means of the HES, while most empirical studies estimate the AES, the CPE or the MES.

The differences in functional form and in the definitions of the elasticity of substitution deserve further examination. Blackorby and Russell (1981) show that the AES, MES and HES are identical if (and only if): a) there are only two inputs to the production function; b) the production function has a Cobb Douglas structure; or c) the production function has a non-nested CES structure. The two-input case is of no interest for our purposes, while the Cobb-Douglas structure is excessively restrictive since it assumes that all inputs are substitutes with an elasticity of substitution equal to unity.[37] Of potentially greater interest is the non-nested CES structure, which combines factor inputs in the following way (McFadden, 1963):

$$Y = (a_K K^{-\rho} + a_L L^{-\rho} + a_E E^{-\rho} + a_M M^{-\rho})^{-1/\rho} \quad (5.1)$$

With this structure, the HES (and also the AES and MES) between all inputs is identical and greater than zero - implying that all inputs are equal substitutes for each other. This appears unlikely in practice and also excludes the possibility of complementarity (i.e. AES<0) between factor inputs.

In order to provide greater flexibility in substitution possibilities (while at the same time ensuring computational feasibility) most CGE models use a nested CES functional form, in which pairs of inputs are combined together in a CES function, which then combines with another factor in a second CES function (Sato, 1967). For example, a KLEM production function could take the form:

$$Y = [a(bK^{\alpha} + (1-b)E^{\alpha})^{\frac{\rho}{a}} + (1-a)(cL^{\beta} + (1-c)M^{\beta})^{\frac{\rho}{\beta}}]^{\frac{1}{\rho}} \quad (5.2)$$

Which may also be written as:

$$Y = g[K^*, L^*] \quad (5.3)$$

This is called a two-level nested CES because it contains CES production functions for two intermediate inputs (K* and L*) embedded within a CES production function for aggregate output (Y). The intermediate inputs in this case are a composite of capital and energy inputs ('utilised capital' – K*) and a composite of labour and materials inputs (L*), which are assumed to combine with each other to provide the final output (Y). The assumption is that producers engage in a two-stage decision process: first optimising the combination of factors required to produce utilised capital and the labour/materials composite, and then optimising the combination of these composite inputs required to produce the final output. The relevant CES functions are:

$$K^* = (bK^{\alpha} + (1-b)E^{\alpha})^{\frac{1}{a}} \quad (5.4)$$

$$L^* = (cL^{\beta} + (1-c)M^{\beta})^{\frac{1}{\beta}} \quad (5.5)$$

$$Y = [a(K^*)^{\rho} + (1-a)(L^*)^{\rho}]^{\frac{1}{\rho}} \quad (5.6)$$

The nesting structure (although not the functional form) is the same as used by Berndt and Wood (1979) and described in the graphical example of Section 2. As discussed in Sections 2 and 3, this type of production function rest on the assumption that utilised capital (K*) is

[37] Despite the obvious limitations of the Cobb Douglas formulation, it is still widely used.

separable from the labour/materials composite (L*), which may not be correct (Berndt and Christensen, 1973). Alternative nesting schemes (such as (KL)(EM) or (EL)(KM)) are available and the choice between them in CGE models may be made on theoretical grounds or (less commonly) tested empirically (Kemfert, 1998).[38] If a distinction is made between different types of capital, labour or energy inputs (e.g. electricity and non-electricity), a multilevel CES can be formed, with more than one function nested within the original one (Chang, 1994).

With a nested CES structure, the AES between a pair of inputs belonging to different nests is equal to the HES between the nests. In fact, the equality of the AES between pairs of inputs in different nests is a general result for a production function in which one nest of inputs is separable from a second group (Berndt and Christensen, 1973). For example, for the (KE)(LM) nesting structure implies that: AESKL = AESKM = AESEL = AESEM.. In the case of a nested CES function, these are also equal to HESK*L*.

In contrast, with a nested CES structure the AES between a pair of inputs belonging to the same nest is not equal to HES between those inputs. Indeed, while two inputs within an individual nest are necessarily HES substitutes, they may at the same time be AES complements. This is precisely what Berndt and Wood (1979) showed in their illustrative example, where capital and energy combine in an individual nest and are shown to be AES complements. The AES between these two inputs is only equal to the HES if the output of the nest is held constant (K*). In Berndt and Wood's terminology, the gross elasticity between these two inputs (with the output of the nest held constant) is different from the net elasticity (where the output of the nest, K*, is allowed to vary, while still holding output, Y, constant). Again, this inequality of gross and net elasticities is a general result for a production function in which one nest of inputs is separable from a second group.

Sato (1967) has shown how the AES between a pair of inputs belonging to the same nest depends upon the HES within the nest, the value share of that nest and the HES between the nests. This is analogous to Equation 2.14, which relates Berndt and Wood's net price elasticity and gross price elasticity (Anderson and Moroney, 1994). For example, in the case of capital and energy, with the production structure given by Equation 5.2 ((KE)(LM)), Sato (1967) shows that:[39]

$$AES_{KE} = HES_{K^*L^*} + \frac{1}{a}(HES_{KE} - HES_{K^*L^*}) \qquad (5.7)$$

Hence, it is possible for AES_{KE} to be negative (i.e. capital and energy to be AES complements) even if HES_{KE} is positive (i.e. capital and energy are HES substitutes in the subfunction) and $HES_{K^*L^*}$ is positive (i.e. utilised capital and the labour/materials composite are HES substitutes in the master function). A necessary condition for this is that $HES_{K^*L^*} > HES_{KE}$ - or in other words, the scope for substitution between utilised capital and the labour/materials composite must be greater than the scope for substitution between energy and capital in the production of utilised capital. A sufficient condition is that:

[38] For example, van der Werf (2006) tests different nesting structures for a *KLE* CES production functions for OECD manufacturing and finds that a *(KL)E* structure is preferred.

