UK ENERGY RESEARCH CENTRE

UKERC Review of Evidence for Global Oil Depletion

Technical Report 6:

Methods of forecasting future oil supply

July 2009: REF UKERC/WP/TPA/2009/021

Adam Brandt

Energy and Resources Group, University of California, Berkeley

THE UK ENERGY RESEARCH CENTRE

The UK Energy Research Centre is the focal point for UK research on sustainable energy. It takes a whole systems approach to energy research, drawing on engineering, economics and the physical, environmental and social sciences.

The Centre's role is to promote cohesion within the overall UK energy research effort. It acts as a bridge between the UK energy research community and the wider world, including business, policymakers and the international energy research community and is the centrepiece of the Research Councils Energy Programme.

www.ukerc.ac.uk

Acknowledgements

This report was improved by the helpful comments of other authors in the UKERC *Review of Evidence for Global Oil Depletion*, especially Steve Sorrell (University of Sussex), Jamie Speirs (Imperial College London), Michael Smith (EnergyFiles), and Roger Bentley (University of Reading).

Numerous discussions with Alex Farrell (University of California, Berkeley), Tad Patzek (University of Texas, Austin), Dick Norgaard (University of California, Berkeley), Andrew Mills (University of California, Berkeley) and Anand Patil (Oxford University) were helpful in improving my understanding of oil depletion by providing a range of (often conflicting) interpretations of the evidence for global oil depletion.

Additionally, the staff of the UC Berkeley library helped obtain some of the references cited here that are not widely available.

The content of this work and its conclusions remain the responsibility of the author.

Preface

This report has been produced by the UK Energy Research Centre's Technology and Policy Assessment (TPA) function.

The TPA was set up to address key controversies in the energy field through comprehensive assessments of the current state of knowledge. It aims to provide authoritative reports that set high standards for rigour and transparency, while explaining results in a way that is useful to policymakers.

This report forms part of the TPA's assessment of evidence for **near-term physical constraints on global oil supply**. The subject of this assessment was chosen after consultation with energy sector stakeholders and upon the recommendation of the TPA Advisory Group, which is comprised of independent experts from government, academia and the private sector. The assessment addresses the following question:

What evidence is there to support the proposition that the global supply of 'conventional oil' will be constrained by physical depletion before 2030?

The results of the project are summarised in a *Main Report*, supported by the following *Technical Reports*:

1. Data sources and issues
2. Definition and interpretation of reserve estimates
3. Nature and importance of reserve growth
4. Decline rates and depletion rates
5. Methods for estimating ultimately recoverable resources
6. Methods for forecasting future oil supply
7. Comparison of global supply forecasts

The assessment was led by the Sussex Energy Group (SEG) at the University of Sussex, with contributions from the Centre for Energy Policy and Technology at Imperial College, the Energy and Resources Group at the University of California (Berkeley) and a number of independent consultants. The assessment was overseen by a panel of experts and is very wide ranging, reviewing more than 500 studies and reports from around the world.

Technical Report 6: Methods of forecasting future oil supply is authored by Adam Brandt of the Energy and Resources Group at UC Berkeley. It describes the different methodologies available for forecasting oil supply, identifies their key assumptions and sensitivities and assesses their strengths and weaknesses. It highlights a convergence in the literature and draws conclusions regarding the predictive value of such models and the confidence that may be placed in the forecasts obtained.

Executive Summary

For over a century, oil depletion has repeatedly surfaced as an issue of concern. The mathematical methods used to understand oil depletion and to predict future oil production have become more sophisticated over time: poorly-quantified concern over future oil resources in the early 20th century has given way to sophisticated simulations of oil discovery and extraction. This systematic review assesses the insight offered by these methodologies and critically evaluates their usefulness in projecting future oil production. It focuses on models that project future rates of oil production, and does not address the modeling or estimation of oil resources (e.g., ultimately recoverable resources, or URR).

Models reviewed include the Hubbert methodology, other curve-fitting methods, simulations of resource discovery and extraction, detailed "bottom-up" models, and theoretical and empirical economic models of oil resource depletion. Important examples of published models are discussed, and the benefits and drawbacks of these models are outlined. I also discuss the physical and economic assumptions that serve as the basis for the studied models.

Simple mathematical models are reviewed first. Resource exhaustion models based on the reserve-to-production ratio have been used since the early 20th century. These models are unfit for understanding even the gross behavior of future oil supplies, as they account for neither reserve growth nor empirically-observed shapes of oil production profiles. Curve-fitting methods began with the early hand-drawn models of Hubbert and Ayres. Hubbert later built his logistic model of oil depletion. The key characteristics of the Hubbert method include: a logistic cumulative discovery function, a symmetric production profile, and the use of ultimately recoverable reserves (URR) as an exogenous constraint.

Hubbert's work inspired a variety of related curve-fitting models which relax one or more of his key assumptions. Some rely on non-logistic functions, such as exponential, linear, or Gaussian curves. Some models exhibit asymmetric rates of increase and decrease, allowing them to closely fit production profiles that have decline rates that differ from their rates of increase. Other models, often called "multi-cycle" models, treat production as the sum of a number of technologically or geologically distinct cycles of development.

There are a variety of arguments made in support of using bell-shaped curves, but these are not rigorous in general, and arguments based on the central limit theorem are especially difficult to justify. Empirical results suggest that bell-shaped functions are useful for fitting historical production profiles, but that they are by no means exclusive in this respect. Also, the increased complexity of multi-cycle models often results in spurious precision of model fit to data because such models lack *a priori* justification for the specific timing or magnitude of any given cycle.

Simulation models of oil depletion differ from curve-fitting models in that they do not assume a given function for oil production curves (e.g., a bell-shaped logistic function) but instead represent physical and economic mechanisms of the discovery and extraction

of oil. Such mechanistic aspects can include discovery probabilities or the increasing difficulty of oil extraction as a function of depletion. The most complex of these simulations model the investment and extraction of a number of competing fuels, such as conventional crude oil and unconventional hydrocarbons like tar sands.

Unfortunately, simulation models are complex and difficult to understand or critique. This hinders a key goal of mathematical modeling: to increase our understanding of how the oil production system functions. As model complexity increases, the required number of data inputs and model parameters increases as well. Often times the primary data on which to base such model inputs are of poor quality or are simply not available. This can result in an increase in the number of assumptions required, thus negating some of the value of their increased detail.

Bottom-up models of oil depletion utilize detailed datasets of reserves and production, often including data at the field level. These models allow relatively straightforward and simple assumptions about field-level production behavior to be summed into aggregate regional or global production curves that exhibit considerable complexity and, in principle, could accurately reproduce observed production profiles. These models are critiqued because they are based on datasets that are not publically available. In addition, their complexity requires many assumptions based on modeler judgment. Both of these characteristics, which are fundamental to the method and give it much of its strength, hinder the ability of other authors to replicate or improve on these models.

There are two primary types of economic oil depletion models: optimal depletion models and econometric models. Theoretical models of optimal resource depletion are created to analytically explore the economic tradeoffs between producing exhaustible resources now or producing them at a later time. They are very simple, which is advantageous because they allow tractable analytical solutions to be developed. Econometric models of oil depletion form the mechanistic counterpart to theoretical models of optimal depletion. These models require significantly more input data than optimal depletion models. Of particular importance are "hybrid" econometric models that include simple representations of physical aspects of oil production (such as the relationship between depletion level and extraction costs) in addition to more traditional economic variables such as the oil price.

There are difficulties with both types of econometric models. Economic optimal depletion models are problematic because they are largely theoretical and have little or no representation of the specifics of oil production. This is a consequence of keeping the models simple so that they remain analytically tractable. In contrast, econometric models can include many variables affecting oil production, and therefore can exhibit excellent fidelity to historical production data. Unfortunately, this fidelity tends not to be robust when used for anything but short-term predictions.

We conclude this review with a number of synthesizing thoughts. We first classify all major reviewed models along four dimensions of variability: 1) their emphasis on physical or economic aspects of oil production, 2) their scale, 3) their degree of representation of mechanistic details of the oil production process, and 4) their complexity. Interestingly, a number of models that are based on quite disparate assumptions (e.g., physical simulation vs. economic optimal depletion) produce approximately bell-shaped production profiles. This is a hopeful convergence, because a good number of observed historical production profiles are approximately bell-shaped.

The mathematical and analytical tools used to determine the quality of fit of models to historical data make it difficult to determine definitively whether one model type is superior. Available empirical data suggest that a number of different functions are useful for fitting historical data. And experience with the results of complex simulation models suggests that fidelity in fitting historical data does not indicate that a model will be successful in forecasting future production.

Skepticism is warranted regarding the ability of simple models to predict with precision the date of peak oil production. Despite this, they are likely useful for making predictions of the decade of peak production for a given URR estimate. I also argue that more detailed models have significant advantages for near-term forecasts, but that the many uncertainties involved reduce this advantage for making long-term predictions. I emphasize that attempting to use any models to make detailed predictions is likely to not be useful, because many aspects of the world are not included in even the most complex models.

Increasing model complexity is often not a useful way to increase model fidelity. And, in fact, increasing model fidelity though additional complexity is detrimental to the other important goal of modeling oil depletion, which is to increase our understanding of the physical and economic processes underlying oil depletion.

Lastly, I argue that forecasting the physical aspects of oil depletion without also including the economics of substitution with alternatives to conventional petroleum (e.g. coal-liquids or the tar sands) results in unrealistic projections of future energy supply. By ignoring this adaptive substitution process, models ignore the important economic and environmental impacts that will arise from this transition to substitutes for conventional petroleum.

Contents

1 Introduction

Concern about the availability of oil emerged soon after the birth of the modern oil industry and has resurfaced repeatedly since, leading to numerous predictions of impending exhaustion of oil resources. And while these projections were generally proven incorrect (sometimes spectacularly so), the future never seemed assured. This is because, as Adelman (1997) argued, the oil industry is fundamentally "a tug-of-war between depletion and knowledge." And although knowledge has won out over depletion for the last 150 years, allowing us to increase oil production almost continuously, there is uncertainty about how much longer this can continue.

There are two key questions facing those who attempt to model oil depletion. First, *how much recoverable oil exists?* Answering this question requires estimating ultimately recoverable resources (URR), the amount of oil that can be economically produced over all time. See a companion report (Sorrell and Speirs 2009) for a discussion of this problem. Secondly, we can ask: *how quickly will this stock of oil be depleted, and what path will production take over time?* This is the problem of how one converts an estimate of URR into an estimate of future rates of oil production. This report addresses this question by reviewing the mathematical methods used to project rates of oil production over time.

Quantitative understanding of oil depletion has increased significantly over the last century. Calculations of the exhaustion time of remaining oil resources were performed as early as 1909, although the methods used were simple (Day 1909). By mid-century, methods of predicting field-level production were used in evaluating the economics of producing fields (Arps 1945), and statistical methods were developed to better project how much oil is likely to be found in a given region (Kaufman 1983). In the 1950s and 1960s, curve-fitting techniques were used to forecast petroleum production, accurately predicting the peak in US oil production in 1970 (Hubbert 1956). After the oil crisis of 1973, the problem of oil depletion received great attention from economists, temporarily elevating the area of resource depletion to a field of vigorous theoretical exploration (Krautkraemer 1998). And finally, the 1970s and 1980s saw increasing focus on econometric modeling of oil discovery and extraction (Walls 1992). Interest in oil depletion waned after the oil price decline of the mid 1980s, resulting in a decline in academic interest until recently.

Oil depletion models vary in many ways, but three dimensions are key (see Figure 1). First, the level of aggregation varies between models: some models project global production, while others model production from individual fields. Second, some models fit a theoretical function to historical data to project future production, while others attempt to model the mechanisms governing oil discovery and extraction. Lastly, some models rely primarily upon on economic reasoning, while others emphasize the physical nature of oil depletion.[1]

[1] This list is not exhaustive, and other important dimensions include: model complexity, forecasting period (near vs. long-term); maturity of the forecasted region (i.e., unexplored, pre-peak, or post-peak); or the production of deterministic (point value) or probabilistic results (Schuenemeyer 1981). Other classification

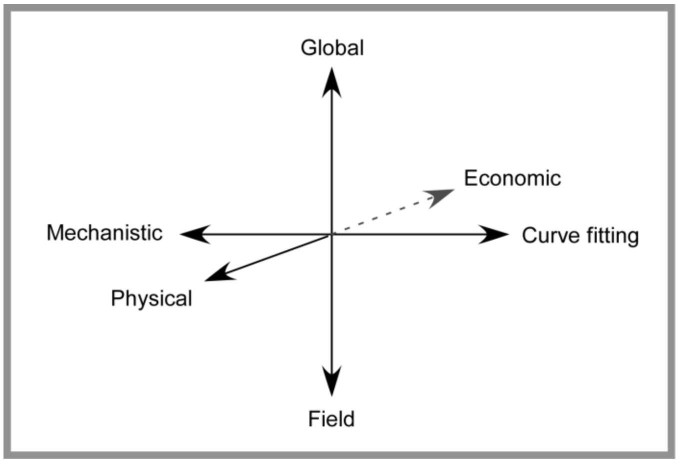


Figure 1. Three of the dimensions along which oil depletion models vary. The level of aggregation is shown by the vertical dimension, the level of mechanistic detail is given by the horizontal dimension, and the intellectual grounding of the model is represented by the depth dimension.

The models reviewed in this report inhabit different portions of the space represented in Figure 1. Some models clearly inhabit one region of this diagram, like a global curve-fitting model that utilizes a physically-based exponential depletion function (Wood, Long *et al.* 2000). Other models attempt to bridge the gap between the extrema of these dimensions, such as a field-level model that "builds up" to a global depletion projection (Miller 2005) or a model that uses probabilistic properties of the energy system to generate the functional forms used in curve fitting (Bardi 2005).

What insight is offered by these methods of modeling oil depletion? Which of these models best reproduce historical data? And, given the uncertainties involved, are any of these methods useful for predicting the course of future oil production? This systematic review attempts to address these questions.

1.1 Outline of report structure

A wide range of mathematical models of oil depletion are reviewed.[2] I start by outlining simple quantitative models (Section 2), followed by curve-fitting methods such as Hubbert's method (Section 3). Next, I review simulations of resource discovery and extraction (Section 4), and "bottom-up" field-level projections of near-term production (Section 5). Lastly, I review economic models of oil depletion (Section 6).

For each modeling approach, I discuss its physical and/or economic basis, including its grounding in observed data. I attempt to critique each model within its original context, and save overarching critiques that apply across classes of models for the synthesizing

systems have been used to group models. Walls (1992) groups models into geologic/engineering and econometric models, while Kaufman (1983) groups models of oil resource estimation into six categories.

[2] This review represents a comprehensive survey of the published, peer-reviewed literature. Some non-peer reviewed industry journals such as *Oil & Gas Journal* are included, as well as a few prominent or particularly original works published in the informal or government literature.

discussion. I do not review models used for predicting production from individual fields (i.e. exponential or hyperbolic decline curves) (Arps 1945). I also do not review statistical "discovery process" models, because they are more typically used to estimate URR (Arps and Roberts 1958; Drew and Schuenemeyer 1993).

I conclude by addressing overarching topics (Section 7). First, I classify the variety of models reviewed in the paper along a number of dimensions and discuss a convergence across a number of model types toward "bell-shaped" production profiles. I then question whether we can determine which model "works best" at fitting historical production profiles. I then summarize what is known about the predictive value of these models, and discuss the role of complexity in the predictive ability of models. I conclude by describing one way in which future oil depletion models could be improved.

1.2 Terminology and mathematical formulation

Some terms are used repeatedly throughout this report. A *production profile* or *production curve* is a plot of oil production with volumes of oil produced plotted on the y-axis and time on the x-axis. The *production cycle* is the complete production profile from the start of production to when the resource is exhausted.

Reserves are the volume of oil estimated to be extractable from known deposits under current technical and market conditions. The level of confidence in these estimates is typically indicated by the terms *proved* reserves (1P), *proved and probable* reserves (2P) and *proved, probable and possible* reserves (3P). Typically, only proved reserves estimates are publicly available.

Cumulative discoveries for a region represent the sum of cumulative production and reserves in known deposits. Estimates of cumulative discoveries tend to grow over time, as a result of improved recovery, additional discoveries and other factors. This is commonly referred to as *reserve growth* although it is more accurately the estimates of cumulative discoveries that are growing, rather than declared reserves. *Ultimately recoverable resources* (URR) are the volume of oil estimated to be economically extractable from a field or region over all time. For known deposits, the URR represents the sum of cumulative discoveries and estimates of future reserve growth. For a geographical region, the URR represents the sum of cumulative discoveries, future reserve growth and *yet to find* resources. The *remaining resources* for a region are all the resources that have yet to be produced – calculated by subtracting cumulative production from the estimate of URR.

Nearly all models presented in this review are of the general mathematical form:[3]

$$y = f(x_1, x_2, x_3 \ldots, \beta_1, \beta_2, \beta_3 \ldots) + \varepsilon \, . \quad \textbf{eq. 1}$$

[3] A good reference for the statistics of model fitting is the NIST Engineering Statistics Handbook. (NIST/SEMATECH 2008).

Table 1. Mathematical notation used in this study

t	Time, typically measured in years
t_0	Initial time period (first year of production or first year of model fit)
t_{peak}	Year of peak oil production
t_{ex}	Year of exhaustion of oil resources
$P(t)$ or P	Oil production in a given year t, equal to $Q'(t)$ or dQ/dt
P_0	Oil production in initial year t_0
P_{peak}	Oil production in year t_{peak}, or maximum oil production rate
$Q(t)$ or Q	Cumulative oil production to year t, equal to sum of $P(t)$ from years t_0 to t
Q_∞	Ultimately recoverable resources (URR), equal to sum of $P(t)$ from years t_0 to ∞.
$D(t)$	Cumulative discoveries to year t
$R(t)$ or R	Current reserves in year t
$M(t)$ or M	Remaining resources in year t, equal to Q_∞ - $Q(t)$. $M(t)$ is larger than $R(t)$ due to undiscovered oil and reserve growth.
r_{inc}	Rate of increase of oil production
r_{dec}	Rate of decrease of oil production

Here the dependent variable y is a function f of a set of input data x_i, parameters β_i, and random error ε.[4] In model fitting, historical values of the dependent variable and input data are used to fit the model so as to solve for the values of the parameters β_i. These fitted values of β_i are then used to make predictions by solving the model for future values of the input data. In nearly all cases described here the dependent variable is P, oil production in a given year, and common input data include the year, URR, or oil price.

In general, I refer to model parameters β_i in this report as *parameters* or *free parameters*. I refer to x_i as the *input data*, or as an *input time series* if a series of observations (e.g., yearly oil price) is used in f. I will often call a given input datum a *constraint* if it is a value that in some instances of the model could be treated as a free parameter, but in this case is assigned a fixed value. For instance, in some simple curve-fitting models, the value of URR can be left free to vary (therefore a free parameter), but in most instances a fixed value is provided for URR in order to constrain the fitting algorithm and improve model results.

Also, in order to make the variety of models outlined here comparable, I standardize their mathematical notation wherever possible (see Table 1). Therefore, notation used here will not always align with notation in the cited work, but mathematical equivalency will be maintained.

[4] Other names for these parts of a model are the response variable (y) and predictor variables (x_i). Because most oil depletion models are not built by statisticians, the random error component is generally not made explicit. Authors of such models generally acknowledge implicitly that there is an error term, through suggestions that their model is, for example, "only approximate."

2 Simple models of oil depletion

The simplest models of oil depletion use estimates of recoverable oil volumes to calculate future availability of oil, often by calculating the exhaustion time of known resources.