[39] Substituting parameter values gives: $AES_{KE} = \frac{1}{1-\alpha} + \frac{1}{a}\left[\frac{1}{1-\alpha} - \frac{1}{1-\rho}\right]$

$\left|(HES_{K^*L^*} - HES_{KE})/a\right| > HES_{K^*L^*}$, which suggests that AES_{KE} is more likely to be negative when the share of utilised capital in the value of output (a) is small.

The main point is that empirical estimates of the AES between two inputs do not easily translate into the HES parameters required for the nested CES production functions used in CGE models. If the separability assumptions were valid, a particular nested CES could be parameterised if the function was estimated directly. But the majority of empirical studies estimate Translog rather than nested CES functions and do not impose separability restrictions. Moreover even if separability restrictions are imposed with a Translog, these do not ensure that estimates of the AES between two factors are invariant to the price of other factors. This requires Frondel and Schmidt's (2004) stricter condition of 'empirical dual separability'. Furthermore, even if empirical dual separability is found to hold, the implied nesting structure may not correspond to that required within a particular energy-economic model. The links between empirical estimates of elasticity the substitution and the values required by CGE models therefore appears to be fairly tenuous.

The diversity of approaches to factor nesting is illustrated in Table 5.1, which compares nesting structures and assumed values of elasticities of substitution in a number of contemporary CGE models. Over half of these models exclude materials inputs, so therefore implicitly assume that these are separable from other inputs. These models vary further in terms of how they disaggregate and nest individual inputs (e.g. fuel and electricity) and how they model technical change. The basis for the assumed values for the HES between different inputs and nests of inputs is rarely made clear and the values chosen appear to vary widely between different models (although in all cases, the HES are assumed to be less than or equal to unity).

Table 5.1 Nesting structures and assumed elasticities of substitution in contemporary CGE models

Authors	Nesting structure	Assumed values for HES
Bosetti et al	(KL)E	HESK,L=1.0 HESKL,E=0.5
Burniaux et al (1992)	(KE)L	HESK,E=0 or 0.8 HESKE,L=0 or 1.0
Edenhofer et al (2005)	KLE	HESK,L,E=0.4
Gerlagh and van der Zwaan (2003)	(KL)E	HESK,L=1.0 HESKL,E=0.4
Goulder and Schneider (1999)	KLEM	HESK,L,E,M=1.0
Kemfert (2002)	(KLM)E	HESKLM,E=0.5
Manne et al (1995)	(KL)E	HESKL=1.0 HESKL,E=0.4
Popp (2004)	KLE	HESK,L,E=1.0
Sue Wing (2003)	(KL)(EM)	HESK,L=0.68 to 0.94 HESE.M=0.7 HESKL,EM=0.7

Source: van der Werf (2006)

5.3 Separability assumptions, nesting structures and energy services

There is a comparable gap between the focus of empirical studies and the requirements of Saunders' theoretical models. Saunders (1992; 2006b) follows Hogan and Manne (1970) in adopting a three factor CES production function in which energy combines with a composite of capital and labour inputs ('value added'). The elasticity of substitution between capital and labour is assumed to be unity (i.e. a Cobb Douglas function) and 'energy efficiency' improvements are represented by a multiplier for energy augmenting technical change ($\upsilon_E(t)$):[40]

$$Y = \upsilon_N \left\{ a\left[(K)^{\gamma}(L)^{1-\gamma})\right]^{\rho} + b(\upsilon_E(t)E)^{\rho} \right\}^{\frac{1}{\rho}} \qquad (5.8)$$

Saunders then examines the conditions under which energy augmenting technical change will lead to absolute reductions in energy consumption and concludes that this will only occur when the magnitude of the HES between energy and the capital-labour composite ($HES_{KL,E} = 1/(1-\rho)$) is less than unity. This conclusion depends upon the particular nesting structure assumed and Saunders (1992) argues that alternative nesting structures of the same function (i.e. (KE)L and (LE)K) will always lead to backfire, regardless of the HES between the nests.

Since capital and energy are in separate nests in this formulation, the AES between them should be equal to the HES between the nests. This suggests that an estimate of the AES between capital and energy could provide some information on the likelihood of backfire. But this result depends (amongst other things) upon the particular nesting structure that is assumed - and hence on the validity of the separability assumptions. The implication of Saunders' nesting structure is that the AES between energy and capital is the same as that between energy and labour (Berndt and Christensen, 1973). But most empirical estimates with Translog cost functions find these parameters to be different, suggesting that the (KL)E nesting structure is a poor representation of actual production relationships.

The implications of this may be made clearer by reconsidering Berndt and Wood's (1979) graphical illustration of energy-capital complementarity that was summarised in Section 2. This assumed a (KE)(LM) nesting structure, since this was the only separability restriction that was supported by Berndt and Wood's (1975) data.[41] In contrast to Equation 5.8, this has energy and capital in the same nest. For the purposes of exposition, this graphical example may be taken as illustrative of the behaviour of a (KE)(LM) nested CES function under the assumption that $\left|(HES_{K^*L^*} - HES_{KE})/a\right| > HES_{K^*L^*}$.