2.1 Early concerns about oil depletion

Estimates of the lifetime of remaining oil resources were developed at least as early as 1883 (Olien and Olien 1993). At that time, US geologists Lesley and Carll predicted exhaustion of oil "in a generation." By the turn of the 20th century, concern about oil depletion began to increase due to rapid growth in automobile use. In 1905 the automobile enthusiast's magazine *Horseless Age* argued that "the available supply of gasoline…is quite limited, and it behooves the farseeing men of the motor car industry to look for likely substitutes" (McCarthy 2001). These concerns remained poorly quantified until the development of simple mathematical models of oil depletion.

2.2 Reserve to production ratio: The simplest "model" of depletion

The simplest mathematical model of oil depletion is the reserve-to-production ratio (R/P ratio). The number of years until reserve exhaustion (t_{ex}) is calculated by dividing an estimate of current reserves (R), or sometimes remaining resources (M), by current production (P):

$$t_{ex} = \frac{R}{P}, \qquad \textbf{eq. 2}$$

or,

$$t_{ex} = \frac{M}{P}. \qquad \textbf{eq. 3}$$

Because M accounts for reserve growth and yet-to-find oil, the estimate of t_{ex} from eq. 2 will be larger. Figure 2 (left) shows the production profile implicitly assumed by R/P models. These bear little relationship to actual production experience. Variations of the R/P methodology have been used since at least the early 1900s (Day 1909). If production grows exponentially at rate r after the initial model year t_0, as shown in Figure 2 (right), then

$$\int_{t_0}^{t_{ex}} Pe^{rt}dt = R, \qquad \textbf{eq. 4}$$

or if we solve for t_{ex},

$$t_{ex} = \frac{1}{r}\ln\left(\frac{Rr}{P}+1\right).^{5} \qquad \textbf{eq. 5}$$

[5] Of course, M could also be used in place of R in this model as well.

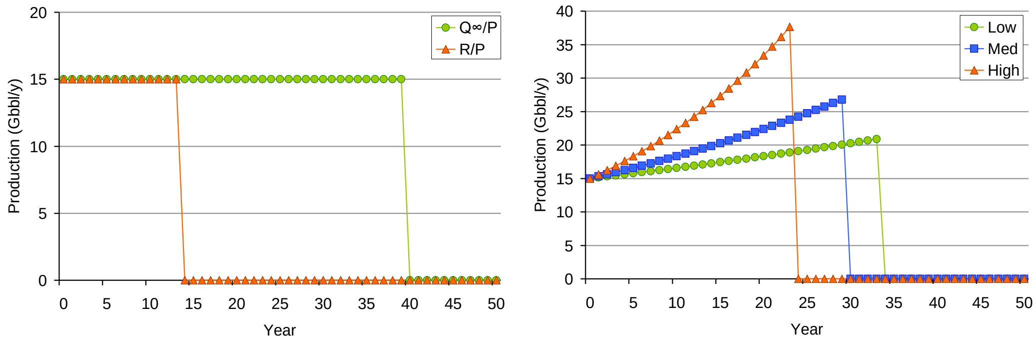


Figure 2. Left: Production profile assumed by simple reserve-to-production ratio model, with *R* ≈ 200 Gbbl (eq. 2) and *M* ≈ 600 Gbbl (eq. 3). Right: Production profiles for reserve-to-production ratio assuming exponential production growth (eq. 4). Each curve has *R* ≈ 600 Gbbl of oil, at growth rates of 0.01, 0.02, and 0.04 y^{-1} in the low, medium and high cases, respectively.

Use of the R/P ratio has a long history. Day (1909; 1909) published such calculations in Congressional reports and popular sources, causing significant concern (Olien and Olien 1993). He argued that oil resources would be depleted in 90 years at then-current rates of production, but that with growth only 25 years would be required. Reserve-to-production methods have been used continuously since, especially in popular and journalistic accounts of oil depletion (most frequently in support of optimistic assessments of resource availability).[6]

Reserve-to-production models are deficient. Comparing reserves to production is "a fallacious approach based on circular reasoning" (McCabe 1998). Reserves, at least in regulated markets, are estimates of what is currently known to be economically producible at a given level of confidence, not the total oil in place. Thus, R/P measures the inventory of discovered and delineated petroleum deposits, not the exhaustion time of the petroleum resource. Also, production does not stay constant and will not decline to zero in a single year. It was acknowledged long ago that production would peak and then decline, not following the implicit R/P profiles shown above. For example, White (1920) calculated an R/P of 14 to 16 years, but argued that the peak would occur "within five years…and possibly within three."[7]

For these reasons, R/P models are of virtually no use in predicting oil availability. Despite this, they continue to be used because they satisfy a "natural reaction that most people have" when they are told a reserve figure: to understand how much oil remains by calculating how many years are left if production remains constant (McCabe 1998).[8]

---

[6] Journalistic use of R/P occurred as early as 1920, when the New York Times (1920) cited a Bureau of Mines calculation that the United States "has only an eighteen year supply."

[7] White (1922) also gives an intriguing discussion of depletion that sounds remarkably similar to modern concerns, despite its formality: "our prodigal spending of our petroleum heritage may cause its too rapid depletion if not its early exhaustion in the midst of our spendthrift career, and at some untoward moment send us as beggars to foreign countries for the precious fluid not only to satisfy our extravagant habits but even to sustain our industrial prosperity, our standards of living, and our civilization."

[8] A recent example is Tony Hayward, CEO of BP who stated in June 2008 that "…Myth number two is that the world is running out of hydrocarbons. Not so. The world has ample resources, with more than 40 years of proven oil reserves…."

3 Curve-fitting models of oil depletion

Mathematical curve-fitting[9] models of oil production have been used since the 1950s. A variety of models exist, but their general approach is as follows:

1. Define a mathematical function to statistically fit to historical production data. These functions do not model causal mechanisms (i.e., production is a function only of time or cumulative extraction). The most common form is a bell-shaped logistic function.
2. Include constraints to improve the quality of projections made from the model fit. The most commonly used constraint is that total production must be less than estimated URR.[10]
3. Fit the constrained model to historical data in order to project future production.[11]

Curve-fitting models vary in the function used, in the use of URR as a constraint and in the assumption (or not) of symmetry of the model function.

3.1 Hubbert's logistic model

The most well-known curve-fitting model is that of M. King Hubbert. Hubbert first produced a schematic of his model of resource depletion in 1949, followed by his projection of future US oil production in 1956 (Hubbert 1949; Hubbert 1956). While some analysts argue that these projections were unprecedented, there were important historical antecedents to Hubbert's methods.

The idea that oil production follows a bell-shaped profile was advanced quite early. Arnold (1916) noted that "the crest of the production curve is not a sharp peak, but is represented by a more or less wavy dome." The work of Hewett (1929) was cited by Hubbert (1972) as a source of inspiration. Hewett applied a life-cycle framework to resource production, arguing that mineral producing regions would undergo a series of smoothed peaks, with the earliest peak in exports, and later peaks in the number of mills and smelters, and in production of metals.

In 1949, Hubbert plotted asymmetrical bell-shaped projections of fossil energy production over time. No function was defined for the curve; he simply argued that "the production curve of any given species of fossil fuel will rise, pass through one or several maxima, and then decline asymptotically to zero" (Hubbert 1949).

Ayres made similar projections a few years later (Ayres 1952; 1953). In 1953 Ayres predicted that United States peak production of oil would occur in 1960 or 1970

[9] This is not the only way to describe these models. (2003) uses the pejorative term "trendology" to refer to curve-fitting methodologies. Wiorkowski (1981) refers to these models as "black-box" models because they subsume all effects to a single trend that is a function of time.

[10] Another constraint used is that the production curve is analogous to the discovery curve but shifted in time.

[11] A related approach fits a mathematical function to historical data on either cumulative production or cumulative discoveries and uses this to *estimate* the URR. These approaches are discussed in detail in a companion report (Sorrell and Speirs 2009).

depending on the level of ultimate recovery (100 or 200 Gbbl URR, respectively). Ayres'prediction predated Hubbert's famous prediction by 3 years, used an identical estimate of URR, and arrived at the same peak date (Ayres 1953). Also, in 1952 the President's Materials Policy Commission study *Resources for Freedom*, an important and widely read study of the day, predicted peaks of 1963 to 1967 in two scenarios (PMPC 1952).

In March of 1956, Hubbert famously predicted that US oil production would peak between about 1965 and 1970 (Hubbert 1956). These two projections differed by the value of URR used to constrain the curve (150 and 200 Gbbl, respectively). This prediction was subsequently shown to be accurate when United States production peaked in 1970. Interestingly, one month earlier than Hubbert's prediction, analysts from Chase Manhattan Bank published a prediction showing a peak in US oil production in 1970 as well (Pogue, Hill *et al.* 1956).[12] Also, in June of 1956, Ion predicted a peak between 1965 and 1975 (Ion 1956).

In 1959, Hubbert first applied the logistic function as a mathematical model for cumulative oil and gas discoveries. By plotting cumulative discoveries as a function of time, a "sigmoid" curve was generated, which he fit with the logistic function. This curve can be extrapolated to find the asymptote of cumulative discovery (or URR). This value of URR can then be used to constrain the production curve, as cumulative production over all time must be less than or equal to cumulative discoveries.[13] He does not give justification for his choice of the logistic function, stating that

> if we plot a curve of cumulative production for any given area, this curve will start from zero with a very low slope, because of the slow rate of initial production, and will then rise more or less exponentially before finally leveling off asymptotically to some ultimate quantity…

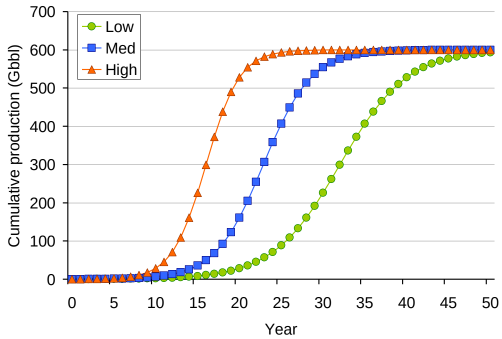
Hubbert published papers until the 1980s utilizing these basic methods. He also developed related analytical methods, including modeling cumulative discovery as a function of exploratory effort (Hubbert 1967). In 1980, he published a full derivation of his logistic model (Hubbert 1980). This includes a derivation of the logistic curve from an assumption that the rate of growth of cumulative production forms a parabola (that is, the growth rate will be zero at the beginning and end of production from a region). The resulting function for cumulative production is

$$Q(t) = \frac{Q_\infty}{\left(1 + N_0 e^{-a(t-t_0)}\right)}. \qquad \textbf{eq. 6}$$

Here a governs the spread of curve and N_0 is a dimensionless factor equal to

$$N_0 = \frac{Q_\infty - Q_0}{Q_0}. \qquad \textbf{eq. 7}$$

[12] Hubbert cites this text in his 1956 paper, so it is known that he read their work. The text of this article states that the peak will be in the "1965-1975 decade" but their plots show the peak in 1970.
[13] Hubbert also shifted the cumulative discovery curve by a fixed time increment to fit cumulative production data. It is unclear how primary this method was in his model fitting.



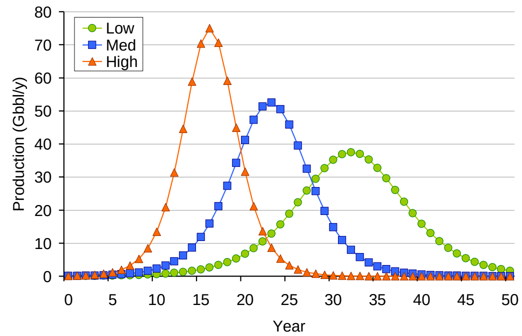


Figure 3. Left: Cumulative production in a logistic oil production model. In all cases URR is 600 Gbbl, and Q_0 is 0.2 Gbbl at time t_0 =1. *a* equals 0.25, 0.35, and 0.5 in the low, medium, and high cases, respectively. Right: Production in a logistic oil production model, using same settings as cumulative production curves.

In this equation, Q_0 equals the cumulative production in year t_0. Production in year t is given by $P(t)$, the derivative of $Q(t)$ with respect to time:

$$P(t) = \frac{dQ(t)}{dt} = Q_\infty \frac{aN_0 e^{-a(t-t_0)}}{\left(1 + N_0 e^{-a(t-t_0)}\right)^2} . \qquad \textbf{eq. 8}$$

Figure 3 plots cumulative production $Q(t)$ (left) and annual production $P(t)$ (right) from Hubbert's logistic model.

In order to solve for the free parameters in the model (Q_∞ and a), Hubbert developed a technique now called "Hubbert linearization" (Deffeyes 2003). He started with an assumption of a parabolic form for the differential equation governing the logistic curve:

$$\frac{dQ}{dt} = aQ - bQ^2 . \qquad \textbf{eq. 9}$$

This parabola has no constant term, because dQ/dt = 0 when Q = 0. We also know that the growth rate must also equal zero when all oil has been produced ($Q = Q_\infty$), so $aQ_\infty - bQ_\infty^2 = 0$, and thus $b = a / Q_\infty$. Therefore,

$$\frac{dQ}{dt} = aQ - \frac{a}{Q_\infty} Q^2 , \qquad \textbf{eq. 10}$$

and we can divide both sides of this equation by Q:

$$\frac{dQ/dt}{Q} = a - \frac{a}{Q_\infty} Q , \qquad \textbf{eq. 11}$$

Hubbert realized that this equation can be plotted as a straight line in $(dQ/dt)/Q$ vs. Q space. Solving this linear equation for the x-intercept (with $(dQ/dt)/Q$ = 0) gives the value of Q_∞.[14] This technique has seen a modern resurgence as a variation of the conventional Hubbert technique (Deffeyes 2003). It simply represents a transformation of the model into a space where logistic behavior appears linear. This improves the ability of the eye to

[14] Similar techniques have long been used to model the production from individual fields (Arps, 1945). However, at the field level, the post-peak decline in production is normally assumed to take an exponential form, leading to a linear relationship between $p(t)$ and $Q(t)$ when the data are transformed into logarithmic space.

spot a logistic trend and deviations from it, but offers no difference in the mathematical properties of the fit.[15]

3.2 Other curve-fitting models

A variety of Hubbert-like curve-fitting models exist. These models share properties of the Hubbert method while relaxing or altering some of its assumptions. Some key assumptions of Hubbert's mathematical method include:[16]

1. Production profile is given by the first derivative of the logistic function;
2. Production profile is symmetric (i.e. maximum production occurs when the resource is half depleted and its functional form is equivalent on both sides of the curve);
3. Production follows discovery with a constant time lag;
4. Production increases and decreases in a single cycle without multiple peaks.

Curve-fitting models that relax some or all of these assumptions are described below.

3.2.1 Gaussian models of oil depletion

In contrast to the logistic function, a Gaussian model of oil production is used by a some researchers (Bartlett 2000; Brandt 2007):

$$P(t) = P_{peak} e^{\left(\frac{-(t-t_{peak})^2}{2\sigma^2}\right)}, \qquad \textbf{eq. 12}$$

where σ is the standard deviation (width of the curve). Brandt (2007) uses an asymmetric version of the Gaussian curve to test for symmetry in production curves:

$$P(t) = P_{peak} e^{\left(\frac{-(t-t_{peak})^2}{2f(t)^2}\right)}, \qquad \textbf{eq. 13}$$

where

$$f(t) = \sigma_{dec} - \frac{\sigma_{dec} - \sigma_{inc}}{1 + e^{k(t-t_{peak})}}. \qquad \textbf{eq. 14}$$

In eq. 14, σ_{inc} and σ_{dec} are the standard deviations on the increasing and decreasing sides of the production curve. k governs the rate at which σ shifts from the increasing to decreasing value. Note that for small values of t, σ_{inc} dominates, while σ_{dec} dominates at large values of t. Schematic plots of conventional and asymmetric Gaussian functions are given in Figure 4.

3.2.2 Exponential models of oil depletion

Exponential decline in production at the field level was noted as early as 1908 (Arps 1945). In 1916 Arnold argued that "the rate of decrease [is] based on the previous year's

[15] Caithamer (2007) argues that fitting a line to linearized data at certain points in production history would result in infinite oil production being projected. While the behavior of the transformed data will eventually settle and allow projection to the axis, it is unclear how soon one can be certain that this has occurred.
[16] Hubbert (e.g.,1982) argued repeatedly and forcefully that the production profile in reality need not be symmetric or bell-shaped in reality, but his mathematical models were always based on this simplified formulation.

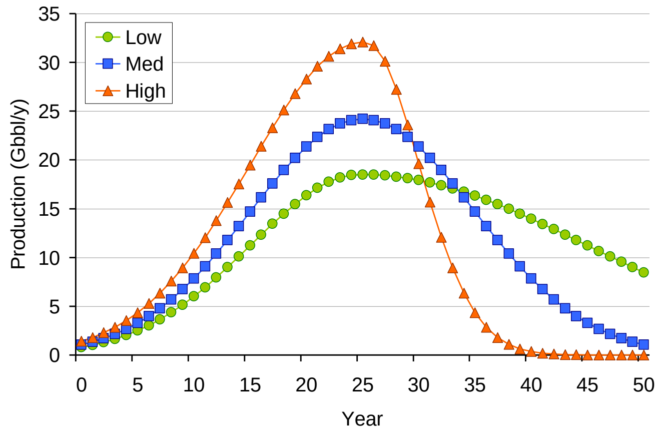


Figure 4. Symmetric (medium) and asymmetric (low, high) Gaussian oil production profiles. Cumulative production over all years ≈ 600 Gbbl for three curves. In all cases σ_{inc} = 10 years and t_{peak} = 25 years. σ_{dec} = 20, 10, and 5 years in low, medium, high cases, respectively.

production, becoming gradually less and less." Exponential regional production curves are often justified by analogy with field-level models of exponential production decline, but it is unclear whether this analogy holds rigorously.[17]

Exponential growth or decay is characterized by a constant percentage change in the rate of production per year. It is given by the equation $P(t) = P_0 e^{r(t-t_0)}$, where production in an initial year (P_0) grows by the rate r after the initial year t_0. A simple model of exponential increase and decrease is defined as follows:

$$P(t) = \begin{Bmatrix} P_0 e^{r_{inc}(t-t_0)} & \text{for } t \le t_{peak} \\ P_{peak} e^{-r_{dec}(t-t_{peak})} & \text{for } t > t_{peak} \end{Bmatrix}. \qquad \textbf{eq. 15}$$

[17] For example, if each of the fields in a region exhibit exponential depletion, will the aggregate production curve for the region also exhibit exponential decline? Depending on the timing of the projects, this is not necessarily the case (see Figure 7). Also, if the actual function followed resembles a combination of functional forms, such as the suggestion by Meng and Bentley (2008) that production is bell-shaped on the increasing side and exponential on the decreasing side, then they need not aggregate to an exponential curve at all.

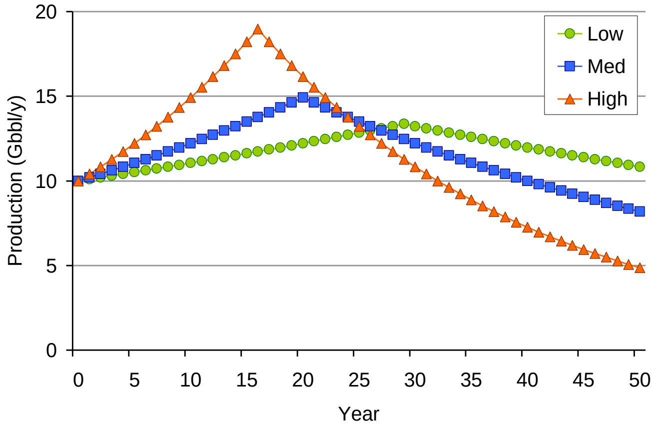


Figure 5. Exponential model of oil production. All profiles have cumulative production of ≈ 600 Gbbl over years shown. All curves are symmetric, with growth rates of 1, 2, and 4% per year in low, medium, and high cases respectively.

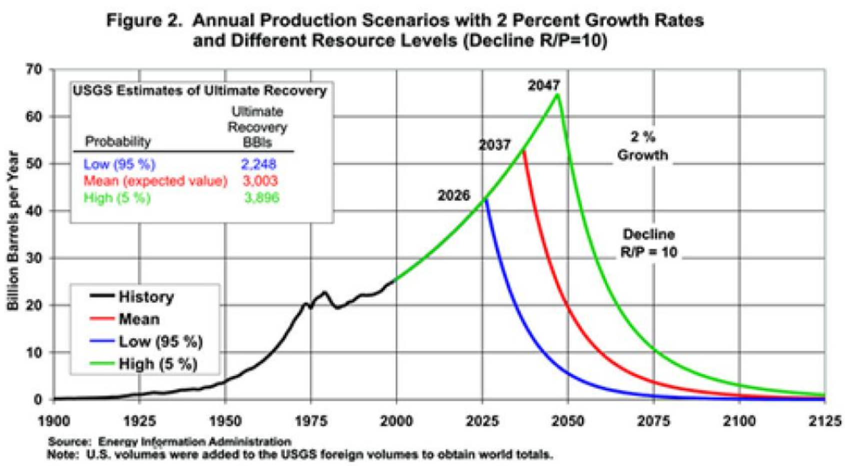


Figure 6. Projections from the Wood *et al*. EIA study (Wood, Long *et al.* 2000). Note the rapid rate of decline in production due to the assumption of decline under a constant R/P of 10 years.

Where r_{inc} and r_{dec} are the absolute values of the rates of exponential increase and decrease. A schematic of the exponential production model is shown in Figure 5.

A notable exponential projection was made by analysts from the US Energy Information Administration (Hakes 2000; Wood, Long *et al.* 2000). The projection utilizes probabilistic USGS estimates of recoverable reserves with a globally-aggregated exponential depletion model to produce profiles with very sharp peaks. They assume that production growth continues at about 2% per year until the global remaining-resources-to-production ratio (M/P) reaches 10 years,[18] at which point production declines in order

[18] This assumption derives from US experience. When US production peaked, the proved reserves-to-production ratio equaled about 10 years. But the USGS data used in the EIA model includes reserves, yet-to-find oil, and reserve growth until 2030 at a given probability (i.e., remaining resources). Thus, their application of the R/P rule actually represents a M/P model.

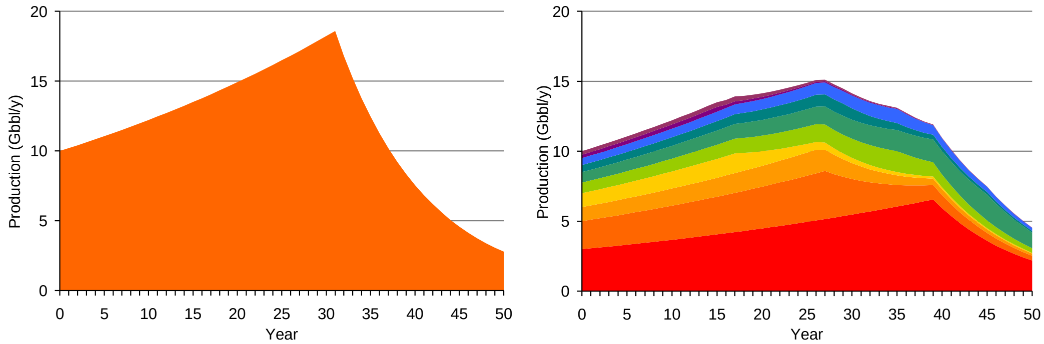


Figure 7. Two exponential production models. In both cases, production for each region increases at 2% per year and decreases at 10% per year, as in Wood *et al.*, and cumulative production in both cases is ≈ 600 Gbbl. Left: production follows the R/P decline rule in the aggregate (as in Wood *et al.*). Right: each of 10 regions individually follows the same R/P decline rule with different sized resource endowments.

to keep M/P equal to 10 years. This results in a 10% decline in production per year. Some results from this projection are shown in Figure 6.

Also, Hallock *et al.* (2004) use a modified exponential methodology. Production increases in each country to meet domestic demand plus an increment for new world demand. The production rate increase for each nation is capped at a different rate depending on the scenario (at 5%, 7.5%, or 15%). Once cumulative production reaches 50% of URR (or 60% in other scenarios), production declines exponentially. The growth rates around the peak (between 45 to 55% depletion) are modulated to smooth the peak.

A key problem with exponential models is the interaction between aggregation and decline rates (Cavallo 2002). In the EIA model, production increases until the *global* R/P ratio reaches the target value. This is very different than a model where each country or region behaves according to the same R/P rule. The difference is illustrated in Figure 7. The left plot shows a globally aggregated exponential depletion model (as in the work of Wood *et al.*), while the right plot shows the same total quantity of oil produced but with production divided between 10 regions that independently follow the same R/P rule. In reality, the appropriate decline rate will vary by scale, and will generally be highest at the field level, with slower decline observed in aggregated regions.

Empirical evidence for exponential behavior exists. Exponential and exponential-like production declines have been observed at the field level for decades (Arps 1945). And there is evidence that production declines exponentially for larger regions as well. For example, Pickering (2008) found that annual production in a given region is often a linear function of the country's proved reserves in that year. This, in essence, represents a fixed R/P production rate, or exponential decline. He found that the slopes of the linear relationship between reserves and production in non-OPEC "fringe" and "small fringe" countries suggested 4.6 and 3% exponential decline. Brandt (2007) found a median value of r_{dec}= 2.6% for 74 post-peak regions. On the other hand, Skrebowski (2005) argues that overall depletion from existing sources is likely 4 - 6%.

3.2.3 Linear models of oil depletion

Linear models of oil production are used infrequently [e.g., (Hirsch, Bezdek *et al.* 2005; Brandt 2007)], although they represent the most simple formulation of a rising and falling production curve. A linear production profile can be defined as

$$P(t) = \begin{Bmatrix} P_0 + S_{inc}(t - t_0) & \text{for } t \leq t_{peak} \\ P_{peak} - S_{dec}(t - t_0) & \text{for } t > t_{peak} \end{Bmatrix}, \qquad \textbf{eq. 16}$$

where S_{inc} and S_{dec} are the absolute values of slopes on the increasing and decreasing sides of the production curve. A special case is the symmetric case where $S_{inc} = S_{dec}$. Despite its simplicity, the linear model provides a relatively good approximation to the production profile of some oil producing regions: Brandt (2007) found that it was the best fitting model for 26 out of 139 studied regions (see Figure 10).

3.2.4 Multi-cycle and multi-function models

In contrast to models where production rises and falls in a single cycle, multi-cycle and multi-function models attempt to recreate the non-smooth production profiles seen empirically. Multi-cycle behavior was noted by Hubbert (1956), who argued that Illinois was a multi-cycle region:

> The first period of discovery, beginning about 1905, was based on surface geology with meager outcrop data. Consequently in about five years most of the discoveries amenable to this method had been discovered...It was well known geologically, however, that the whole Illinois basin was potentially oil bearing, which was later verified when a new cycle of exploration using the seismograph was initiated in 1937.

Multi-cycle models have been developed by Laherrere and Patzek (Laherrère 1999; 2000; 2003; Patzek 2008). These models fit the sum of a number of independent logistic production cycles to the overall production data, as shown in Figure 8. Ideally, each additional cycle represents the production of a well-defined resource that can be differentiated from the main body of production. Laherrere argues that "almost every country can be modeled by at most four cycles in which discovery peaks are correlated with corresponding production peaks after a time-lag giving the best fit" (Laherrère 1999). He shows two examples of this, but has not illustrated its generality.

Mohr and Evans (2007; 2008) built a multi-function model that uses a bell curve but models disruptions by postponing the bell curve at the point of a disruption. The disrupted period is modeled with one or more simple polynomials, after which the bell curve resumes, shifted to account for the additional cumulative production that occurred during the period of disrupted production:

$$P(t) = \begin{Bmatrix} P_{bell}(t) \text{ for } t < t_{a0} \\ f_1(t) \text{ for } t_{a0} \leq t < t_{a1} \\ f_i(t) \text{ for } t_{ai-1} \leq t < t_{ai} \quad (\text{for disruptions } i = 2 \ldots n) \\ P_{bell}(t + (t_{an} - t_{a0})) \text{ for } t \geq t_{an} \end{Bmatrix}. \qquad \textbf{eq. 17}$$

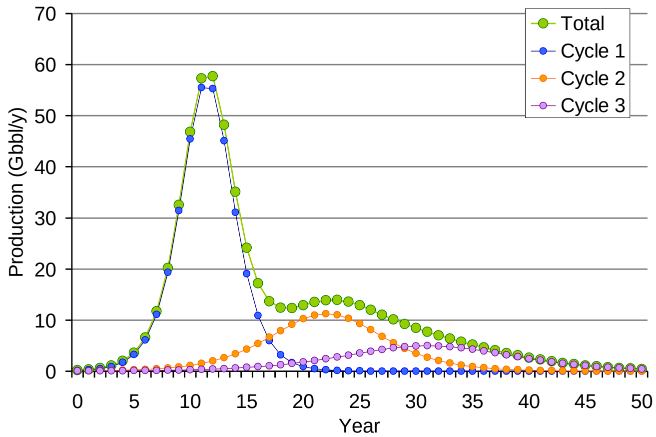


Figure 8. Multi-cycle production model with cumulative production over all years of ≈ 600 Gbbl of oil. Three cycles have 350, 150 and 100 Gbbl ultimate production each. These might represent, for example, production from primary, secondary, and tertiary (EOR) technologies.

Here P_{bell} is production from a bell-shaped curve model. The f_i are a series of n polynomials anchored on each side at $n+1$ disruption points, which occur at times t_{a0} to t_{an}. After the period of disrupted production, the underlying bell-shaped model is shifted to the point where the disruptions end. Guseo and Dalla Valle (2005; 2007) also include exogenous shocks in a model of technological diffusion that they apply to oil depletion modeling.

One problem with these methods is the arbitrary addition of production cycles. The quality of model fit to data can be improved by adding more cycles, but the better fit of the more complex model is often not justified by the additional model complexity. For example, In Mohr and Evans (2008) the bell curve is interrupted by two first-order polynomials, one second order polynomial, and one third order polynomial. This adds between 11 and 16 free parameters to the model, depending on if the breakpoints t_{ai} are also chosen by algorithm, resulting in danger of spurious overfitting. One approach to address this concern is with information-theoretic approaches, as discussed in Section 7 (Burnham and Anderson 2002; Motulsky and Christopoulos 2004).

The second problem is that additional cycles are inherently unpredictable. They might represent new discovery cycles enabled by advances in exploration technology, such as the case with Illinois and the introduction of the seismograph; or by new production technologies, such as thermal EOR applied to heavy oil deposits; or by the discovery of a new type of deposit that was previously unknown, such as very deep offshore oil. Lynch (2003) argues that this technique "destroys the explanatory value of the bell curve" because "not knowing whether any given peak is the final one renders [predictions] useless." At the least, without *a priori* justification for additional cycles such a modeling approach can quickly degrade into what Burnham and Anderson (2002) call "data dredging." These concerns, and the broader principle of parsimony, are discussed in Section 7.

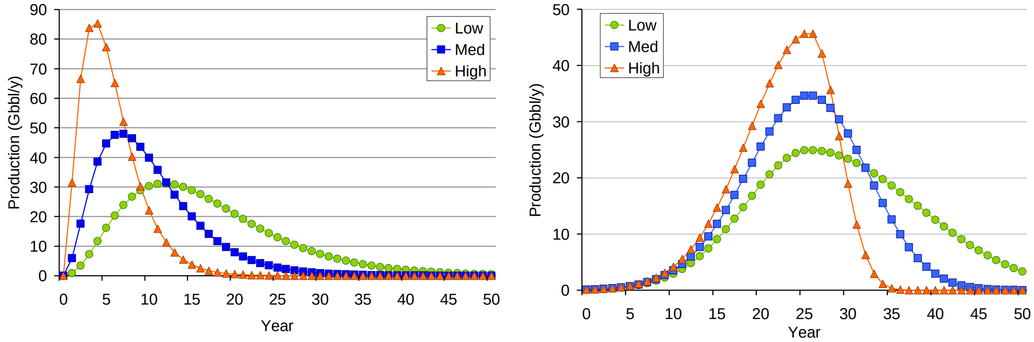


Figure 9. Two asymmetric bell-shaped models. Left: Production profile of the skew normal production profile (SNPP) model of Hammond and Mackay (1993). All curves have total cumulative production over all years of ≈ 600 Gbbl. Parameter settings for low, medium, and high cases are A = (1, 8, 52); n = (2.35, 2, 1.8); and α = (0.2, 0.3, 0.5), respectively. Right: Asymmetric growth model of Kaufmann and Shiers (2008). Cumulative production sums to ≈ 600 Gbbl in all cases. Growth and decay rates (r_{inc}, r_{dec}) are (0.51, 0.165), (0.545, 0.545) and (0.575, 2) for low, medium and high cases, respectively.

3.2.5 Asymmetric smooth curve-fitting models

A variety of asymmetric smooth curve-fitting based models of production have been developed but not widely used. Not long after the work of Hubbert, Moore (1962; 1966) used the Gompertz curve to fit cumulative production and discovery data:

$$Q(t) = Q_{\infty} a^{b^t},$$ **eq. 18**

where a and b are fitting constants (a is defined as the ratio of the initial value of cumulative production Q_0, to the value of ultimate production Q_{∞}, which allows $Q = Q_0$ when t = 0). The Gompertz curve is asymmetric: the decline is slower than the increase. Wiorkowski (1981) argued that the Gompertz curve used by Moore resulted in a poor fit to historical data.

Figure 9 (left) shows the skew normal production profile, or SNPP, developed by Hammond and Mackay (1993) for use in projecting UK oil production:

$$P(t) = At^n e^{-\alpha t},$$ **eq. 19**

where A is a scaling coefficient, n is a shape factor (larger n results in low peak delayed in time), and α is the "peak position factor" (larger α value shifts peak forward). This model was used to project UK oil and gas production, although it resulted in overly pessimistic projections of the peak in UK oil production. The same function was also used by Feng *et al.* (2008) to forecast Chinese oil production, although they refer to it as the Generalized Weng equation.

More recently, Kaufmann and Shiers (2008) built an asymmetric bell-shaped model that continues directly from historical production data (see Figure 9, right). It is solved iteratively as a set of 3 equations. The growth equations can be simplified as follows:

$$P(t)=\begin{cases} P_{t-1}\left(1+r_{inc}-t\left(\frac{r_{inc}}{t_{peak}+1}\right)\right) & \text{for } t\le t_{peak} \\ P_{t-1}\left(1+r_{dec}-t\left(\frac{r_{dec}}{t_{peak}+1}\right)\right) & \text{for } t> t_{peak} \end{cases}. \qquad \textbf{eq. 20}$$

These equations are constrained by the requirement that cumulative production is always less than URR. The resulting production curve (Figure 9, right) is used to generate projections of the required rate of development of substitutes for conventional petroleum (SCPs).

Berg and Korte (2008) built three models (each a system of differential equations) that expand upon Hubbert's logistic differential equation. They add aspects of simulation models, as described in Section 4. Their first model adds demand to the Hubbert model. Another model adds a third differential equation governing the dynamics of additions to reserves. Their differential equation governing reserves forces discoveries to decline as cumulative reserves additions increase,[19] but does not account for technological change that also occurs as reserves are depleted.

3.3 Difficulties with curve-fitting models

Curve-fitting models are simple models that reduce many complex phenomena, to a small number of equations. This results in difficulties that are widely discussed in the literature (e.g. Lynch 2003), include a reliance on estimates of URR, assumptions of symmetry, and assumptions of bell-shaped production profiles.

3.3.1 Is URR a reliable constraint?

Estimates of URR are a key input to curve-fitting models. Caithamer (2007) notes that unconstrained fitting of the logistic model to pre-1970 global production data predicts 2.57×10^{15} bbl URR, about 3 orders of magnitude greater than USGS resource estimates. This is because there is not enough information in pre-peak production data to generate stable values of model parameters without the aid of constraints.[20] (This situation improves the further into the production cycle a region becomes, as the possibilities for divergent futures become increasingly narrow.)

However, using URR to constrain curve-fitting models is problematic. First, estimates of URR have been too low in the past. This type of error has plagued curve-fitting models, especially with regard to projections of world production (Lynch 1999; Lynch 2003).[21]

---

[19] $dD/dT \propto (1-D)$ where D is cumulative reserves discovered, normalized to reach a maximum value of 1.

[20] This is because 1970 was at least 35 years before the peak in production, meaning that there was still significant uncertainty with respect to any signal provided by the data. Also, arguments of this type should not be taken too far: unconstrained models of 3 or 6 parameters will not be used in practice because we do have additional information to bring to bear on the problem (e.g., we know to ignore results suggesting URR of 10^{15} bbl).

[21] For example, Lynch (2003) notes that Campbell increased his value of global URR from 1,575 Gbbl in 1989 to 1,950 Gbbl in 2002.

Estimates of URR can be too low for two reasons: they can underestimate new discoveries and they can underestimate reserve growth. Both of these problems have traditionally been observed.

Another more fundamental problem is that while URR is used as a fixed constraint in curve-fitting models, there are serious problems with viewing URR as a static value. Adelman and Lynch (1997) argue that "the mistake is not mere underestimating of URR. It is the concept of URR as a fixed amount, rather than a dynamic variable." The reason is that URR is fundamentally a hybrid economic-physical concept. If oil resources suddenly became less valuable, then the same physical endowment of resource deposits would result in a lower value of URR. Or alternatively, if consumers are willing to sacrifice more (i.e., pay more) for a unit of petroleum, then URR will grow with no changes in physical properties of the resource. The differences between physical and economic views of resources are discussed in greater detail in Section 7.

The seriousness of the URR problem is a point of disagreement among researchers. Some argue that increases in estimates of global URR have leveled off, and that we are asymptotically reaching the true value of URR, especially if consistent 2P reserves data are used (Bentley, Mannan *et al.* 2007).[22] Others disagree, suggesting that URR values will likely continue to grow (McCabe 1998), and that hydrocarbons will be produced from resources that are currently not included in estimates of URR.[23]

3.3.2 Are bell-shaped models better than other model types?

Hubbert's use of a bell-shaped production profile, together with its reasonable fit to many regions (most notably the US), has created a desire to justify the use of such curves with rigorous scientific reasoning. While Hubbert (1980) noted the roots of the logistic equation in 19th century population biology, he never gave a detailed explanation for his choice of this model.

Recently, Rehrl and Freidrich (2006) described a simple thought experiment that generates logistic behavior from the interaction of information and depletion on oil discovery rates. First, they assume that geologic and technical information, I, is directly proportional to cumulative discoveries, D:[24]

$$I \propto D \, . \qquad \textbf{eq. 21}$$

Next, they assume that the rate of discovery is also directly proportional to the amount of information that we have, and by the equation above, therefore proportional to discoveries:

[22] 2P reserves are said to represent more realistic and stable reserve estimates than the proved reserves commonly reported in the United States and other countries. This conservatism is argued to be the reason behind the significant reserve growth observed in the US.

[23] Given that current estimates of URR neglect some 60-70% of known oil in place (oil that will be left behind by current extraction technologies), Lynch (1999) argues that stable URR estimates make the "unrealistic assumption that technological progress will effectively cease."

[24] They note that this model is simple, as there is "no justification why the regarded proportionalities should be linear to the first power," and proceed given that this is the simplest possible relationship.

$$\frac{dD}{dt} \propto I, \qquad \textbf{eq. 22}$$

and so

$$\frac{dD}{dt} \propto D. \qquad \textbf{eq. 23}$$

This is the differential form of exponential growth. They call this the "information effect" whereby an increase in cumulative discoveries further increases our ability to find more oil.