Although Berndt and Wood (1979) do not use the term, their graphical exposition also provides a useful illustration of rebound effects. Their independent variable is a reduction in the cost of capital, such as could be achieved through the introduction of investment tax credits or accelerated depreciation allowances. This encourages a substitution between capital and energy in the production of 'utilised capital' (Figure 2.4). If energy efficiency (or energy productivity) for this sub-function is defined as the ratio of utilised capital output to energy inputs (E/K*), then the energy productivity of the sub-function has increased as a

[40] See Technical Report 5 for definitions and further discussion.

[41] Various separability assumptions may be tested by imposing restrictions upon the parameter values in the Translog cost function.

result. But an improvement in some measure of energy efficiency, or energy productivity, is also the relevant independent variable for the rebound effect.[42]

The reduction in the cost of capital leads to a corresponding reduction in the cost of utilised capital and hence an increase in demand for utilised capital – even while output is held constant. The corresponding increase in energy consumption offsets the reduction in energy consumption brought about by the energy efficiency improvement. In Berndt and Wood's example, overall energy demand is increased since the expansion elasticity is greater in magnitude than the gross price elasticity. This means that energy productivity (Y/E) for the producer as a whole has fallen, even though energy productivity in the production of utilised capital (K*/E) has increased. Put another way, their results suggest that a particular type of energy efficiency improvement – represented by and increase in K*/E and stimulated by policies such as investment tax credits - is likely to lead to backfire.[43]

In Berndt and Wood's example, if substitution between capital and energy is large (small) compared to the substitution between utilised capital and the labour/materials composite, then the rebound effect (holding output constant) should be small (large). Put another way, if the gross price elasticity is large (small) compared to the expansion elasticity, then the rebound effect should be small (large). The expansion elasticity, in turn, is given by the product of the own price elasticity for utilised capital (assuming fixed output) and the cost share of the factor whose price has changed (Equation 2.14). This relates to the more general conclusion from the rebound literature, that rebound effects are large when the demand for 'energy services' is elastic. In this case, 'energy services' is another name for the capital/energy composite that Berndt and Wood term utilised capital (K*).

Nonetheless, while the Berndt-Wood example gives some insight into the determinants of the rebound effect, it appears to contradict Saunders' conclusion that rebound effects will be small (large) when the scope for substitution between energy and other factor inputs is also small (large). In the example given above, the scope for substitution between energy and capital appears to be small (since they are AES complements), but at the same time the rebound effect is large.

The main reason for the difference is that Saunders' bases his conclusion on a different nesting structure which relies upon different assumptions about separability. In Saunders' (KL)E case, the key variable is the HES between energy and a composite of capital and labour inputs. Given the separability assumptions, this is the same as the HES between energy and capital which in turn is the same as the AES between energy and capital. In contrast, in Berndt and Wood's (KE)(LM) example, the key variable is the magnitude of the HES between energy and capital relative to the HES between utilised capital and the labour/materials composite. Taken together, these two variables determine the AES (or net elasticity) between capital and energy and this is different from the HES (or gross elasticity) between capital and energy in the utilised capital nest.

In Berndt and Wood's example, the large HES between utilised capital and the labour/materials composite leads to a large increase in the demand for utilised capital

[42] In practice, this may have been achieved through the use of technologies which are more thermodynamically efficient than those used previously (e.g. energy efficient motors), or it may been achieved through other means.

[43] The results are based on data for US manufacturing over the period 1947-71 and therefore exclude the energy price rises following the oil shocks of the 1970s.

following the reduction in capital costs. The resulting increase in energy demand is more than sufficient to offset the original reduction in demand brought about by the substitution of capital for energy inputs (i.e. the improvement in energy productivity in sub-function for utilised capital). But the high degree of substitution between utilised capital and the labour/materials composite is also what is responsible for the finding of energy-capital complementarity. Hence, in these circumstances empirical findings of energy-capital complementarity for a particular sector appear consistent with the potential for large rebound effects, or even backfire.

A second reason for the difference lies with the appropriate choice of independent variable for defining the rebound effect - namely, a change in energy efficiency. In Saunders theoretical work, the relevant independent variable is represented by the parameter $\upsilon_E(t)$, which defines energy augmenting technical change. But in broader discussions of the rebound effect, the independent variable is taken to be any relevant ratio of energy inputs to useful outputs. This ratio may apply to different levels of aggregation and may involve different ways of measuring energy inputs and useful outputs (e.g. in thermodynamic, physical or economic terms).[44] If the ratio of energy inputs to utilised capital outputs (E/K*) is taken as the appropriate independent variable for the rebound effect, Berndt and Wood's example can be interpreted as an illustration of backfire. The improvement in this ratio derives from the substitution of capital for energy inputs following a reduction in the price of capital (as a result of investment subsidies), and does not involve any change in Saunder's independent variable ($\upsilon_E(t)$). However, the relative magnitude of the various elasticities of substitution may be expected to also determine the estimated response of the production function to technical change.

Saunders' conclusion may perhaps be reconciled with Berndt and Wood's example if the former is reinterpreted as applying to the HES between 'energy services' and a composite of other inputs. If energy services are highly substitutable for the composite of other inputs, then reductions in the cost of energy services will lead to substantial increases in the demand for energy services. The magnitude of the rebound effect will be determined in part by the own price elasticity of energy services (holding output constant), which in a nested CES function will be determined by the HES between energy services and the composite of other inputs. However, just as Saunders' original conclusion only applies if energy is separable from other inputs, this revised interpretation only applies if energy services are separable from other inputs