Next, they describe the opposing "depletion effect," whereby discoveries hamper future discovery by reducing the amount of oil left to be found. They argue that a reasonable form of this relationship is

$$\frac{dD}{dt} \propto \left(Q_\infty - D\right), \qquad \textbf{eq. 24}$$

which suggests that discoveries drop to zero as D approaches Q_∞. We can combine these two statements of proportionality with an arbitrary constant *a*:

$$\frac{dD}{dt} = aD(Q_\infty - D) = aQ_\infty D - aD^2. \qquad \textbf{eq. 25}$$

Note that eq. 25 is the differential form of the logistic curve (this time in discoveries instead of production). Thus, they argue that simple relationships can generate a logistic *discovery* function. But, it does not necessarily follow that *production* would follow a logistic path as well. In free market conditions under ample demand, it is reasonable to assume that oil discovered will be promptly brought into production,[25] resulting in a consistent lag between discovery and production. In reality, economic and political constraints can alter investment and production.

Other authors use the central limit theorem (CLT) to justify bell-shaped models. In 1952 Ayres foreshadowed this argument: "For some single oil fields the peaks can be relatively sharp, but for the sum of effects of many oil fields in many countries the peak can be expected to be blunt." [26] There is unfortunately little basis for applying the CLT to oil production curves in general (McCabe 1998; Babusiaux, Barreau *et al.* 2004; Brandt 2007). The CLT acts to generate a Gaussian distribution when distributions that are *independent* of one another are *summed* or *averaged*. Dice provide a good example. Rolling one fair die will result in equal probability of obtaining the values 1 to 6 — a uniform distribution. Rolling 5 dice and summing the results will give a range from 5 to 30, but the distribution will not be uniform. The value 5 will rarely be obtained, while the value 15 will be obtained much more commonly (because many combinations of the dice result in a sum of 15). If the dice were rolled and summed many times, a plot of the distribution of results would appear Gaussian.

---

[25] Since so much of the cost, and risk, in oil and gas development is in the exploration stages, producers have little incentive to hold back production once the riskiest portion of the process has resulted in a successful discovery.

[26] Ayres (1952) cited the insights of statistical physics as a justification: "All of modern physics is founded upon the conception of predictability of the sum of large numbers of unpredictable events. The larger the number of events, the greater the probability of prediction."

The crucial difference is that while field-level production curves *are* summed to produce an aggregate curve, they are *not* independent. Production from an oil field is determined at least in part by the decisions of the producers. And producers across regions, nations, and even at a global level, face common stimuli. At a regional level these include transport costs, availability of refining capacity for a given type of crude, and regulatory pressures (such as state or provincial environmental mandates). At a national level, politics can alter production decisions, particularly in nations with central control over production, but even in nations with independent oil companies. And, globally, both long and short-term trends (such as the modern spread of automobility to Asian nations or demand reductions caused by the 1970s oil crises) influence producers simultaneously across the globe.

Given that there is some degree of independence between producers, some smoothing clearly occurs with aggregation. This amount of smoothing varies by region. But, the shape of the global historical production curve is so non-Gaussian that it is very unlikely to have arisen from the summation of tens of thousands of truly independent production profiles.

Other arguments for bell-shaped curves exist.[27] Bentley *et al.* (2000) note that a sum of triangular field-level profiles generates a bell-shaped curve if the largest fields are found first. These ideas are discussed in Section 5 and shown in Figure 20. Also, Cleveland and Kaufmann (1991) suggest that the bell-shaped production profile is the result of physical changes in the resource base causing the long-run cost of production to increase. This increase in cost induces the peak and decline in production. These ideas are discussed in Section 6.

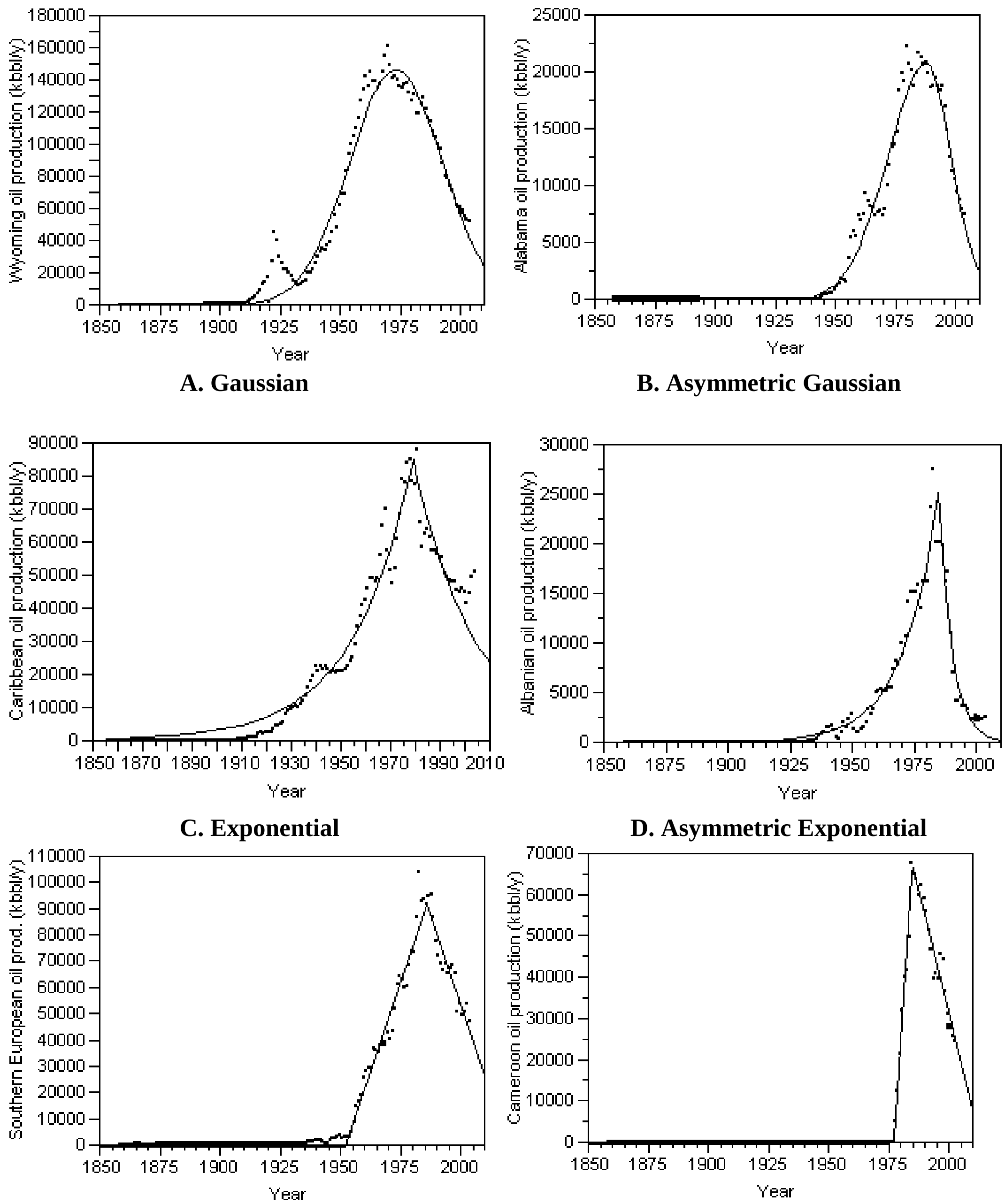
Thankfully, the historical record provides us with some evidence regarding the usefulness of bell-shaped curves. While many production curves are well-approximated by bell-shaped profiles, a good number of production profiles deviate from bell-curve-like shapes: some are significantly more pointed than the bell-curve (Hirsch 2005; Brandt 2007), while others have multiple peaks separated by significant amounts of time (Hubbert 1956; Laherrere 2003; Patzek 2008).

Brandt (2007) compared the fit of six simple (3 and 4 parameter) curve-fitting models to 139 oil production curves at a variety of scales (US states and regions, countries and multi-country regions) Geological definitions for regions (e.g., basins or plays) were not used because of lack of data. He compared symmetric and asymmetric versions of a Gaussian bell-shaped model, a linear model and an exponential model. This comparison

[27] Also, Guseo and Dalla Valle (2005; 2007) argue a model of technological diffusion containing "early adopters" and "word of mouth" adopters as the basis for the bell-shaped profile. On the other hand, Weiner and Abrams (2007) give a physical explanation for logistic production curves. They build a simple physical model of oil depletion as draw-down of fluid from a sealed container containing gas and fluid under pressure. This conceptually appealing approach does not account for other factors affecting the rate of oil production from a deposit over time (water influx, multi-phase flow effects, etc.)

used Akaike's Information Criterion (see Section 7 for more information) to account for the different functional forms and numbers of parameters across the models.

Most generally, he found that these simple models worked reasonably well as a group: out of 139 production curves analyzed, only 16 were found to be significantly non-conforming to any of these simple models. But he found less strong evidence to choose one functional form over another. If only symmetric models were allowed, then the Gaussian model was found to be the most successful, fitting best in 59 regions, while only 26 regions were best fit by each of the exponential and linear models. If symmetry was not required, the asymmetric exponential model was found to be best fitting in 25 regions, compared to 14 regions for the asymmetric Gaussian model, which was the next most successful. To illustrate how excellent the fit to these disparate models can be, Figure 10 shows six regional production curves that were each fit best by one of the six tested models.



A. Gaussian

B. Asymmetric Gaussian

C. Exponential

D. Asymmetric Exponential

E. Linear

F. Asymmetric Linear

Figure 10. Six model types applied to regions where they were found to be the best fitting model.

3.3.3 Are production profiles really symmetric?

Lastly, we can ask whether production profiles are best represented with symmetric models. The motivation for symmetric production models is typically that they are more parsimonious than asymmetric models. Is this simplification supported by available data?

The historical record again provides insight here. PFC Energy studied 18 post-peak countries, finding an average level of depletion of 54% at the start of decline, with the majority of regions peaking at between 40 and 60% of estimated URR (PFC 2004). These percentages are likely to decline, because they were calculated relative to estimates of URR, which are more likely to increase than to decrease.[28]

Other indications from historical production data suggest that production tends to be asymmetric, with the decline slower than the increase. Brandt (2007) found that the production-weighted mean rate of exponential decline was approximately 4% less than the production-weighted mean rate of increase (≈ 2% vs ≈ 6% increase) for 74 post peak regions, ranging in size from US states to sub-continents. In nearly all cases (67 out of 74 regions) the rate difference ($r_{inc} - r_{dec}$) was found to be positive.

This observation aligns with the intuition that improvements in technology will make the production decline less steep than the increase. This was noted as far back as Hubbert (1956), who argued that "a more probable effect of improved recovery will be to reduce the rate of decline after the culmination…". Renshaw (1981) also gives a theoretical reason for this. If a single cumulative production sigmoid is instead replaced with, say, the sum of 5 sigmoids which represent resources of differing difficulty of extraction, the net effect on the summed curve is commonly to slow the rate of decline. That is, harder-to-extract resources are accessed more slowly and act more to lessen the decline in production rather than to postpone the peak.[29]

These data suggest that production profiles tend to be slightly asymmetric, with slower rates of decline than rates of increase. Whether this complexity should be included in a model of oil depletion depends on the purpose for which a model is being developed and the level of detail that is required by such a purpose. This broader issue of model complexity is analyzed in Section 7.

[28] Thus, a region that was originally thought to have peaked at 54% depletion of URR might instead later be recalculated to have actually peaked at 49% of URR, if URR grows between the first and second estimate.
[29] Interestingly, Renshaw draws this result from the psychological work of Thurstone in the 1930s on the learning function as applied to learning tasks of varying difficulty.

4 Systems simulation: resources, discovery rates, and technologies

Simulation models differ in modeling philosophy from curve-fitting models. Instead of fitting a simple pre-defined function to historical data in order to project future production, simulation models explicitly represent physical and/or economic mechanisms that govern oil discovery and extraction. They then let broader behavior of the oil production system (such as the shape of the production profile) emerge from this underlying structure.

The simulation approach remedies a key problem of curve-fitting models noted by Taylor (1998): "No cause-and-effect relationship exists between time and the exploitation of crude oil." Simulation modelers would argue instead that a complex causal chain connects time and oil extraction. Population growth and economic expansion caused increased demand for oil. This has induced the oil industry to search for and produce oil. Exploration affects our prospects for finding oil in the future by simultaneously increasing our knowledge of the subsurface and by removing another oil field from the stock of yet-to-find deposits. And all the while, scientific advances in technologies ranging from seismography to steel alloys have altered the oil industry immeasurably over the last century.

Simulation models attempt to capture some of this causal structure, but they exhibit a wide range of complexity. Some are quite simple "toy models" with a few equations governing discovery probabilities and demand growth. Others are built from dozens of equations and sub-modules, and are therefore difficult to describe in succinct mathematical form.

4.1 Simple simulations of exploration and extraction

The simplest simulations of the oil finding process are scarcely more complex than curve-fitting models. For example, Bardi (2005) built a simple model based on the work of Reynolds (1999). Reynolds characterized the resource finding and extraction process as the activity of agents searching for resources over a number of model years.[30] Bardi builds on this model, defining the probability of finding unit of resource in a given model year *t* as *pr(t)*:

$$pr(t) = \frac{k(t)}{Q_\infty}[Q_\infty - Q(t)] \qquad \textbf{eq. 26}$$

where *k(t)* is an "ability factor."[31] In each cycle of the model, the agents consume some of their resources to survive and to fuel the search process. Bardi's model includes population growth, whereby agents with a surplus of resources can reproduce

[30] In the Reynolds model the agents are shipwrecked islanders, and the resource is defined as tins of provisions washed ashore and buried in the sand.

[31] In the simplest version of the model, *k(t)* is set to a constant. It can also vary with time, i.e., to increase over time due to technological learning. This would therefore increase the probability of finding a unit of resource in opposition to the effect of cumulative extraction.

(representing economic growth). Agents are removed from the simulation as undiscovered resources are depleted and their stock of resource drops to zero.[32]

One version of Bardi's model[33] also includes a simple representation of exploration technology, multiplying the term *exp[-P(t)/L(t)]* by the finding probability *pr(t)*. As *P(t)* increases, this term decreases, representing the exhaustion of known prospects within an exploration cycle. Over the long-term, the function *L(t)* increases, representing the progress of technology and reducing the negative impact of this effect on the finding rate.[34]

In Bardi's simplest case, the model produces a roughly symmetrical bell-shaped production profile [the decline is somewhat steeper than the incline, similar to the findings of Reynolds (1999)]. His more complex cases produce exponential-like production profiles, with steeper rates of decline than rates of increase.

4.2 Complex simulations of finding and extraction

Davis (1958) produced the earliest complex simulation of the oil finding and development process.[35] He linked a series of correlations to project future reserves additions based on exploratory effort, and assumed that production in each year is limited to a set fraction of reserves. His model iterated through a number of steps in each year. By assuming that production is limited to a fraction of existing reserves, Davis essentially adopted a depletion formulation similar to that of Wood *et al* described above (the fixed R/P exponential method). But, instead of this function being constrained by an exogenous fixed estimate of URR, it is fully incorporated into an economic model of discovery and production. As reserves continue to be discovered along the constrained production path, the sharp peaks of the Wood *et al.* model do not occur. Davis' model predicted a peak and decline in US oil production between 1964-1973.

4.2.1 System dynamics and oil depletion: A series of models

The simulation approach of Davis (1958) was echoed in later models that used the methods of *system dynamics* to model the extraction and depletion of resources. System dynamics models focus on the importance of time, rates of change, and system feedbacks as they model the interaction of the many parts of the oil extraction process. While a

---

[32] Bardi notes that a drawback of this model is that the units of resource are all the same size, clearly a simplification given the variation in crude oil deposit size.

[33] This is Bardi's model 3, which incorporates explicit "technology factors" into the success rates. It is not clear from the article if this factor is in addition to the "ability factor" *k(t)*, or if this acts in the place of the ability factor.

[34] In the Reynolds model, *L(t)* is the Hubbert logistic function, in effect hard-wiring the dynamics of the Hubbert curve into the finding probability. In the Bardi model, a simpler linear increase in technology as a function of time is used.

[35] This was early in the computing age. The obvious novelty of computer modeling is illustrated by the inclusion of an appendix which shows photographic reproductions of punch cards used and gives a complete flow chart of equations. It also notes that "the running time to calculate and punch approximately 6,500 output cards for one run is about 4 1/2 hours."

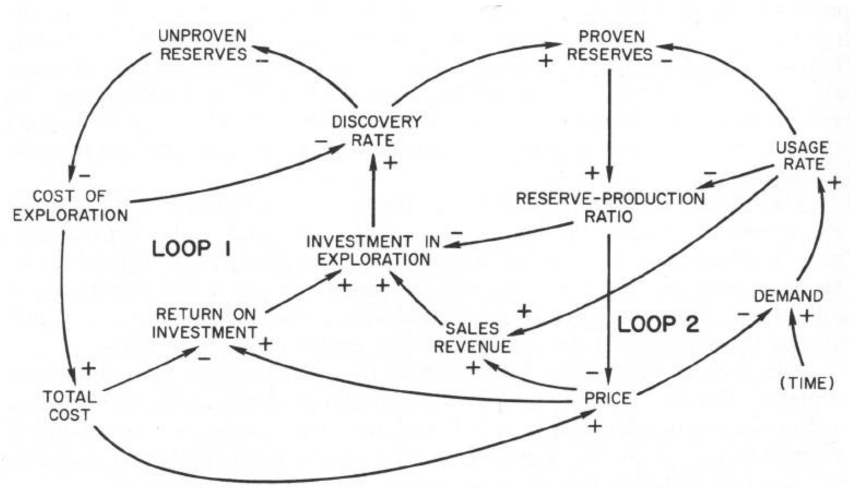


Figure 11. Causal-loop diagram showing all major relationships in Naill's (1973) model of natural gas exploration and production. Loop 1 represents the finding loop, while Loop 2 represents the extraction loop.

number of these models were developed over the course of the 1970s and 1980s, only a few representatives will be discussed (www.hubbertpeak.com 2008).[36]

Naill developed an early system dynamics model of natural gas discovery and production (Naill 1973). The model is based around two main state variables: *unproven reserves* and *proven reserves* (see Figure 11, top). As oil is discovered, it is moved from unproven reserves (a fixed quantity defined at the model outset) to proven reserves. As resources are consumed, they are removed from proven reserves. As unproven reserves decrease, the cost of exploration increases, reducing the return on investment and therefore the incentive for exploration. On the other hand, as proven reserves increase, the R/P ratio increases, reducing the oil price and reducing investment in exploration. A reduction in price reduces revenue to producing companies, which serves to dampen exploration. This feedback serves to stabilize the model.

Figure 11 shows all major causal relationships in the model. An arrow connecting quantities represents their interaction, with the sign attached representing the "polarity" of influence: a positive sign indicates that an increase in one factor leads to an increase in the other, while a negative sign indicates the opposite. Loops with an overall positive sign represent positive feedbacks, while those with a negative sign represent negative feedbacks.

Each causal interaction (arrow in the diagram) is quantified with a graphical correlation, mathematical function, or small group of functions. For example, the correlation between unproven reserves and the cost of exploration is shown in Figure 12. The resulting

[36] These models had their roots in the MIT System Dynamics Group, which produced the World3 model for the Club of Rome, the basis of the book *The Limits to Growth* and its successors (Meadows, Randers et al. 2004). These models included the Naill natural gas model as described below, its extension to the larger energy system called COAL1, later modifications COAL2, and FOSSIL1. FOSSIL2 was later developed by Naill and others at the US Department of Energy (DOE). FOSSIL2 has been improved and renamed IDEAS and is still maintained for the DOE (www.hubbertpeak.com 2008).

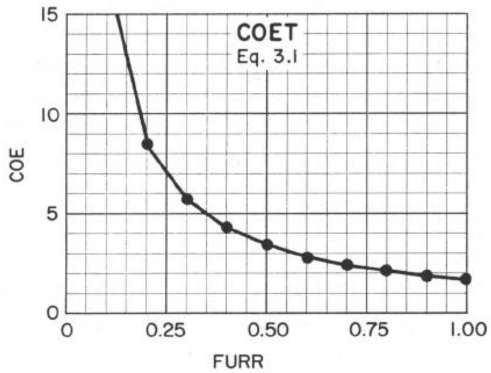


Figure 12. Relationship between fraction of resources remaining to be discovered (FURR) and the cost of exploration (COE) in Naill's model, corresponding to relationship between unproven reserves and extraction cost shown in upper-left corner of Figure 11.

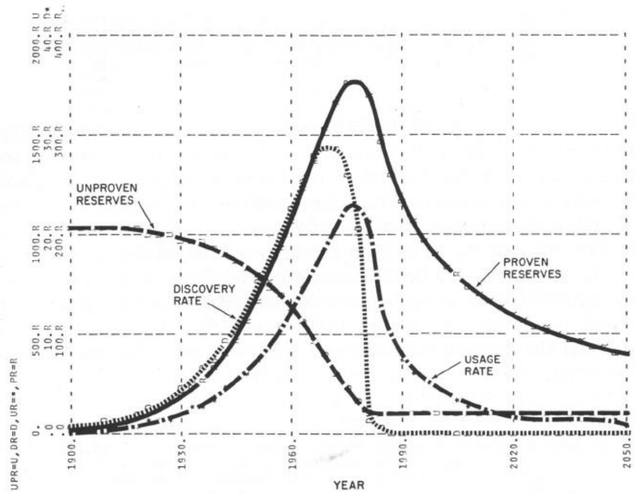


Figure 13. Natural gas production (labeled "usage rate"), proven reserves, unproven reserves, and discovery rate from Naill's model of natural gas exploration and extraction (Naill 1973).

production profile from Naill's baseline run is shown in Figure 13. Note that production ("usage rate") drops precipitously from the peak, resulting in much steeper decline rates than have been observed empirically. Also, note that the rate of production drops more quickly than the level of reserves. This causes a spike in the R/P ratio after the production peak (not plotted in this figure), which has not been empirically observed to this author's knowledge.[37]

Later extensions to the system dynamics models of resource depletion were made by Sterman and others (Sterman 1983; Sterman and Richardson 1985; Sterman, Richardson *et al.* 1988; Davidsen, Sterman *et al.* 1990). These models included investment in technology, imports, and synthetic fuels. The complexity of these models is evident in the exploration portion of the Davidsen *et al.* model, shown in Figure 14. This more complex causal loop is functionally analogous to Loop 1 in Naill's model. Note that undiscovered

[37] In the case of the United States, the R/P ratio has undergone a relatively smooth decline from about 18 years to about 10 years from 1918 to 1994, with no increase after peak production in 1970 (McCabe 1998).

oil is now not a static quantity, but is the product of the total resource base and the parameter called "fraction discoverable." The fraction discoverable is an endogenous technological parameter that is a positive function of oil company revenues. The result is that as revenues increase oil companies invest in technologies that increase the fraction of oil recoverable.[38] Loops of comparable complexity govern the extraction and price formation portions of the model.

This model results in impressive reproduction of US historical data. This is illustrated by historical and modeled demand for conventional petroleum, shown in Figure 15 (left). But, as Figure 15 (right) shows, the model predicted significant production of synthetic fuels by the year 2000, which was not realized. The authors claim that the only significant exogenous data inputs are GNP and the price of imported oil (Davidsen, Sterman *et al.* 1990). But, some tens of major input data are required for the model to operate, as are a number of functional relationships between model parameters and input data (Sterman and Richardson 1983).

[38] A similar technological parameter affects the extraction process, increasing the fraction of discovered oil that is technically recoverable.

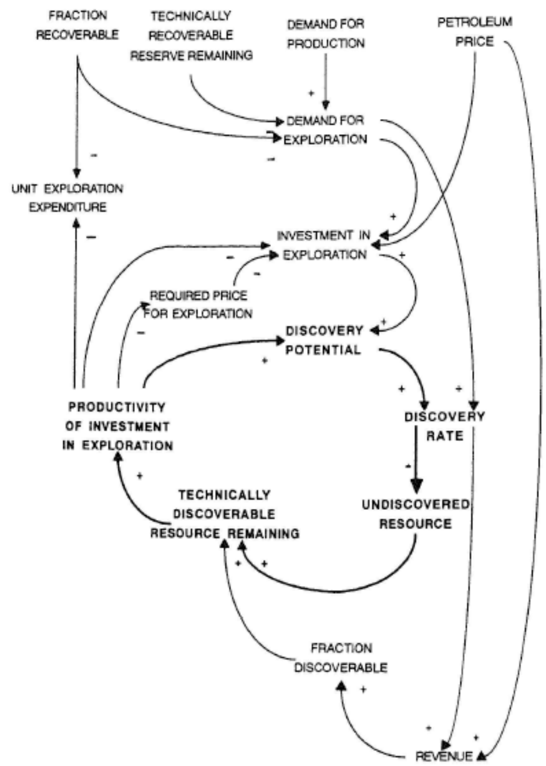


Figure 14. Causal diagram of exploration loop in model of Davidsen *et al*. (1990). Similar causal loops are reported in other papers by the Sterman group.

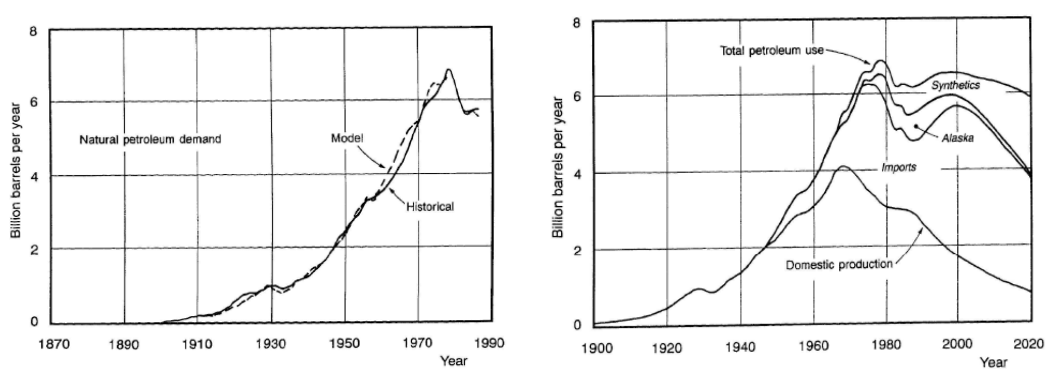


Figure 15. Results from Davidsen *et al.* (1990) system dynamics model of US petroleum exploration and extraction. Left: Excellent reproduction of observed demand for conventional oil. Right: Modeled breakdown between domestic production, imports, and synthetic fuel production. Note the significant production of synthetic fuels by the year 2000, which did not come to pass.

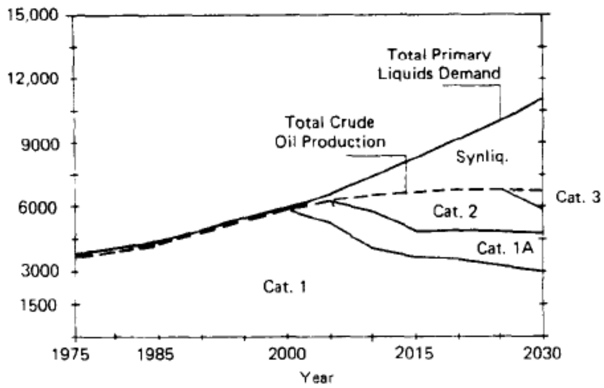


Figure 16. High scenario projection from Basile and Papin (1981), units are GWyr/yr. Category 1 oil is low-cost, conventional oil, while categories 1A, 2 and 3 represent increasingly expensive hydrocarbon sources (e.g., deep offshore, tar sands). "Synliq" represents synthetic refined fuels produced from coal.

4.2.2 Simulating the oil transition: Depletion as a starting point

The simulation models described above raise a fundamental question: as oil depletion progresses, what will we use instead of oil? Answering this question requires simulating not just oil extraction, but simulating the production of oil and a variety of competing substitutes for conventional petroleum (SCPs).[39]

The earliest projections of a transition to SCPs were by Ayres (1952; 1953), who discussed a transition to coal-based synfuels (although they were not mathematically based). The earliest mathematical models of the oil transition were the system dynamics models of Naill *et al.*, which modeled the transition from oil and gas (www.hubbertpeak.com 2008). In the economic literature, this problem was framed as a transition to "backstop" resources. This terminology was popularized in Nordhaus' classic model of transitions to nuclear power in the electricity sector and synthetic fuels in the transport sector (Nordhaus 1973). Later, Basile and Papin (1981) and Edmonds and Reilly (1983) built models of oil production within the context of the larger energy system.

The Basile and Papin (1981) projections utilized the MESSAGE energy systems model, which divided the world into 7 regions. They project the "maximum potential production" — not the actual production of fuels — by region, subject to a number of constraints. A variety of SCPs are included, such as enhanced oil recovery, tar sands, deep offshore, shale oil, and synthetic liquids from coal. These constraints include limits on reserve additions (likely limited by a URR-like quantity, although not specified), the rate of capacity build up, and production ceilings for some fuels (e.g., oil shale) representing environmental constraints.

[39] Note that models that incorporate aspects of the transition to SCPs are discussed elsewhere in this review, including those of Kaufmann and Shiers (2008), Hirsch et al. (2005), and Sterman, Davidsen and others (e.g., Davidsen, Sterman et al. 1990).

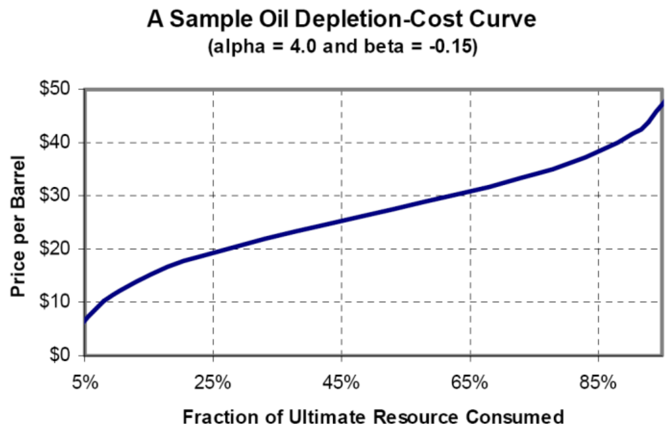


Figure 17. Relationship between production cost and resource depletion from Greene *et al*. (2003). Note that the function is undefined for values below 5% and above 95%, as the logistic function asymptotically approaches -∞ or ∞.

In this model, declining production of a resource is triggered by the resource reaching a minimum R/P value, at which point production undergoes exponential decline. The minimum R/P is different for different resources: 15:1 for conventional oil, 7.5:1 for EOR, and 25:1 for heavy oil. No justification is given for these assumptions of different constraining R/P ratios. This model predicted the peak of low-cost conventional oil around the year 2000, with successive waves of more expensive resources introduced as low-cost oil production declined (see Figure 16).

Greene and others recently developed a model of the transition from oil to SCPs (Greene, Hopson *et al.* 2003; Greene, Hopson *et al.* 2004; Greene, Hopson *et al.* 2006). This model projects demand for liquid fuels using a recursive demand function, such that demand for a given year is dependent on demand in the previous year. This allows simultaneous solution of all years of the model, and results in a smooth transition to SCPs. The relationship between cost and depletion is modeled using a logistic function[40] which causes an increase in production cost as a resource becomes depleted:

$$p_{rt} = \frac{\ln\left(\frac{1}{Q_{rt}/Q_{\infty r}} - 1\right) - \alpha_r}{\beta}, \qquad \textbf{eq. 27}$$

where p_{rt} is the price of the resource in region r, year t, Q_{rt} is cumulative production, $Q_{\infty r}$ is URR in region r, and α_r and β are fitting constants.[41] This function is plotted in Figure 17. Note that this function performs an analogous role to the correlation shown for the Naill model above (Figure 12): it defines the relationship between resource depletion and increases in the cost of production. The resulting reference case projections for hydrocarbon production are shown in Figure 18.

[40] This model is a sigmoid function with its axes reversed. This functional form for cost as a function of depletion was adapted by Greene from Rogner (1997).

[41] α_r is the intercept, which varies by region and is set by initializing data, while the slope parameter β governs how quickly cost increases as a function of depletion level. β is equal in all regions and for all fuels due to lack of data on depletion slopes for individual regions.

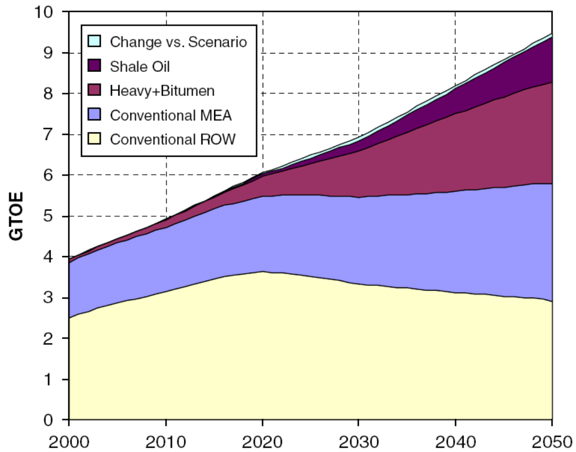


Figure 18. Production of conventional oil and oil substitutes in the Reference/USGS case of Greene *et al*. (2003). Note the smooth transition to heavy oil and bitumen after the peak in non-Middle East production.

Brandt and Farrell (2008) recently modeled oil depletion and the transition to unconventional resources in a framework similar to that of the above models. It includes tar sands, oil shale, and synthetic fuels from natural gas and coal. In this model, production of a given resource in each of 17 model regions is limited by a minimum ratio of remaining resources to production (M/P). When this minimum M/P is reached, production must decline to keep M/P from dropping, resulting in exponential decline.[42]

[42] Production that maintains a constant M/P ratio under decline is equivalent to exponential decline, with the percentage decline given by the reciprocal of the M/P ratio. For example, if M/P is held constant at 10 and in year 1 there are 100 units of remaining resources, then p will be 10. In the next year p will be 9 based on M of 90, followed by p of 8.1 based on M of 81, etc.

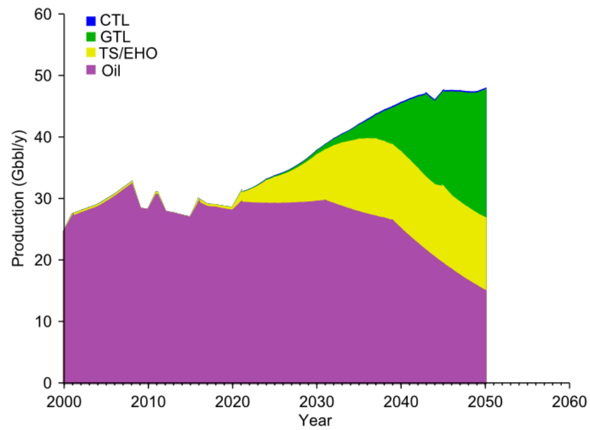


Figure 19. Transition from conventional oil to substitutes for oil in high supply/medium demand scenario from Brandt and Farrell (2008). Left: production of oil, tar sands/extra-heavy oil, gas-to-liquid synfuels and coal-to-liquid synfuels (bottom to top). Note the drop in demand resulting from the spike in price necessary to induce investment in alternatives.

Depletion in the Brandt and Farrell model is based on the logistic depletion function of Greene *et al.* (eq. 27). In this model, each year in the model is solved independently without foresight (the solving algorithm cannot see future demand or resource endowments as it is solving the model for a given year). Because the rate of investment in SCP capacity is limited, the result is a non-smooth transition to SCPs, as shown in Figure 19. Oil prices increase rapidly around the time of peak production to induce investors to build SCP production capacity. This results in reduced demand and delayed investment in alternatives to petroleum, causing a temporary drop in demand around the peak in production.

4.3 Difficulties with simulation models

In contrast with the difficulties faced by curve-fitting models – which largely stem from their extreme simplicity – the difficulties encountered by simulation models generally arise from their complexity.

Systems dynamics models, because of their complexity, require the quantification of a large number of relationships and correlations, often in the face of conflicting or nonexistent data. Consider, for example, the "fraction discoverable" parameter of the Davidsen *et al.* model. This is the fraction of oil deposits that are theoretically discoverable as a function of cumulative investment in exploration technology. Such a relationship would ideally be empirically derived, but this is likely impossible. Even given an accurate time series of monetary investment in exploration technology, one would require historical data on the fraction of unknown deposits that would have been discoverable (in theory) with a given level of technology. It is difficult to imagine how one might construct such a series. Certainly reasonable assumptions can be made despite this lack of data, but such dependence on assumptions certainly reduces some of the utility of building a more complex model.

Second, systems dynamics models tend to be finely balanced between positive and negative feedbacks. If positive feedbacks in the model are too strong, the result can be rapid growth and sudden, wrenching decline. This effect is evident in Naill's results, and is quite common in system dynamics models of resource depletion.[43] The gentle decline observed in real-world production data is due to mitigating factors (e.g. negative feedbacks) that exist in the real world but are clearly absent from such models.

Models of the transition to SCPs suffer from similar problems. They are complex, with data inputs that are difficult to obtain. They have historically tended to overestimate the ease and speed of a transition to SCPs (as seen in both Sterman *et al.* and Basile and Papin models). This is likely because of the similar errors to those in system dynamics models: they likely do not include negative feedbacks that exist in the real fuel production system and that result in systemic inertia. This systemic inertia makes the shift to SCPs more slow than is predicted by these models.

[43] The classic example of "overshoot and collapse" is the World3 model (Meadows, Randers et al. 2004). Other resource depletion models built in this framework suffer from similar problems, such as the general natural resource utilization model of Behrens (1973).

5 Bottom-up models: Building up oil depletion from the field level

5.1 Near-term predictions using detailed datasets

Bottom-up models use detailed knowledge of existing and newly discovered oil fields and/or their associated development projects to "build up" projections of production from larger regions (such as a basin, nation, or the world). They tend to focus on the largest and most important oil fields. They typically include simple models of decline from existing fields, and they often include new fields using estimates of plateau production rates.

Bottom-up models were suggested as early as the 1960s by Moore (1966), who argued that "acceptable techniques are available for short-term projections based on current and planned activities." Bottom-up modeling has become an increasingly prominent method as discoveries have slowed and an increasing fraction of future oil is expected to come from already-discovered fields (Bentley and Boyle 2007).

The most widely published model with bottom-up characteristics is that of Campbell and co-authors, produced since the mid-1990s (starting with Campbell and Laherrere 1995). This model was originally produced for Petroconsultants SA, using detailed proprietary data, including reserves and production data at the field level for significant global fields, adjusted using the judgment of the authors (Bentley and Boyle 2007). Estimates of yet-to-find oil were added based on statistical approaches such as regional creaming curves.[44] This resulted in country-level projections of future production. A number of publications have been generated using the same core model, updated as additional data become available (Campbell 1995; 1996; 1997; 1998; 2000; 2004).

Campbell's (1995) model combines bottom-up and curve-fitting methods. The basic approach is as follows:

1. A field-level database is modified with judgment by the authors to estimate country-level values of URR.
2. Countries are classified into those which have produced more than 50% of their URR (post-midpoint) and those which have not (pre-midpoint).
3. Production from pre-midpoint countries rises until they reach their midpoint. After the midpoint, production declines at the depletion rate prevailing at the midpoint. Countries past midpoint are assumed to continue to decline exponentially at the current decline rate.

[44] A creaming curve plots volumes discovered as a function of new-field wildcat wells. This is used to project undiscovered oil by assessing the asymptote of the discovery curve. This technique is controversial for a number of reasons, and has been criticized (Lynch 2003). See the discussion in Sorrell and Speirs (2009).