In the Berndt and Wood example, 'energy services' are derived from a composite of capital and energy inputs, for which they use the term 'utilised capital'. This framework allows energy and capital to be found to be AES complements, while the high substitutability between energy services and a composite of other inputs leads to high rebound effects – as Saunders predicts. But in general, the provision of 'energy services' (S) is likely to require dedicated capital (KS), labour (LS), materials (MS)and energy (E) inputs, with additional capital (K), labour (L) and materials (M) being required for the provision of final output (Y). For example, a combination of the boiler and fuel inputs may provide steam (energy services) to drive separate production processes. This suggests a more general nested production function of the form:

[44] See *Technical Report 5* for a comprehensive discussion of the relevant independent and dependent variables for the rebound effect.

$$Y = f[g(K,L,M),S(K_S,L_S,M_S,E)] \quad (5.9)$$

In principle, this framework allows more targeted policies to be explored. For example, suppose we were interested in the effect on energy demand of an investment tax credit targeted on the capital equipment used to provide energy services (KS). Using Equation 2.14, the effect of this tax credit on energy demand, holding output constant, will be given by:

$$CPE_{EK} = CPE^{S}_{EK_S} + s_{K_S,S} * CPE_{SS} \quad (5.10)$$

$CPE^{S}_{EK_S}$: price elasticity of energy with respect to the price of energy services capital (P_{K_S}), holding energy services demand (S) constant.

CPE_{SS} : own price elasticity of energy services, holding output (Y) constant

s_{K,K^*} : share of capital costs in the total cost of energy services.

Then backfire will occur when $s_{K_S,S} * CPE_{SS} > CPE^{S}_{EK_S}$

In principle an empirical test of Saunders prediction could be made through a measurement of the AES between energy services and a composite of other inputs under conditions where the (KLM)S separability assumptions are supported by the data (i.e. AESSK = AESSL = AESSM.). But existing empirical studies measure energy rather than energy services and estimate the AES between energy and individual inputs without imposing any separability restrictions. This means that the available measures of the relevant elasticities of substitution do not provide a definitive guide to the likely magnitude of rebound effects.

Once separability restrictions are removed, the relationship between the measured elasticities of substitution and the magnitude of the rebound effect becomes more complex. In recent work, Saunders (2007) has shown that the magnitude of the rebound effect with a Translog cost function is a complex function of the elasticities of substitution between each pair of inputs, together with the cost shares of each input:

$$g[s_K, s_L, s_E, \sigma_{KK}, \sigma_{KL}, \sigma_{KE}, \sigma_{LL}, \sigma_{LE}, \sigma_{EE})] \quad (5.11)$$

In other words, the magnitude of the elasticity of substitution between each pair of inputs plays a role in determining the behaviour of the Translog - in contrast to the (KL)E CES, where only the elasticity between energy and a composite of capital and labour inputs appears relevant. Saunders (2007) shows that similar results apply to other, less common types of production function, including the Symmetrical Generalised Barnett and Gallant (Fourier). This suggests that Saunders early results for the CES may have led researchers to focus inappropriately upon one particular parameter.[45]

[45] Restrictions normally have to be imposed upon the parameter values in a Translog cost function to ensure that its behaviour is consistent with basic economic theory. In particular, the cost function must be concave, implying that the marginal product of each input declines with increasing use of that input. In many applications, such as CGE modelling, these conditions need to be satisfied for all input combinations, but empirically estimated cost functions sometimes violate these conditions (Diewert and Wales, 1987). In the most recent version of his working paper, Saunders (2007) finds that imposing a global concavity restricition means that the Translog production function always leads to backfire. However, Ryan and Wales (2000) show that if concavity is imposed locally at a suitably chosen reference point, the restriction may be satisfied at most all of the data points in the sample. Under these circumstances, the Translog may be able to represent different types of rebound effect for particular data sets – but only if it can be empirically verified that concavity is honoured across the domain of measurement.

In summary, the conclusions that may be drawn from neoclassical production theory regarding the scale of rebound effects may be less straightforward than is suggested by a number of statements in the rebound literature. Notably:

The AES between each pair of inputs may be relevant and not just that between energy and a composite of other inputs.

A finding that energy is a weak AES substitute for another factor, or even a complement to that factor, is not necessarily inconsistent with the potential for large rebound effects, or even backfire from particular types of energy efficiency improvement.

5.4 Technical change and 'effective energy'

The importance of distinguishing between 'energy services' and raw energy inputs is further complicated by the appropriate treatment of technical change.

The Berndt and Wood example of rebound effects describes a substitution between (all types of) capital and energy in the production of utilised capital, brought about by a reduction in capital costs. The reframing of this example in terms of energy services shows how it may be relevant to energy efficiency policies that target investment subsidies at energy efficient equipment. But such policies are only desirable if they overcome the market failures inhibiting the adoption of energy efficient technologies cost effectively. If not, they may be may be costly to the economy overall. More generally, the concern is with the rebound effects that may result from energy augmenting technical change.[46] These improvements are assumed to occur independently of any changes in relative prices and are considered desirable since they increase aggregate income. While the substitution effects brought about by the investment subsidy may be represented by a movement along an isoquant of the production function for utilised capital, 'technical change' refers to the development of new technologies and methods of organisation that shift the isoquant to the left, allowing the same level of utilised capital to be produced from a lower level of energy and/or capital inputs. Energy augmenting technical change reduces the amount of energy inputs required to produce a unit of utilised capital, but has no effect on the amount of capital required. In Equation 5.8, it is represented by the parameter υ_E.