4. Persian Gulf producers are swing producers. The difference between projected demand and supply from the rest of the world is made up by swing producers until they reach their midpoints.

In the 1995 version of Campbell's model, the peak is projected before the year 2000, while more recent versions of the model predict peak production in 2010 or slightly beyond (Campbell 2003).

Another bottom-up model is maintained by Smith of EnergyFiles Ltd. (Smith 2008). This model is based on field-level data where available, and otherwise on a variety of data sources aggregated at the operator, basin, or country level, depending on data availability.[45] These projections include all fossil hydrocarbon liquids excepting synthetic fuels produced from coal, natural gas, and oil shale.

In the EnergyFiles model, production at the field level is projected using historical field decline rates (including expected EOR projects), and announced plateau levels for new fields. For fields not yet in decline, decline rates vary with field characteristics: 5% decline rates for onshore fields 15% for offshore fields and 3% for regions where data are unavailable (Smith 2008).[46] Undeveloped fields are modeled using estimated plateau production rates and decline rates. Thus, a simple "midpoint peak" assumption is not used.

A useful schematic of the bottom-up approach is presented in Figure 20 (Smith 2006; Smith 2008). On the left is an idealized representation of bottom-up modeling of an offshore basin: some 40 oil fields are discovered in order of size, developed with a 2-3 year lag, produce at a plateau level for a short time, and then decline exponentially over time. This produces a smooth, bell-shaped curve. On the right is a graph of actual field-level offshore UK oil production, where we see divergence from this ideal: fields were not discovered strictly in order of size, and significant disruptions to production occur.

Miller (2005) built a bottom-up model that uses field-level reserves and production data, as well as published production plans for the largest oil fields. Field-level data are used for approximately 70% of global supply, with the remainder aggregated at, e.g., the state or province level (Bentley and Boyle 2007).[47] Production from significant oil fields in each country is extrapolated to 2030, using reserves as a constraint and continuing the historical decline rate if the field is in decline. Small fields are aggregated and projected in aggregate.

[45] Bentley and Boyle (2007) state that the underlying data for reserves and historical production are based on publically-available data, while Smith (2008) seems to suggest that some proprietary data are used. Nevertheless, it appears that most of the EnergyFiles data were collected from a wide range of publically available sources.

[46] Smith notes that there is subjective judgment involved in building a bottom-up model. Therefore these decline rates are not used for a field if other information suggests a more suitable value.

[47] Bentley and Boyle (2007) suggest that 15% of production is modeled at state or provincial level, and 15% from small fields aggregated as "other production."

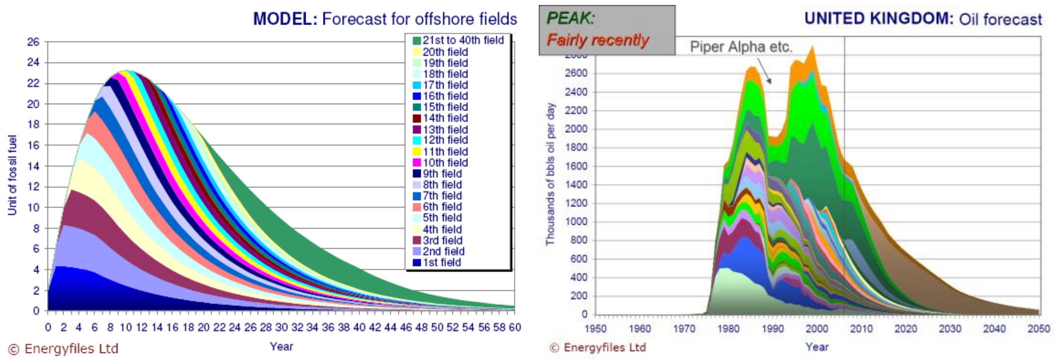


Figure 20. Left: Idealized bottom-up model with field-level offshore production summed to provide aggregate production profile (Smith 2008). Right: Actual production profile generated from (largely) offshore production in the UK, showing that fields were not discovered in order of size and that production can be disrupted (Smith 2006).

Technological change is included in Miller's model, addressing one of the criticisms of other models. It is represented as an increase in URR per field at a uniform rate of 0.2% per year.[48] Since this is a relatively low estimate of possible reserve growth, it could contribute to relatively pessimistic supply forecasts. Conversely, political and economic delays in investment are not included, which could make the projections optimistic in the rate at which undiscovered oil can be converted into yearly production (Miller 2005). Miller's model predicts a peak in non-OPEC production around 2009, with a global supply shortfall arriving by 2020, dependent upon OPEC investment patterns.

Skrebowski (2004; 2005; 2006; 2007) maintains a database of oil field "megaprojects" — oil field development projects above a threshold size.[49] As large projects provide the majority of new oil output, this approach provides insight into short-term capacity increases. Despite the inclusion of gross output from hundreds of projects in the database, the *net* result of new capacity additions is difficult to calculate. This is because aggregate production statistics do not allow easy separation of the effects of new projects from depletion at existing fields (Skrebowski 2007).

Skrebowski's approach removes some uncertainty in the 3-5 year period of capacity addition (2004). Unfortunately, new fields that are smaller than the threshold size are not included in the database, therefore causing potential underestimation of production. This effect is likely small because large oil fields provide the majority of production.[50] Another acknowledged difficulty is that additions to capacity from new discoveries must be projected based on development timelines. These timelines often slip, an effect that can only be estimated. Since these uncertainties work in opposing directions, they may partially cancel each other out.

[48] As technological change is not the only source of reserve growth, it is unclear how Miller accounts for other sources of reserve growth.
[49] The threshold field size for this database was lowered from 100 kbbl/d in 2004 to 40 kbbl/d in 2007, allowing for the inclusion of smaller projects and therefore reducing the potential for error.
[50] A future with many more small oil fields and fewer large ones might change the importance of this uncertainty.

Cambridge Energy Research Associates (CERA) also use a similar approach to projecting future supply, relying on detailed proprietary databases of oilfield projects (Dittrick 2006; Jackson 2006). Jackson argues that their bottom-up methodology reduces the level of uncertainty by "an order of magnitude" when compared to curve-fitting methods (Jackson 2006). Because their methods, data, and assumptions are not made public, their results are difficult to critique. PFC Energy, another consulting group, has also developed a bottom-up model, but they too provide little detail on methodologies presented (PFC 2004). It is notable, however, that PFC forecasts an early peak in oil production, in a similar manner to Smith and Miller, while CERA's forecasts are comparatively optimistic.

5.2 Opportunities and difficulties with bottom-up models

Of the model types described in this report, bottom-up methods seem to hold the most promise, in principle, for accurate projections of future production. This is especially true for the short- to medium-term projections. By accounting for decline and investment at the field level, bottom up models allow the modeler to directly build up aggregated growth and decline rates. This removes uncertainty about the proper functional form for the aggregated production profile: the profile emerges directly from summing individual field-level production data. These models could also allow sensitivity analyses of great detail, such as estimating how much slowing of aggregate decline rates would result from a given investment in EOR projects.

While bottom-up models clearly have advantages over other reviewed model types, the literature describing them is generally of poor quality. This is for a number of reasons. For one, most articles are not peer reviewed. This is largely because they rely fundamentally on proprietary databases augmented with the modeler's judgment and experience. While these characteristics are the source of the advantages of bottom-up models, they simultaneously make the models susceptible to criticism and difficult or impossible to reproduce (Lynch 2003). This situation is worsened by the fact that the methods used in the models are typically not explained in detail.

More fundamentally, uncertainty about the future is not removed by modeling at the field level. The proliferation of data that gives these models their strength also results in the need for many more assumptions: what is the decline rate for each field? What is the discovery trend in each region, and is this likely to change with additional investment? As an example, Lynch notes that Campbell increased his value of global URR from 1,575 Gbbl in 1989 to 1,950 Gbbl in 2002, despite his reliance on a field-level dataset. Also, it is difficult to include projects that lessen the decline rate at existing fields, such as infill drilling, workovers, or EOR projects. This "stealth oil" is difficult to track because it is a distributed response across tens of thousands of global oil fields in response to higher oil prices.[51] Many such projects will, by necessity, be left out of even the most complete

[51] Additions to supply from existing fields are likely to manifest as a decline in the absolute value of the decline rate. Skrebowski (2004) assumes a 4% decline for the ≈ 20 Mbbl of oil production that is in decline.

database, and it is unclear how a modeler might accurately estimate the number of such projects that might occur.

Some of this uncertainty about improved recovery is closely related to the issue of reserve growth (Thompson, Sorrell et al. 2009). Reserve growth is the appreciation in reserves over time as fields become better understood, conservative early estimates are revised upwards, and tertiary production technology is installed. Reserve growth clearly differs from the case where improved recovery projects or workovers speed rates of production but do not affect the overall recoverable volume. It is unclear in these models how modeled development projects can be rigorously classified as either causing reserve growth or not.

So, numerous questions remain for bottom-up models: what are the decline rates in each field or group of fields? What equations or methods of informal judgment are used to generate yet-to-find volumes of oil by region? How exactly is EOR modeled? More transparency in these areas would certainly improve the usefulness of bottom-up models.

Projects to improve flow in these fields could result in the overall rate of decline of these fields dropping to, for example, 3.5%.

6 Economic models of oil depletion

Economists understand natural resources and natural resource depletion differently than natural scientists. Rather than focusing on physical aspects such as depletion rates or field-size distributions, economists focus on investment, responses to changes in the oil price, optimal extraction paths, and substitution of different energy resources. Economic models of resource depletion can either be theoretical or empirical.

The first section below describes economic optimal depletion theory (ODT). A more thorough review of the voluminous optimal depletion literature is beyond the scope of this paper, but Krautkraemer provides accessible reviews (Krautkraemer 1998; Krautkraemer and Toman 2003). I then describe empirical econometric models of oil depletion. Again, the large amount of literature makes this topic difficult to cover in detail. Kaufman (1983) and Walls (1992) provide useful, if less recent, reviews. [52]

6.1 Economic optimal depletion theory

The primary concern of exhaustible resource economics is resource allocation over time. That is, *do we consume resources now or consume them later*? Or, by consuming resources today, do we damage our prospects of maintaining a high quality of life tomorrow?[53]

The chief theory of nonrenewable resource extraction over time is called optimal depletion theory (ODT). This theory has as its foundation the work of Hotelling (1931), who formalized the key insight underlying ODT: rational resource producers should equate the future value of a unit of resource in the ground with the value they would receive if the resource was sold and the profits invested.[54] This suggests that the value of a unit of resource in the ground, less marginal extraction costs, should rise at the rate of interest.[55] The problem can be stated most simply as a problem of maximizing the net present value (NPV) of welfare, *W*, over a number of time periods:

$$\max_q W = \int_{t=0}^{\infty} [pq - c(q)] e^{-\delta t} dt, \qquad \textbf{eq. 28}$$

where price p times quantity consumed q is the utility gained from consumption of resources, while $c(q)$ is the cost of producing the resource and δ is the discount rate which

---

[52] This discussion will not address the views of "cornucopians" who reject the concept of scarcity and do not recognize inviolable tradeoffs between consuming resources now or later (Simon 1996; Huber and Mills 2005). They see increasing returns to human effort and little human-induced danger to ecological systems.

[53] In this question, "tomorrow" is generally (although not always) defined to include future generations. It should be noted that there are significant difficulties with the way that future generations are represented in traditional optimal depletion theory (Howarth and Norgaard 1990).

[54] Formally, producers should account for demand growth affecting the demand curve in future periods. This aspect is typically absent in the simplest models, although see discussions in Watkins (2006) and Holland (2008).

[55] Hotelling further argued that this rational behavior results in the socially optimal path of extraction; that the net present value of resource extraction over time is maximized by producers acting in their self interest.

converts the stream of benefits to their present value. Note the difference in economic notation: p is the price and q is the amount produced (equivalent to P in models reviewed above). The stock of resource, x, is constrained as follows:

$$\frac{dx}{dt} = q, \quad \lim_{t\to\infty} x(t) \geq 0 , \qquad \textbf{eq. 29}$$

Or, in words, the change in the stock of resource is equal to that extracted in a year, and consumption over all time must not result in negative quantities of resources remaining. This is therefore the problem of depleting a known deposit of a single resource over a finite amount of time. $H = pq - c(q) - \mu(t)q$

This problem can be solved by creating a (current value) Hamiltonian[56] function from the integrand (Conrad and Clark 1987, p. 123):

$$H = pq - c(q) - \mu(t)q . \qquad \textbf{eq. 30}$$

In this equation $\mu(t) = \lambda e^{\delta t}$, or the present value of the "shadow price" λ of the resource (the value of a unit of resource in the ground). The two first-order conditions for optimality are found by differentiating H with respect to the rate of production (q) and the stock of the resource (x) such that:

$$\frac{dH}{dq} = 0 \text{, and } \frac{dH}{dx} = \delta\mu - \dot{\mu} . \qquad \textbf{eq. 31}$$

Thus from the first condition,

$$p - \frac{dc(q)}{dq} - \mu(t) = 0 \text{, or } \mu(t) = p - \frac{dc(q)}{dq} , \qquad \textbf{eq. 32}$$

and from the second,

$$0 = \delta\mu - \dot{\mu} \text{, or } \frac{\dot{\mu}}{\mu} = \delta , \qquad \textbf{eq. 33}$$

because the derivative of H with respect to x is zero. These two results are the primary results of ODT: the value of a unit of resource in the ground (μ) is equal to its price less the marginal cost of extraction (the additional cost of extracting one more unit of resource, or $dc(q)/dq$); and that μ grows at the rate of interest.

[56] The Hamiltonian function is adapted from classical mechanics. It is generally useful in constrained optimization problems, here being equal to the objective function, or the quantity to be maximized, *pq-c(q)*, less the shadow price multiplied by the constraint governing the stock of the resource ($dx/dt = q$).

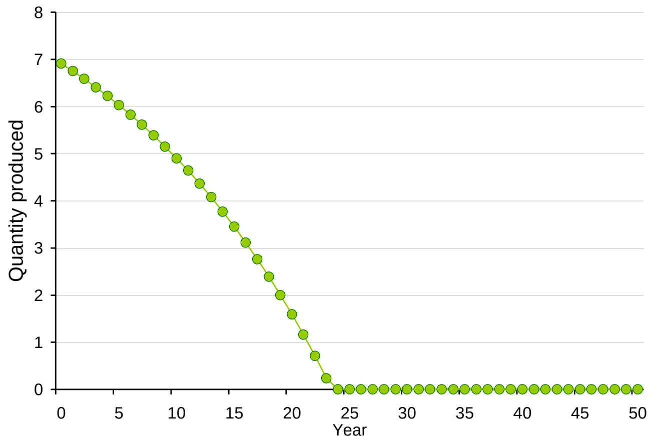


Figure 21. Modeled time path of extraction in optimal depletion model of a single deposit. The total resource size is 100 units, the discount rate is 5%, and cost is directly proportional to quantity produced. Price is constant at $15 per unit. The extraction cost function *c(x)* is equal to 1x*x (cost per unit increases linearly with rate of production, total cost increases quadratically).

Most attention in the exhaustible resource literature has focused on the optimal path of prices, not production profiles (Krautkraemer and Toman 2003). A typical example of an optimal production path for a single resource deposit is shown in Figure 21. Production starts high and declines over time due to the declining value (in present value terms) of production in future years. Such a path obviously does not reproduce historically observed behavior.

6.1.1 Extensions to optimal depletion theory

A number of extensions have been made to ODT. These were developed chiefly during the 1970s and 1980s because of increased interest in resource depletion resulting from high oil prices. Some of these extensions have the effect of generating more realistic production paths with peaks.

Modeling production costs

If depletion increases the marginal cost of production (that is, producing an additional barrel becomes costlier as the resource is depleted) then *dH/dx* does not equal zero as in eq. 33 because *c(q)* is replaced with *c(q,x)*. If *dc/dx* is negative (extraction costs increase as the resource is depleted), then there is additional incentive to hold the resource in the ground, and its value increases at less than the rate of interest (Fisher 1979).

An important variant is provided by Slade, who replaces *c(q,x)* with *c(q,x,t)*, such that production cost varies with time in addition to the level of cumulative extraction. Such behavior might result from technological change in the oil industry. If cost declines with time ($dc/dt < 0$), this model results in price paths that are "U-shaped" and production profiles that can exhibit peaks. (Slade 1982).[57] U-shaped paths arise when technological change brings down production costs rapidly the early years of an industry. After a relatively long period of low production costs, depletion takes hold, causing cost increases as good exploration prospects are exhausted and production from existing fields drops. Cleveland (1991) found such a long-term trend in US long-run average cost of oil production, with the low point of the curve being the mid 1960s, just before the 1970 peak in US production. Cleveland and Kaufmann argue that this U-shaped cost trend (as predicted by the theory of Slade) is responsible Hubbert-type behavior in general.

Exploration and depletion of multiple deposits

Rather than maximizing the value of a single deposit of known size, oil producers face a complex problem of exploring for and depleting a number of deposits of uncertain size. Pindyk (1978) built an ODT model that includes exploration as well as extraction of resources:

$$\max_{q,w} W = \int_{t=0}^{\infty} [pq - C_1(R)q - C_2(w)]e^{-\delta t}dt, \qquad \textbf{eq. 34}$$

where C_1 is the cost of production as a function of reserves R, and C_2 is the cost of exploration as a function of exploratory effort w. This maximization is subject to two constraints:

$$\dot{R} = \dot{x} - q \text{ and } \dot{x} = f(w, x), \qquad \textbf{eq. 35}$$

where x equals cumulative additions to reserves, and therefore the rate of change of R is equal to the difference between additions to reserves and extraction q. Note that the rate of change of x is a function both of exploratory effort (increasing) and cumulative additions to reserves (decreasing). The analytical solution to this problem is beyond the scope of this review. This solution has interesting properties: the oil price path is U-shaped, and if the initial reserves are small (as in an industry like the oil industry where deposits require significant exploration), then the optimal time path of extraction first increases and then decreases after a peak. Note again the contrast to the optimal path of depletion of a single deposit, as shown in Figure 21.

Focus on the backstop: depletion as a transition

Given economists' interest in the substitution of energy resources, it should not be surprising that ODT models have been developed that explicitly allow a transition to SCPs (e.g. tar sands or oil shale). In economic nomenclature, these substitutes are called "backstop" resources.

Optimal depletion models with backstop resources began with the seminal work by Nordhaus on long-term energy transitions (1973). Other models include those with polluting backstop technologies (Hoel 1978), and pollution-free backstop technologies [numerous, see citations in Tahvonen (1997)]. A common result of models with backstop resources is that resources are consumed in order of increasing cost.

[57] In the U-shaped path, prices are high in the early years of resource extraction, low throughout most years of production, and high again in later years as depletion outweighs the ability of technical change to reduce the cost of extraction

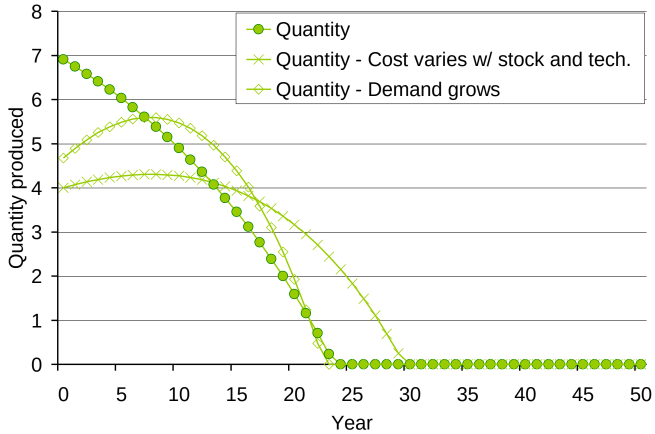


Figure 22. Optimal depletion of a fixed resource under conditions that produce a "peak." Production path labeled "quantity" is the same as Figure 21. Other paths show quantity produced when cost varies with stock or when demand grows.