While Saunders (1992) refers simply to energy or fuel, Saunders (2006a) refers to the product $\upsilon_E(t)E$ in Equation 5.8 as 'energy services'. This suggests a distinction between the energy inputs to a conversion device (e.g. the coal used by a boiler) and the 'energy service', or 'useful work' outputs (e.g. the steam that is produced). However, in thermodynamic terms this is potentially misleading since the 'energy service' outputs ($\upsilon_E(t)E$) are greater in magnitude than the energy inputs ($\upsilon_E(t)E \geq E$ since $\upsilon_E(t) \geq 1$). An alternative terminology, frequently used with CGE models is to refer to the product $\upsilon_E E$ as 'effective energy' ($\tilde{E}$). The improvement in the productivity of energy inputs (E) that result from technical change is then expressed as an increase in the quantity of effective energy inputs ($\tilde{E}$). Hence, to examine the implications of technical change and to be consistent

[46] The terms 'energy augmenting technical change' and 'energy saving technical change' have precise definitions in the literature and are not necessarily equivalent. For a full discussion, see *Technical Report 5*.

with standard terminology, the term 'energy services' in the preceding section should really be replaced with 'effective energy'. This gives the following production function:

$$Y = f[g(K, L, M), \tilde{E}] \quad (5.12)$$

With this approach, the production function represented by Equation 5.9 maps the relationship between effective factor inputs and economic output rather than real factor inputs and economic output. The location and shape of the individual isoquants are assumed to be fixed over time, with energy augmenting technical change reducing the amount of real energy required to produce a unit of effective energy. Since this reduces the cost of effective energy, technical change leads to substitution between effective energy and other inputs. The factor augmenting perspective therefore allows technical change to be viewed as movement along an isoquant of a production function (i.e. substitution) as opposed to a leftwards shift of an isoquant.

The implicit assumption of this perspective is that energy augmenting technical change is costless and is achieved without labour and capital inputs. But as suggested above, the provision of 'effective energy' is likely to require dedicated capital, labour and energy inputs and may better be represented by a sub-function. Howarth (1997) explores the implications of this framework by simulating the provision of effective energy inputs with a Leontief (i.e. fixed proportions) production function. The results suggest that the size of the rebound effect depends upon the share of effective energy in total output costs, and the share of energy costs in the total cost of effective energy. Since both of these are likely to be small, Howarth concludes that rebound effects are also likely to be small – even if the HES between (separable) effective energy and other inputs equals (or exceeds) unity. However, Saunders (2000a) shows that these results stem entirely from Howarth's assumption of a Leontief production function for the provision of effective energy. If the production function is assumed instead to take a Cobb Douglas form, energy productivity improvements are found to lead to backfire.

The implications of technical change therefore depends upon the scope for substitution between effective energy and other inputs (assuming these are separable) and therefore on the estimated value of AESS,KLM. However, empirical estimates of elasticities of substitution depend upon the assumptions made for the magnitude and bias of technical change – i.e. the relative contribution of neutral, labour augmenting, capital augmenting and energy augmenting technical change. The estimated value of AESS,KLM will only be accurate if the bias is estimated accurately. As an illustration, Smithson (1979) estimates a Translog cost function for the Canadian mining industry and finds that energy and capital appear to be more complementary if neutral technical change is imposed then if factor augmenting technical change is allowed. In practice, however, nearly two thirds of the empirical studies reviewed in Section 4 either (implicitly or explicitly) assume neutral technical change. This suggests that the resulting estimates of the AES between different inputs could be further biased by the failure to correctly specify the bias of technical change. This may further limit the usefulness of empirical studies for either parameterising CGE models or for providing insights on the likely magnitude of rebound effects in different sectors.

5.5 Summary

The implications of the preceding sections are rather disappointing, since they suggest that the empirical literature on elasticities of substitution may be of relatively little value in either parameterising CGE models or in providing guidance on the likely magnitude of rebound effects.

With regard to the requirements of CGE models, most empirical studies differ with regard to the assumed functional form, the assumptions regarding separability, the associated nesting of production factors, the definitions of elasticity of substitution, the aggregation of individual factor inputs and the aggregation of individual sectors. Combined with the fact that the process of compiling CGE parameter values is rarely transparent and sensitivity tests are uncommon, this suggests that the results of such models should be treated with great caution - quite apart from the range of other theoretical and practical difficulties associated with the CGE approach (see Technical Report 4).

The relationship between empirical estimates of elasticities of substitution and the magnitude of rebound effects is also more complex than is generally assumed. Saunders' statement that "…the ease with which fuel can substitute for other factors of production (such as capital and labour) has a strong influence on how much rebound will be experienced" is found to be somewhat misleading. A more precise statement would, first, refer to 'energy services' (or 'effective energy') rather than fuel; second, clarify that the elasticity in question is the AES between energy services and a composite of other inputs; third, include the qualification that this only applies when energy services are separable from this composite; and fourth, clarify that this conclusion derives from a particular nesting structure in a CES production function. Since the majority of empirical studies use Translog cost functions, measure energy rather than energy services, do not impose any separability restrictions and estimate the AES between energy and individual inputs, they do not provide a direct test of this proposition.

In more recent work with a Translog cost function, Saunders (2006b) has shown that the magnitude of the elasticities of substitution between each pair of inputs may play an important role in determining the magnitude of any rebound effects. But not only does this describe a considerably more complex situation than suggested by the above quote, it also suggests that a finding that energy is a weak AES substitute for another factor, or even a complement to that factor, is not necessarily inconsistent with the potential for large rebound effects. This is arguably consistent with Berndt and Wood's (1979) explanation of how energy and capital may be AES complements rather than substitutes. Although not previously recognised as such, this explanation (which is not universally acknowledged) effectively describes how a particular type of energy efficiency improvement (namely the substitution of capital for energy stimulated by investment credits) may lead to backfire.

These conclusions suggest that our survey of empirical estimates of the elasticity of substitution between energy and capital may have provided relatively limited insight into the likely magnitude of rebound effects. It also suggests that the discussion about elasticities of substitution in the literature may have obscured the real issue, which is the own-price elasticity of energy services in different contexts. While these are determined by elasticities of substitution, the relationship is far from straightforward once the straitjacket of a nested CES is removed. Also, the discussion regarding substitution elasticities may have obscured the important point that rebound effects are also determined by the price elasticity of output in the sector in which the energy efficiency improvement is achieved.

However, the results of this survey do potentially reinforce one of the main conclusions of Technical Report 5 - namely that the scope for substituting capital for energy may be less than is commonly assumed. While it is very difficult to draw any general conclusions, the results of Section 4 suggest that energy and capital are at best weak substitutes and possibly may be complements. Arguably, this suggests that higher energy prices may reduce energy consumption and capital formation, increase energy and capital productivity, reduce labour productivity, and reduce economic output to a larger extent than is suggested by the share of energy in total costs. This suggests the possibility of a strong link between energy consumption and economic output and potentially high costs associated with reducing energy consumption. At the same time, this is not necessarily incompatible with the potential for large rebound effects from energy efficiency improvements. However, such conclusions must be treated with great caution, given the numerous limitations of the evidence base described above.

6 Summary and implications

6.1 Summary

This report has investigated the role that elasticities of substitution may play in determining the magnitude of rebound effects. It has also conducted a survey of empirical estimates of the elasticity of substitution between energy and capital and sought explanations for the widely varying results.

The results are rather mixed. On the one hand, the review has clarified a complex and surprisingly confused area of literature and provided an overview of an important question within energy economics: namely, whether energy and capital may be considered as substitutes or complements. This in turn, is closely related to a key theme of Technical Report 5, namely the relationship between energy consumption and economic growth. On the other hand, the review has concluded that the relationship between elasticities of substitution and the rebound effect is more subtle than is commonly assumed. As a result, it appears that empirical estimates of elasticities of substitution may tell us relatively little about the likely magnitude of rebound effects.

The review has shown that there are at least four definitions of the elasticity of substitution in common use and several others that appear less frequently. The lack of consistency in the use of these definitions and the lack of clarity in the relationship between them, combine to make the empirical literature inaccessible to a non-specialist and arguably both confusing and contradictory. The majority of existing empirical studies use the sign of the Allen-Uzawa elasticity of substitution (AES) to classify inputs as substitutes or complements; however it can be argued that this measure has a number of drawbacks and consequently its quantitative value may lack meaning. In many cases, the Cross Price Elasticity (CPE) or Morishima Elasticity of Substitution (MES) measures could be more appropriate, but these have yet to gain widespread use. Furthermore, the sign of the MES is less useful as a means of classifying substitutes and complements, since in nearly all cases the MES is positive

While most empirical studies estimate the AES between different inputs, energy-economic models normally require assumptions about the HES. The two can be difficult to relate, owing to differences in functional form, separability assumptions, the treatment of technical change, nesting structures, sectoral aggregation and so on. Given these multiple differences it is argued that it raises serious questions regarding the empirical basis of CGE models and the usefulness of their results.

The survey of empirical literature on the elasticity of substitution between energy and capital (EoSKE) has reached no firm conclusions. Although the majority of studies use a static time series, single equation approach, there is a wide dispersion in their results. This could be because the different studies genuinely reflect the different type and degree of substitutability within the various sectors, countries and/or time periods studied. However, the results also appear to be strongly influenced by the type of data used, the definition and measurement of capital inputs, the assumptions about separability, technical change and returns to scale and numerous other factors. The results demonstrate considerable variability even when the same or similar data sets are employed. As a result, despite more than three decades of research, no consensus has emerged on either the sign of EoSKE (i.e.

whether energy and capital are complements or substitutes) or on the magnitude of EoSKE, (although most studies suggest that it is less than unity). If a general conclusion can be drawn, it is that energy and capital appear at best to be weak substitutes (i.e. AESKE<0.5) and possibly may be complements (i.e. AESKE<0.0). But very little confidence can be placed in this statement.

It is also observed that:

The issue of the distinction between short-run and long-run measures of EoSKE has not been resolved, despite being raised in the 1970s. Furthermore, it is surprising that static models still predominate, despite the advances in time series econometrics over the last 25 years.

While assumptions regarding separability, technical change and other factors appear to influence the empirical results, there is no consistent approach to testing these assumptions in order to 'let the data speak.

The number and variety of factors influencing the empirical results and the apparent sensitivity of the results to these factors indicates that the empirical evidence on E-K substitutability is anything but robust.

The lack of up to date and consistent UK evidence makes it impossible to conclude on the sign and size of EoSKE for the UK.

On the basis of this review, it is recommended that:

A full meta-analysis is undertaken to attempt to better ascertain the effect of the different functional forms, types of data, countries, type of separability, form of technical change, etc. on the estimated EoSKE's;[47]

A new UK study is undertaken for the whole economy and various sectors of the economy using time series analysis of a general KLEM Translog specification that explores the effect of allowing for a dynamic model and non-neutral technical change and develops a formal statistical framework to test the general model for restrictions such as Hicks neutral technical change, separability of E from KLE, etc. By 'letting the data speak'; this should produce more robust estimates of EoSKE.for the UK that may be used to help understand the rebound effect.