6.1.2 Reconciling optimal depletion theory with observed behavior

Observers have noted that predictions from simple ODT models bear little resemblance to actual natural resource extraction paths (Krautkraemer 1998). However, Holland (2008) illustrated that the extended models described above can production paths with peaks. He argues that at least four model features can cause peaking behavior:

1. Demand increases over time, inducing producers to delay extraction to later time periods even given the disincentive resulting from discounting of future profits.[58]
2. Technological change results in exogenous decreases in production cost that temporarily outweigh the disincentive due to discounting (e.g., Slade 1982);
3. Exploration for new deposits occurs (e.g., Pindyck 1978);
4. Production can move to new sites or new resource types over time.

Given that all four of these causal features exist in the oil industry, this development bridges a gap between what economic theory suggests is optimal and what has occurred in reality. To illustrate these types of impacts, Figure 22 shows the impact of adding some of these features to the simple model that generated Figure 21.[59]

[58] Watkins (2006) notes the importance of the demand function on the production path: "An oft neglected aspect of Hotelling's seminal paper was the role of his demand function, which set a maximum price, reached as output approached zero…In general, the higher the price anticipated when the rate of production becomes small, the more protracted the period of operation." Or, in the case of Figure 22, the increase in demand results in production that rises to a peak before falling.

[59] First, we alter the production cost function so that production cost increases as the stock is depleted and decreases as time passes (as in Slade 1982): $C(t) = ap(t) + bQ(t) - ct$, where $C(t)$ is the cost to extract one unit of resource, $p(t)$ is production in year t, and $Q(t)$ is cumulative production, and a, b, and c are fitting parameters. For the plot above, $a = 2$, $b = 0.05$, and $c = 1$. Second, we model demand growth by increasing the real price received by the resource producer by 1.5 units per year t. This incentivizes production in later years.

6.2 Econometric models of oil depletion

Econometric models are data-rich statistical models that project supply and demand using economic variables such as price and extraction cost (Kaufman 1983; Pindyk and Rubinfeld 1998). Econometric oil supply models project the volumes of oil or gas produced over time as a function of the oil price (typically linear or log-linear), extraction costs, number of exploratory wells drilled, or other economic variables that vary over time. A group of "hybrid" econometric models have also been developed that incorporate non-economic and physical/geologic aspects of oil supply.

6.2.1 Early econometric models of oil and gas supply

Econometric models of oil and gas supply were first developed by Fisher (1964). A number of models from the 1970s and 1980s followed the general form of the Fisher model, but I will only describe one example here. See the reviews of Kaufman (1983), Dahl (1998) and Walls (1989) for more information.

The model of Erickson and Spann (1971) is an early example of a Fisher-type model. They model the quantity of oil and gas discovered (in district j, year t) as the product of the number of wildcat wells drilled, W_{jt}, the success ratio F_{jt}, and the average size of new discoveries, S_{jt} or N_{jt} for oil and gas, respectively. Thus, discoveries of oil D_{jt} are given by $D_{jt} = W_{jt} \times F_{jt} \times S_{jt}$. The three equations for W, F, and S are log-linear. For example, the average size of new oil discoveries S_{jt} is modeled as:

$$\begin{aligned} \log S_{jt} &= \beta_0 + \beta_1 \log p_{jt} + \beta_2 \log g_{jt} + \beta_3 \log F_{j,t-1} \\ &+ \beta_4 X_{jt1} + \beta_5 X_{jt2} + \beta_6 Z_1 + \beta_7 Z_2 + \beta_8 Z_4 \end{aligned}, \qquad \textbf{eq. 36}$$

where β_0 to β_8 are fitting parameters, p_{it} and g_{it} are input oil and gas prices, X_{jti} are variables representing Texas prorationing, and Z_i are dummy variables that distinguish between four studied districts. The other equations share similar functional forms.

Fisher-type models have serious weaknesses. Most glaring is that they include no information about the geological nature of oil resources.[60] For example, the above equation implies that with a positive value of β_1, high enough oil and gas prices mean that the average size of discovered oil and gas deposits could continue to increase indefinitely through time, which is clearly false (i.e., the size of discoveries is modeled as a log-linear function of price, with no ability to account for long-term trends like declining exploration prospects). Also, these models are specified in a somewhat ad-hoc fashion, without justification from the insights of optimal depletion theory (Walls 1992).

In practice, the performance of these models was poor. In the Erickson model some parameters had unexpected magnitudes and signs (Erickson and Spann 1971; Walls

[60] A formulation that is highly dependent on price makes much more sense for a product that is produced in a factory in response to a given demand. Oil and gas are clearly different in that they are derived from deposits laid in place over geologic time. No price increase will result in the deposition of new hydrocarbon deposits - it can only induce use of more intensive extraction technologies or the conversion of other fuels into petroleum substitutes. The topic of substitution is discussed in Section 7.

1992). The results were often not robust, as exemplified by Pindyk finding significantly different results when fitting the Erickson model later to another dataset (Kaufman 1983; Dahl and Duggan 1998). Lastly, results vary widely across models of this type. For example, estimates of the short-run price elasticity of oil supply vary from 0.31 to 3.9 across 7 Fisher-type models surveyed by Dahl (1998).[61]

6.2.2 Models derived from theories of intertemporal optimization

In the 1980s, econometric models were developed that had functions derived from the insights of economic ODT (Walls 1994). Such models were originally produced by Epple (1980; 1983), Walls (1989) and others. Because their form is suggested by theory, such models are preferable in principle to the more ad-hoc models described above.

Pesaran (1990) gives an example of this ODT-based approach (although his model also models some physical aspects of oil production). First, he directly derives his estimated equations from an ODT model with exploration. Producers are assumed to maximize expected discounted profits subject to a given information set Ω:

$$\max_{q_t, x_t} E\left\{\sum_{t=0}^{\infty}[p_t q_t - C(q_t, R_{t-1}) - w_t x_t]\beta^t \left|\Omega_{t-1}\right.\right\}. \quad \textbf{eq. 37}$$

This is very similar to Pindyk's (1978) model with exploration described above. Again, $p_t q_t$ is revenue in year t, $C(q_t,R_{t-1})$ is extraction cost, and $w_t x_t$ is exploration cost (w_t is the cost of a unit of exploration activity and x_t is the amount of exploration). Because this is a discrete time optimal depletion model, β is defined as $1/(1 + r)$, with r being the rate of interest.

Pesaran's model attempts to capture some physical aspects of oil production. He defines the production cost as a function of quantity produced (q_t) and the reserves of the previous year (R_{t-1}):

$$C(q_t, R_{t-1}) = \delta_0 + \delta_1 q_t + \frac{1}{2}\delta_2 q_t^{\,2} + \frac{1}{2}\delta_3 \frac{q_t^{\,2}}{R_{t-1}} + \varepsilon_t q_t\,, \quad \textbf{eq. 38}$$

where the δ_i are fitting parameters.[62] The inverse relationship between costs and the previous year's reserves (in the δ_3 term) is included to account for the impact of "pressure dynamics of the petroleum reservoirs on production costs." Also, Pesaran includes in his exploration model aspects of the discovery process by using an exponential discovery function inspired by Hubbert (1969) and Uhler (1976). Thus, discoveries F are a function of exploration effort x_t in year t and cumulative exploration effort, X_{t-1}:

---

[61] Such disagreement is not unexpected given that the studies likely covered different time periods or regions. But variation in modeled elasticities of more than an order of magnitude is clearly problematic if the goal is finding values useful for practical applications.

[62] A problem with this model's use of reserves (as well as in the Pindyk and other models) is that the commonly reported reserves (1P) do not represent the total oil in place, and therefore might not be useful as a signal of impending depletion.

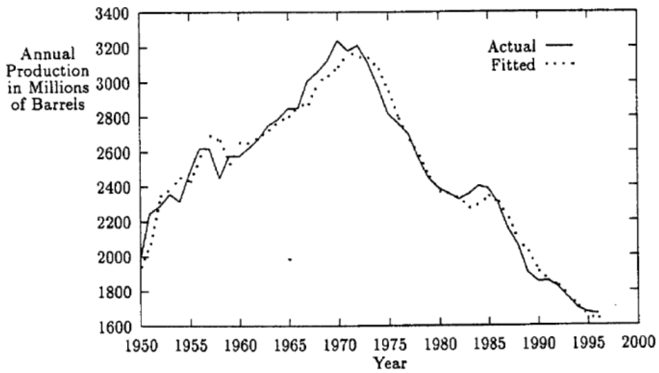


Figure 23. Agreement between model predictions and data from Model IV of Moroney and Berg (1999). Model IV contains a lagged production term to account for the inertia in production systems. This did not completely remove the serial correlation problems seen in their other models (note runs of consecutive positive and negative residuals).

$$F(x_t, X_{t-1}) = Ax_t^{\rho} \times e^{(b_1 X_{t-1} - b_2 X_{t-1}^2)} . \qquad \textbf{eq. 39}$$

Note that discoveries are an increasing function of exploration effort x_t (with ρ <1, so diminishing returns apply), and an increasing-then-decreasing function of cumulative exploration X_{t-1}. Therefore, the initial period of exploration leads to increased knowledge and therefore more discoveries, but eventually the depletion effect dominates and discoveries decline.

The derivation of Pesaran's econometric functions is beyond the scope of this paper. One interesting result from this model is that the price responsiveness of oil supply declines as reserves decline. Later work critiqued Pesaran's model for its aggregation level (Pickering 2002). For example, R_t in his model are yearly aggregated UK proven reserves, obscuring any differences that might result from field or producer heterogeneity.

6.2.3 Hybrid models of oil and gas supply

Pesaran's (1990) model above can be considered a "hybrid" econometric model. Hybrid modeling combines traditional economic variables like the oil price with variables that represent non-economic aspects of oil supply. Such variables include the sizes and distributions of oil reserves, or the correlation between depletion and extraction costs. Hybrid models vary in the extent to which they are based on economic or geologic/physical factors. Walls (1994) and Dahl (1998) survey these models in detail.

Some hybrid models are structured as comparatively minor modifications of traditional econometric models, such as the Pesaran (1990) model above. Another model of this type was constructed by Moroney and Berg (1999). They built four models in which US oil production is a simple log-linear combination of economic and non-economic variables:[63]

[63] Here we present their Model IV, with Model I including just the geophysical elements (linear term plus reserves), Model II including just the price and regulatory elements (real oil price and Texas Railroad

$$\ln(q_t) = \gamma_0 + \gamma_1 \ln(RESRV_{t-1}) + \gamma_2 \ln(RP_t) + \gamma_3 TRC_c + \gamma_4 dum1 + \gamma_5 \ln(q_{t-1}) + \omega_t , \quad \textbf{eq. 40}$$

where *RESRV* represents oil reserves, *RP* is the real price of oil, *TRC* accounts for the prorationing decisions of the Texas Railroad Commission, and *dum*1 accounts for possible changes due to a shift in data sources. This model reproduces historical data nicely, as shown in Figure 23. Three of their models experienced severe problems with serial correlation of the residuals, while Model IV presented in Figure 23 is improved in this respect but still exhibits serial correlation. This can be seen in the fact that the model consistently under- or over-predicts production over consecutive years.[64]

Other hybrid models act by augmenting curve-fitting models with economic variables. The first such attempts were by Uri (1982) who converted the Hubbert model and Gompertz models to econometric hybrid models. For example, in Hubbert's logistic differential equation

$$\frac{dQ}{dt} = cQ - \frac{c}{Q_\infty} Q^2 , \quad \textbf{eq. 41}$$

Uri modifies Q_∞ to address the economic critique that URR is not a static value but depends on the oil price and technological change:

$$\frac{dQ}{dt} = bQ - \frac{bQ^2}{\sum_{i=0}^{k} \alpha_{1i} p_{t-i} + \alpha_2 T_t} . \quad \textbf{eq. 42}$$

Here α_i and b are fitted parameters, p_t are lagged prices and T is a measure of technological change. Through experimentation, it was found that $k = 2$ provided the best fit. Since no clear metric for technological change was available, T_t is replaced with $1/e^{(t-t_0)}$:

$$\frac{dQ}{dt} = bQ - \frac{bQ^2}{\alpha_{10} p_t + \alpha_{11} p_{t-1} + \alpha_{12} p_{t-2} + \alpha_2 \frac{1}{e^{(t-t_0)}}} . \quad \textbf{eq. 43}$$

Kaufmann (1991) critiques this model because URR depends solely on price and a decaying technology parameter. Thus, in the long-run URR is essentially linearly related to price, implying that no limit to oil resources exists given a high enough oil price.

Later, Kaufmann (1991) built another hybrid model based on Hubbert's work. His procedure has two parts: first he fits a logistic Hubbert curve to oil production data in the lower-48 states. He then attempts to account for deviation of the data from the Hubbert model by fitting the residuals (the deviations between the predicted and actual values) to an econometric model that accounts for economic and political factors. His equation models the residual in each year *t*, R_t, as a function of economic and political variables:

$$R_t = \alpha + \beta_1 RP_{(1-2)} + \beta_2 RP_{(3-5)} + \beta_3 OG_t + \beta_4 TRC_t + \beta_5 PC' , \quad \textbf{eq. 44}$$

Commission prorationing) and Model III being the same as Model IV but without the term accounting for lagged production.

[64] The Durbin-Watson statistic improves from ≈ 0.4 (severe serial correlation) to 1.5 (moderate serial correlation).

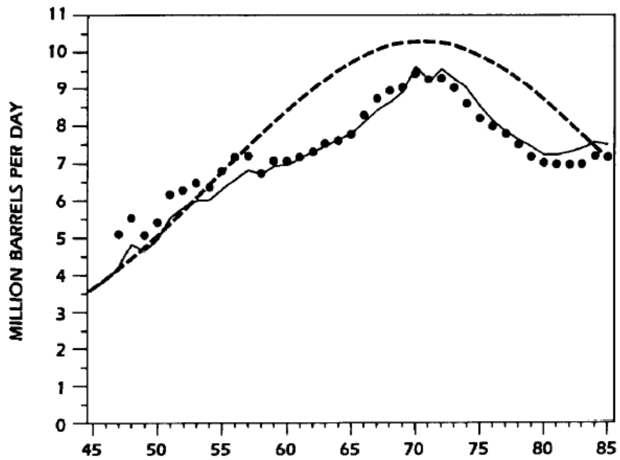


Figure 24. Fit of Kaufmann's (1991) hybrid econometric model (solid line) to lower-48 states US production data (dots) as compared to traditional Hubbert model (dashed line).

where α and β_i are the coefficients of the linear regression, and time series variables including: the running average of real oil prices lagged 1 to 2 years ($RP_{(1\text{-}2)}$) and 3 to 5 years ($RP_{(3\text{-}5)}$), the price of oil relative to gas (OG_t), the fraction of allowable production in Texas under the Texas Railroad Commission (TRC_t), and the first difference of the production curve after its peak (*PC'*, included to account for asymmetry).

Such a hybrid formulation assumes, in effect, that geologic and physical factors cause oil production to broadly rise and fall, while economic variables alter the observed path around this underlying trend. They argue that the bell-shaped curve "mimics physical changes in the resource base" caused by depletion. This model accounts quite nicely for the deviations from the Hubbert model, as shown in Figure 24.[65]

[65] In a complementary effort, Cleveland and Kaufmann (1991) also altered Hubbert's model of discoveries as a function of exploratory effort to include economic variables. Thus, Hubbert's model suggested that $YPE = YPE_0 e^{-\lambda h}$, where *YPE* is the yield of oil per unit of effort (bbl/ft drilled) and *h* is cumulative footage drilled. Cleveland and Kaufmann augmented this with economic factor such that $YPE = YPE_0 e^{-\lambda h} e^{\delta p} e^{-\phi r}$, where *p* is the wellhead price of oil and *r* is the rate of exploratory drilling.

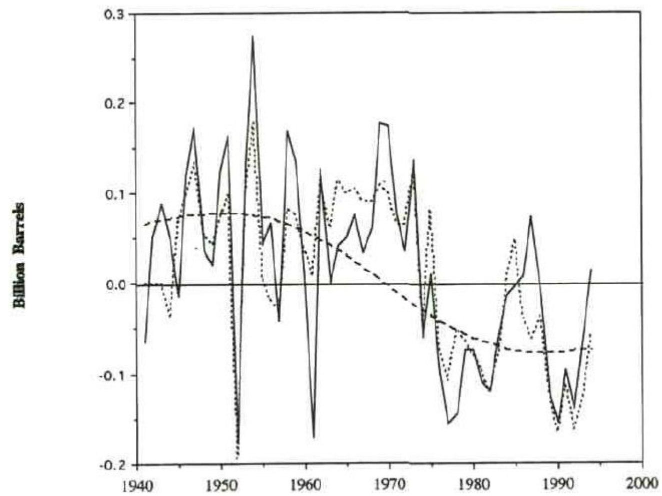


Figure 25. Results of Kaufmann and Cleveland's (2001) econometric model. Annual change in oil production (solid line) is compared to model output (dotted line) and predictions from Hubbert's model (smooth dashed line).

More recently, Kaufmann and Cleveland (2001) built a model that does not rely on Hubbert's logistic function: it instead includes the average real cost of production from Cleveland's (1991) long-term historical analysis. The inclusion of average production cost data (which rise sharply shortly before the peak in production in 1970) obviates the need for an exogenous bell-shaped function as in Kaufmann's (1991) analysis. This model also allows for asymmetric responses to price increases and decreases (e.g., a price maximum can have effects that are not undone by later price reductions).

Figure 25 shows that this model reproduces historical rates of change of production (the first difference of the production curve) with much greater fidelity than the simple Hubbert model. They claim that this shows that "Hubbert got lucky" in his prediction of the 1970 peak and that the peak in his model fortuitously aligned with the increases in production cost that occurred before 1970. But it is unclear how this production cost increase can be causally distinguished from the effects of depletion (in other words, the production cost increase could be the mechanism by which depletion acts to cause a bell-shaped peak).

6.3 Difficulties with economic models

The difficulties and problems associated with economic models of oil depletion parallel closely in form the difficulties with curve-fitting and simulation models. Optimal depletion models are overly simple, as was discussed with curve-fitting models. And econometric models tend toward complexity that makes them difficult to formulate and less robust than a simpler model.

6.3.1 Optimal depletion theory: Simplicity as a benefit and a difficulty

The models of optimal depletion theory are intentionally simple. By keeping the models simple, causation in the model can be traced and understood, and they can be solved analytically by a skilled mathematician. This simplicity can be contrasted with the more complex models, which are generally solved by optimization algorithms and are can be difficult to understand.

A result of this simplicity is that these models lack empirical grounding. For example, Tahvonen finds “eleven optimal regime combinations” in his model (Tahvonen 1997). This proliferation of solution regimes suggests the need for empirically-derived functions and parameters to reduce uncertainty. Alternatively, Withagen argues that models of the oil industry need “considerable further work…to understand the micro-foundations of the industry cost function” (Withagen 1998).

A good example of the difficulties of simple models is given by the production cost functions used in ODT models. For example, Pindyk models per-unit extraction costs as inversely proportional to reserves, $c = m/R$, where m is a free parameter. In this way, as reserves decrease, costs increase. Ruth and Cleveland (1993) use historical data to fit parameters (econometrically) to Pindyk’s equations of optimal exploration and depletion. They generalize somewhat from Pindyk’s equation, setting $c = \alpha R^{\beta}$, where β is < 0 (cost is a decreasing function of the reserve size).[66] Unfortunately, this equation fits observed data with an R^2 of only 0.26. Discussing this poor fit, they note that “extraction costs are influenced by many other factors than stock size, which itself is only an approximation of depletion.” Or, put differently, there is no reason to believe that costs should be a simple inverse function of the reserve size.