6.2 Implications

The relationship between empirical estimates of elasticities of substitution and the magnitude of rebound effects appears to be more complex than is generally assumed. Saunders' statement that "…the ease with which fuel can substitute for other factors of production (such as capital and labour) has a strong influence on how much rebound will be experienced" is therefore potentially misleading. A more precise statement should arguably, first, refer to 'energy services' (or 'effective energy') rather than fuel; second, clarify that

[47] As stated above, since staring this review Koetse *et al* (2006) have produced a meta-analysis of capital-energy substitution. However, this uses under 40 previous studies; hence the suggestion that a *full* meta analysis is undertaken utilising as many as possible of the previous studies covered in this review.

the elasticity in question is the AES between energy services and a composite of other inputs; third, include the qualification that this only applies when energy services can be considered to be separable from this composite; and fourth, clarify that this conclusion derives from a particular nesting structure in a CES production function. Since the majority of empirical studies use Translog cost functions, measure energy rather than energy services, do not impose the same (or sometimes any) separability restrictions and estimate the AES between energy and individual inputs, they do not provide a direct test of Saunders proposition. For similar reasons, such studies appear to be of little value in parameterising CGE models.

On the basis of both this review and Saunders more recent work on Translog cost functions it is concluded that:

> The AES between each pair of inputs is relevant to the magnitude of rebound effects and not just that between energy and a composite of other inputs.
>
> A finding that energy is a weak AES substitute for another factor, or even a complement to that factor, is not necessarily inconsistent with the potential for large rebound effects, or even backfire from certain types of energy efficiency improvement.

This suggests that our survey of empirical estimates of EoSKE has provided only limited insight into the likely magnitude of rebound effects. However, the results do arguably reinforce one of the main conclusions of Technical Report 5 - namely that the scope for substituting capital for energy may be less than is commonly assumed. While it is very difficult to draw any general conclusions, the results of Section 4 suggest that energy and capital are at best weak substitutes and possibly may be complements. This arguably suggests the possibility of a strong link between energy consumption and economic output and potentially high costs associated with reducing energy consumption. At the same time, a limited scope for substitution between energy and capital may not necessarily be incompatible with the potential for large rebound effects from certain types of energy efficiency improvements. However, such conclusions must be treated with great caution, given the numerous limitations of the evidence base described above.

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Annex 1: Functional forms

Empirical studies assume a particular functional form for the production function and estimate the parameters of that form econometrically. It follows that the empirical results may depend very much on the particular form that is chosen. Various functional forms appear in the literature, of which the most common are:

the Cobb-Douglas, due to Cobb and Douglas (1928);
the Constant Elasticity of Substitution, defined by Solow (1956) and Arrow et al (1961);
the Transcendental Logarithmic (Translog) form of Christensen et al (1973).

Each of these functional forms will be discussed briefly in turn using a three factor (KLE) example.[48]

6.2.1.1 The Cobb-Douglas production function

A three-factor Cobb-Douglas production function (see for instance Varian (1996), p. 317) takes the form:

$$Y = K^{\alpha} L^{\beta} E^{1-\alpha-\beta} \qquad \text{(A1.1)}$$

Where α is the proportion of capital which is used in the production of Y, β is the proportion of labour used and 1-α-β, is the proportion of energy. It is assumed that α+β +(1-α-β) = 1, such that the proportions of all factor inputs used, sum to one. Empirically, the assumption that factor proportions sum to unity can either be imposed or tested for using standard statistical techniques. This functional form satisfies the assumption of constant returns to scale (i.e. it is homogeneous of degree 1) and assumes a constant and unitary Hicks elasticity of substitution between inputs ($HES_{ij} = 1.0$). Hence, a percentage reduction in one input can be fully compensated by a percentage increase in another input.

6.2.1.2 The Constant Elasticity of Substitution (CES) production function.

The CES function, originally due to Solow (1956) and subsequently developed by Arrow et al (1961) as its name indicates, assumes that there exists a constant (Hicks) elasticity of substitution between factor inputs

This functional form can be for a two input production function:

$$Y = (aK^{\rho} + bL^{\rho})^{\frac{1}{\rho}} \qquad \text{(A1.2)}$$

Where a and b are positive constants and $\rho = \frac{\sigma - 1}{\sigma} \Leftrightarrow \sigma = \frac{1}{1-\rho}$, with σ the Hicks elasticity of substitution between capital and labour (HESKL).

Energy may be included in a CES function as follows:

$$Y = [a(bK^{-a} + (1-b)E^{-a})^{\frac{\rho}{a}} + (1-a)L^{-\rho}]^{-\frac{1}{\rho}} \qquad \text{(A1.3)}$$

[48] This exposition is based on Saunders (2006).

This form of a 3-input CES is called "nested" because the original CES function contains another CES for only two factors - energy and capital. This functional form implies at the same time: a) a constant elasticity of substitution between energy and capital; and b) a constant elasticity of substitution between the KE composite (assumed to be derived from a combination of energy and capital) and labour. It rests on the assumption that labour is separable from capital and energy inputs

In the same fashion, it is possible to have a multi-nested CES function that allow the inclusion of further inputs like materials. Following Manne & Richels (1990), the value of the EoS in the capital-energy nest can be restricted to be unity. This restriction gives a Cobb-Douglas within the CES function:

$$Y = [a(K^{\alpha} L^{1-\alpha})^{\rho} + b(E)^{\rho}]^{1/\rho} \qquad \text{(A1.4)}$$

Where $0 < \delta < 1,\ -1 < \rho \neq 0$

Here, a and b are the share (or distribution) parameters, and indicates the proportions of each factor input being used to obtain the level of output Y, thus b should be equivalent to 1-a (see, for instance, Chiang (1984), p.426). This particular specification assumes separability between energy and other inputs, by specifying the relationship between capital and labour to be Cobb-Douglas, hence allowing for a different EoS between these two factors. Thus, in this specification, the (Hicks) elasticity of substitution between energy and capital is the same as that for energy and labour, though alternative nested relationships could equally be expressed.

6.2.1.3 The Translog Production (and Cost) Function

The Transcendental Logarithmic function, commonly referred to as the Translog function was first introduced by Christensen et al (1973). It was conceived in an effort to define a less restrictive, more flexible functional form for production functions, which offers particular advantages when there are more than two factors of production under consideration. The term 'transcendental' derives from the fact that no factor can be solved for explicitly without being a function of itself (i.e. a 'transcendental' equation). Importantly, and unlike the Cobb-Douglas and CES, the Translog does not impose any restrictions on the substitutability between different factor inputs. For three factors, it is expressed as;

$$\begin{aligned} \ln Y = \quad & \alpha_0 + \alpha_1 \ln K + \alpha_2 \ln L + \alpha_3 \ln E \\ & + \frac{1}{2}[\gamma_{11}(\ln K)^2 + \gamma_{12} \ln K \ln L + \gamma_{13} \ln K \ln E \\ & + \gamma_{21} \ln L \ln K + \gamma_{22}(\ln L)^2 + \gamma_{23} \ln L \ln E \\ & + \gamma_{23} \ln E \ln K + \gamma_{32} \ln E \ln L + \gamma_{33}(\ln E)^2] \end{aligned} \qquad \text{(A1.5)}$$

Although this expression appears somewhat lengthy, it amounts to assigning parameters to all individual factors, as well as to all feasible cross products. If all the γ coefficients are zero, then the above form is exactly equivalent to a Cobb-Douglas.

The majority of empirical studies use a Translog cost function, rather than a production function. Assuming that agents are rational and efficient and acting in equilibrium, the dual

function of maximising production can be specified as cost minimisation. Then the Translog cost function may be written as:

$$\begin{aligned}\ln C = \quad & \alpha_0 + \alpha_1 \ln P_K + \alpha_2 \ln P_L + \alpha_3 \ln P_E \\ & + \frac{1}{2}[\gamma_{11}(\ln P_K)^2 + \gamma_{12} \ln P_K \ln P_L + \gamma_{13} \ln P_K \ln P_E \\ & + \gamma_{21} \ln P_L \ln P_K + \gamma_{22}(\ln P_L)^2 + \gamma_{23} \ln P_L \ln P_E \\ & + \gamma_{23} \ln P_E \ln P_K + \gamma_{32} \ln P_E \ln P_L + \gamma_{33}(\ln P_E)^2]\end{aligned} \qquad \text{(A1.6)}^{49}$$

where PK, PL and PE represent the real prices of capital, labour and energy respectively. As stated, this is the main basis for the empirical estimates reviewed in the main text. By differentiating Equation A.1.6 with respect to each factor input (i.e. K, L and E) and then applying Shephard's Lemma, cost share equations for each factor input can be derived (see Berndt and Wood, 1975). These may be estimated econometrically to give the key parameters in Equation A.1.6 and then used to derive the measures of the elasticity of substitution discussed above.[50] In particular, the AES is given by

$$AES_{ij} = \frac{\gamma_{ij} + S_i S_j}{S_i S_j} \qquad \text{(A1.7)}$$

Where $i = j = K, L, E$ and Si represents the cost share of factor i.

The MES is given by:

$$MES_{ij} = CPE_{ji} - OPE_{ii} \qquad \text{(A1.8)}$$

$$MES_{ji} = CPE_{ij} - OPE_{jj} \qquad \text{(A1.9)}$$

Where: $CPE_{ij} = S_j AES_{ij}$, $OPE_{ii} = S_j AES_{ii}$, and $AES_{ii} = \frac{\gamma_{ii} + S_i^2 - S_i}{S_i^2}$.

[49] Note this is specification for the Translog cost function with neutral technical progress, to include non-neutral technical progress the following terms are added to equation A1.6: $\gamma_t t + \gamma_{tt} t^2 + \gamma_{Kt}\, t\, lnP_K + \gamma_{Lt}\, t\, lnP_L + \gamma_{Et}\, t\, lnP_E$.

[50] This formulation allows for convenient estimation of the Translog function via the share equations which may be estimated by the Seemingly Unrelated Regression method of Zellner (1962) or Maximum likelihood. Details of the estimation are not discussed here since it is outside the current remit but may be found in the many empirical papers reviewed above.

Annex 2: Search criteria

The search was conducted by initially using the ISI Web of Science online database to obtain a near comprehensive list of the academic journals to-date, which had referenced the seminal work of Berndt and Wood (1975). Subsequently to this, the Google/Google Scholar search engine was used to search the internet for suitable references using search terms such as;

KLE,
KLEM,
Production,
Production functions,
Capital energy,
substitutability,
Elasticity of substitution,
Multifactor production,
etc..

Combinations of these words were defined using Boolean operators to constrain search results to be within more clearly defined parameters. Furthermore, the papers obtained from the above approach were further consulted to identify any further relevant studies which may have been overlooked.

It should be noted that in addition to studies of KLE or KLEM production functions there are a number of studies that analyse, interfuel substitution instead of, or as well as, factor substitution (see for instance Urga and Walters (2003) for a recent example). Those that analysed interfuel substitution only and/or did not openly present a measure of EoSKE were not included in the empirical papers reviewed. Similarly, any paper that discussed the issue of E-K being substitutes or complements (such as Hunt and Lynk (1992)) but did not openly present a measure of EoSKE were also not included. No papers were rejected on the grounds of estimation method, data type, level of aggregation, etc.