Slade’s (1982) model suggests an improvement to this simple formulation by making cost a function of time, so as to allow technological change. But, our understanding of the effects of technological change on oil depletion is provided by only a sparse set of papers. Cuddington and Moss (2001) study exploration using a metric of the raw *number* of technologies developed, but not their effectiveness at increasing exploration success. Livernois (1987) generated econometric cost functions for water-injection under depletion. And Norgaard (Norgaard 1971; Norgaard and Leu 1986) analyzed the role of technology in mitigating cost increases associated with increased drilling depths.

Another fundamental difficulty of ODT is the assumption that resource producers are knowledgeable about the extent and nature of resource deposits. This assumption is implicit in the fact that in ODT resource producers optimize net present value over all models years simultaneously (Howarth and Norgaard 1990). In contrast, Reynolds’ (1999) model of uncertain exploration suggests that the oil price can increase very rapidly after the peak in production, as depletion can “sneak up” on producers with incomplete knowledge of resource availability.

[66] They find that $\beta = -1.49$, so cost declines somewhat faster as a function of the size of reserves than in the Pindyk model, where $\beta = -1$.

6.3.2 Difficulties with econometric modeling of oil supply

Despite the quite impressive agreement between econometric models and data shown above, these models have their own set of shortcomings. It is easiest to dismiss the econometric models with no inclusion of geology or technology, especially for use in long-term projections. Kaufman (1983) argues that these models “have not done well” at predicting production more than few years in advance, most likely because “the functional forms employed in most econometric models do not conform closely to the physical character of exploration, discovery and production.”

One uncertainty is the role of costs and prices in governing production of fuels. Kaufmann (1991) notes that while real oil prices were declining between 1947 and 1970, US oil production increased significantly, but during the dramatic increase in world oil prices between 1970 and 1985, US oil production dropped significantly. If we assume that US producers could freely respond to market prices, this is the opposite reaction from what one would expect. As an explanation, Kaufmann suggests that depletion-induced cost increases dominated the economics of production. He argues that a U-shaped production cost path caused an inverse reaction in production: production rises and then peaks and falls as costs drop and then rise again. After such paths were suggested by Slade (1982), Cleveland (1991) found evidence for a U-shaped long-run average production cost curve in the US.

Lynch (2002), generally a staunch advocate of economic analysis, finds little hope for econometric models of oil depletion: he goes so far as to call their forecasting performance “abysmally bad.”[67] In practice he argues that they have been too pessimistic, because depletion is often included but technological change is not included to compensate. Even if this bias were removed, he argues that improved econometric models would be very data intensive in an industry with notoriously poor data availability.

And, despite the good match between the models above and historical data, this fidelity is generally fragile. This is because predictions made with these models typically fare poorly only a few years beyond the fitted data. This is because the large number of parameters included in most econometric models allows for good flexibility in fitting the model to data. But such a good fit is fragile because the parameters included are dwarfed by a huge number of omitted variables (for which data are likely not available). The values of these variables omitted variables will change in future years, stymieing predictions based on the old fitted parameters (Lynch 2002).

Also, Krautkraemer (2003) notes that econometric models “have not coped well with what appear to be basic nonlinearities in the relationship between unit supply cost and

[67] This is not to single out econometric models in particular, as Lynch levels as strong or stronger critiques against other oil depletion models, particularly curve-fitting models.

reserves." This critique can be applied more generally: there is no reason to suspect that the relationships in exploring for and producing oil should generally be linear or log-linear. Analysts make such simplifications to allow tractable econometric models to be built, but they can only approximate real world behavior.[68]

It is vital to note that these critiques are very similar to those that have been applied to the non-economic models above. This confluence of difficulties across model types is discussed next in Section 7.

[68] Kaufman (1983) also argues that "much precision is lost in translation of a descriptively rich verbal argument to models of the form employed by most econometric model builders to date."

7 Synthesizing thoughts: What have we learned?

This section presents synthesizing thoughts about oil depletion models reviewed here. First, we compare the characteristics of the reviewed models along a number of dimensions, describing trends in variation and convergence across model types. Next we discuss using these models for prediction, including evaluating the quality of model fit to data and the role of complexity in prediction. We conclude with thoughts on one way of improving future oil depletion models.

7.1 A variety of model types

The model reviewed here vary in specificity, phenomena represented, time scale, and geographic scope. Importantly, they also vary in disciplinary worldview and in underlying assumptions about how the world works. For the purposes of this discussion, we will emphasize 4 "dimensions" of variability. Three of these dimensions were shown in Figure 1: intellectual orientation (physical or economic), scale (field-level or global), and degree of model detail (theoretical or mechanistic). We present this diagram again (see Figure 26) to provide a basis for this discussion. The fourth dimension discussed here is model complexity (not presented in Figure 26).

The models included in this review occupy nearly all quadrants of the model space shown in Figure 26. Example models A and B each reside solely in a single quadrant of the figure. Model A is a global, theoretical model that is based on physical aspects of oil production. An example of this type is Hubbert's global logistic model (Hubbert 1980). Model B, on the other hand, is a mechanistic model that operates at the sub-national level, built within an economic framework. An example is Fisher's sub-national econometric model of oil and gas exploration (Fisher 1964).

Model C, on the other hand, "builds" from one quadrant to another. An example is Smith's (2008) bottom-up model. His model is physical in nature, and more mechanistic than, for example, the Hubbert model. It uses data at the field level to build up a global projection of oil availability. Alternatively, Model D includes both economic and physical characteristics simultaneously in a mechanistic framework. An example of this might be the simulation model of Greene *et al.* (2004), which forecasts production in 11 multi-national regions, incorporating both economic and physical aspects of oil production.

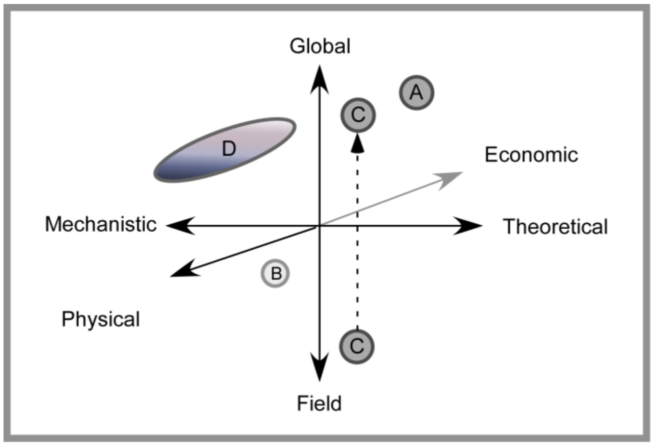


Figure 26. Different model types placed in depletion modeling space. Models A and B reside in one model quadrant. Model C builds up from a field-level basis to project global production. Model D straddles economic and physical portions of the diagram. See text for further discussion.

Models also vary in their complexity. Models can be very simple, such as the R/P model, which solves for one free parameter (exhaustion time) by dividing two input data points (current reserves and current production). They can also be very complex, such as the Davidsen *et al.* (1990) model which requires numerous data inputs and projects production of multiple fuels as well as crude oil imports.

All major models discussed in this review are classified below in Table 2 along these four dimensions of variability. Precise classification of a model is often difficult, and each of these axes represents a spectrum along which models can exist at a number of points. The terms used in Table 2 are defined in the text box below.

Dimensions of model variation
The symbols used in Table 2 (presented in **bold**) are defined as follows:

Models have mechanistic or theoretical character:

- Mechanistic (**M**) models have functions representing detailed economic, technological, or physical aspects of the oil production process.
- Theoretical (**T**) models rely on general, simple functions relating oil production to a small number of input parameters, generally with time as the key independent variable.

Models can also be physically or economically based:

- Physical (**P**) models focus on physical, geological, technological, or engineering aspects of oil production. Examples include models that use empirically-based decline functions.
- Economic (**E**) models rely on or are justified by economic aspects of oil production, such as oil price, demand for oil and oil products, and extraction costs.

Models are built at a variety of scales:

- Field-level (**F**) models project production at individual oil fields, while basin-level (**B**) models project production from a geologically-defined basin. Sub-national (**Sub-N**) models forecast production from geologically arbitrary sub-national regions (e.g., states or provinces), while national (**N**) models project production from a single nation. Multi-national regional (**R**) models forecast production from a region comprising multiple nations (e.g., the North Sea) while global (**G**) models project global production.

Models exhibit a wide range of complexity:

- Very simple (**VS**) models require at most 1 input data series (e.g., historical production data), 1-3 other data inputs (e.g., an estimate of Q_∞) and have 1-3 free parameters that are solved (e.g., t_{peak}). Simple (**S**) models are similar to VS models except they have up to 6 data inputs and and 6 free parameters. Models of moderate (**M**) complexity still require only one time series data input, but they require more than 6 input data parameters or solve for more than 6 free parameters.
- Complex (**C**) models require more than one time series data input (e.g., yearly production data plus yearly oil price) and any number of additional input data, but they only model future oil production. Very complex (**VC**) models are similar to complex models except that they model production of more than one resource type (e.g., crude oil, tar sands, and oil shale).

Some models exhibit characteristics of multiple portions of the diagram simultaneously:

- Models can build from one type of analysis to another. This is represented with an arrow (**→)**. Examples include models that build global production projections from field-level modeling (e.g. **F→G)**
- Models can also integrate multiple model characteristics (**+**). For example, a model might include both physical and economic reasoning simultaneously (e.g., **P+E**).

Table 2. Classification of reviewed models. See text box above for model classification definitions. Author name is first author of first paper for a model type. Year is year of first paper for a model type. See text and references for details for each model.

Author	Year	Model form	Mech. / Theor.	Phys. / Econ.	Scale	Complex.
Pre-curve-fitting models						
Day	1909	Reserve-to-production (R/P)	T	P	N	VS[a]
Day	1909	R/P w/ exponential growth	T	P	N	VS[a]
Ayres	1952	Hand-drawn bell-shaped	T	P	N, G	--[b]
Hubbert	1956	Hand-drawn bell-shaped	T	P	N, G	--[b]
Curve-fitting models						
Hubbert	1959	Logistic cumulative prod.	T	P	N	VS
Moore	1966	Gompertz cumulative prod.	T	P	N	VS
Bartlett	1978	Exponential resource exhaustion	T	P	G	VS
Hammond	1993	Skew-normal production prof.	T	P	N	VS
Laherrere	1999	Multi-cycle logistic	T	P	N	S, M[c]
Hakes	2000	Exponential growth + decline	T	P	G	VS
Hallock	2004	Smoothed exponential	T	P	N→G	S[d]
Hirsch	2005	Linear	T	P	Sub-N, N	VS
Guseo	2005	Technological diffusion	T	P+E	N, G	M
Rerhl	2006	Logistic-curve justification	M→T	P	--	VS
Brandt	2007	Asymmetric curve-fitting	T	P	Sub-N, N, R, G	VS, S
Mohr	2007	Multi-function disrupted	M+T	P	N, R→G	M[c]
Berg	2008	Expanded differential model	M+T	P+E	--	M
Kaufmann	2008	Flexible inverted U-shaped	T	P	G	VS[a]
Simulation models						
Davis	1958	Iterative economic sim.	M	P+E	N	C[d]
Naill	1973	Natural gas production sim.	M	P+E	N	C
Basile	1981	Sim. crude oil and substitutes	M	P+E	R→G	VC[d]
Sterman	1983	Sim. crude oil exploration and production	M	P+E	N	C[d]
Davidsen	1990	Sim. of crude oil, synthetics production plus imports	M	P+E	N	VC[d]
Reynolds	1999	Simple sim. of finding buried resources	M→T	P	--	M
Greene	2003	Regional sim. of producing oil and oil substitutes	M	P+E	R→G	VC
Bardi	2005	Simple sim. of finding and extracting resources	M→T	P	--	M[d]
Brandt	2008	Regional sim. of producing oil and oil susbstitues	M	P+E	R→G	VC
Bottom-up models						
Campbell	1995	Bottom-up model w/ proprietary database	M	P	F, R→G	C[d]
Skrebowski	2004	Database of projects in development	M	P	Sub-F, F→G	C[d]
Miller	2005	Bottom-up model w/ proprietary database	M	P	F, R→G	C[d]
Smith	2006	Bottom-up model w/ mostly public database	M	P	F, R→G	C[d]
CERA	2006	Bottom-up model w/ proprietary database	M	P+E?	F→G	C[d]

PFC Energy	2004	Bottom-up model w/ proprietary database	M	P+E?	F→G	C[d]
Economic models						
Hotelling	1931	Optimal depletion theory	T	E	--	VS
Fisher	1964	Econometric model of oil and gas discovery	M	E	Sub-N	C[f]
Nordhaus	1973	Economic transition to backstop oil substitutes	T	E	R→G	VC
Uhler	1976	Hybrid economic-physical model of exploration	M+T	E+P	Sub-N[e]	C
Pindyk	1978	Optimal depletion with exploration	T	E	--	M
Uri	1982	Hubbert-type economic hybrid	T	E+P	N	C
Slade	1982	Optimal depletion with technological change	T	E	--	C[f]
Pesaran	1990	Hybrid econometric model	M+T	E+P	R	C
Kaufmann	1991	Hybrid econometric/Hubbert model	M+T	E+P	Sub-N	C
Moroney	1999	Hybrid econometric model	M	E+P	N	C
Kaufmann	2001	Econometric model with decomposed oil price series	M	E	Sub-N	C

a – R/P models are, in fact, simpler than our VS definition because they do not require a historical time series to fit the data, but instead only require estimates of reserves and current production. The model of Kaufmann and Shiers does not require the full time series of historical production, only cumulative production to date.
b – Ayres and Hubbert did not specify functional forms for their first projections, making their classification by complexity impossible.
c – The complexity of multi-cycle models depends on the number of cycles modeled (Laherrere) or the number of disruptions allowed in the model (Mohr and Evans)
d – Full equations were not given, so best estimate of complexity is made
e – Uhler makes the claim that the regions of Alberta used in his study, although not geologically defined, actually constitute separate geologic plays.
f – Some economic models (e.g., Fisher or Slade) are applied to multiple resources. They are still classified as complex because each resource is modeled with a separate application of the model. This can be compared to a very complex model like Greene *et al.*, which uses the same model to simultaneously predict the production of a number of competing resources.

7.1.1 Trends suggested by model classification

Table 2 suggests trends across the variety of reviewed models. For example, models have tended to become somewhat more complex over time (see curve fitting models in particular). Also, modern models exhibit more integration of contrasting modeling perspectives. This is especially true in the development of models that mix physical and economic characteristics (P+E). Despite an early pioneering effort by Davis (1958), few models integrated these perspectives until the 1980s. Another promising trend is the recent development of models that attempt to generate assumed theoretical behavior (e.g., bell-shaped profiles) from the mechanisms of oil and gas exploration (M → T).

Another important trend relates to model complexity. Simulation and econometric models tend to be both more complex and more mechanistic than curve-fitting or ODT models. These two dimensions are causally related: if a model assumes that production will follow a bell-shaped curve, essentially no mechanistic modeling is required and therefore there is no need for complexity. Conversely, every aspect added a model to increase its mechanistic specificity requires additional model parameters.

Lastly, a hopeful trend emerges: there is a convergence across models with divergent intellectual underpinnings toward the production of approximately “bell-shaped” production profiles. Such convergence is desirable: many observed profiles have been approximately bell-shaped, and so we should expect models to generate this general behavior regardless of their basis. The following (quite divergent) model types each produce production profiles with generally rounded peaks:

1. System dynamics models of resource extraction and depletion show rounded, often slightly-skewed peaks (see work of Naill or Sterman *et al.*). These models generate peaking as the result of a drawdown of undiscovered oil deposits (a physical effect) which then increases the difficulty of finding oil, which reduces the return on investment in oil exploration (an economic effect).

2. Oil transition simulations under perfect foresight (for example, Basile and Papin or Greene *et al.*) show smooth peaks that tend to be somewhat less bell-shaped. These models generate peaking either through R/P type constraints that limit the rate at which the stock of resources can be depleted in a given year, or through increases in production cost through a depletion cost multiplier.

3. Capacity investment simulation under uncertainty also shows peaking behavior (e.g., Brandt and Farrell). In this case depletion multipliers increase the cost of the marginal barrel of oil or oil substitute, resulting in oil price increases and demand reductions. Also, R/P type constraints limit the rate of extraction.

4. Probabilistic finding models (see work of Bardi or Reynolds) show peaking behavior due to the declining probability of new discoveries as the stock of undiscovered resources dwindles and search effort is limited.

5. Some economic optimal depletion models show optimal production profiles with peaks. These peaks occur in models that include exploration, exogenous technical change, or demand growth (see papers by Holland, Pindyk, Slade, and Tahvonen).

Note that I am excluding from this list all models where peaking is explicitly defined in primary model functions, as in nearly all curve-fitting models reviewed above.

7.2 Prediction, understanding, and complexity

Put plainly, existing models have a poor record of predicting global oil production, and many predictions of a global peak have come and gone without confirmation. The recent stagnation in global output could be the sign of an impending peak in production — thus vindicating a number of recent predictions — but this will not be known for certain for a number of years.

Such uncertainty results not so much from the problems of oil depletion models, but from the general difficulty of forecasting. As Smil (2003) argues, "for more than 100 years long-term forecasts of energy affairs…have, save for a few proverbial exceptions confirming the rule, a manifest record of failure." Clearly, prediction is a much more difficult endeavor than fitting functions to historical data. For this reason, it remains uncertain how useful any of the models reviewed above — even those that appear to fit historical data well — will be for predicting future oil production.

7.2.1 Which model fits historical data best?

In assessing the usefulness of a model for forecasting, a common approach is to compare its fit to historical production data. This is because it is assumed (perhaps reasonably) that models which fit historical data more closely will better predict future production. Unfortunately, the task of determining which model best fits a historical dataset is not straightforward.

Let us again look at the general form of oil depletion models presented:

$$y = f(x_1, x_2, x_3 \ldots, \beta_1, \beta_2, \beta_3 \ldots) + \varepsilon \, . \qquad \textbf{eq. 45}$$

Again, here y is the dependent variable (generally P), x_i are data inputs and β_i are free parameters. The model fitting process consists of searching for values of free parameters that reduce the overall error when fitting the model to historical values of y. This often involves minimizing the sum of squared error terms ε. Generally, oil depletion models are nonlinear, so nonlinear least squares regression is used (NIST/SEMATECH 2008).

The quality of the fit of a model cannot be judged entirely with measures of overall mathematical fit. This is the danger of using metrics such as R^2, which can be misleading. R^2 will nearly always increase when a model is made more complex by adding additional parameters (NIST/SEMATECH 2008). This is because each free parameter adds more flexibility to the model and therefore virtually guarantees that it will fit the data points better. Therefore, statistical measures used to compare models must take into account the complexity of each model (Motulsky and Christopoulos 2004). Such methods include Adjusted R^2, which accounts for the number of parameters in the model, or Akaike's Information Criterion (AIC), which is a result of information theory that allows for the comparison of model fit across divergent models (i.e., models that have different numbers of parameters and are not similar mathematically).[69] Some additional characteristics of good model fits are described in the text box, *What qualities do we look for in a good model fit?*

There are more subtle criteria that apply to the fitting process. Ideally, there should be *a priori* scientific justification for using a model. This is because models are most useful when they can explain what they are modeling.[70] If there is no theoretical basis for a

[69] "Similar" here has a specific meaning. Some tests (e.g., F-tests or *extra sum of squares*) require that the models being compared are mathematically nested, that is, that the simpler models can be written as special cases of the more complex models (Motulsky and Christopoulos 2004).

[70] As Ramsey (1980) argued about oil depletion models, "the obtaining of a good fit to a set of historical data by some statistical expression not generated by a theory only reflects on the ingenuity of the data fitter and says nothing else."

What qualities do we look for in a good model fit?

- An ideal model fit has a number of characteristics. A primary criterion is that we want the smallest divergence between the model and the data points being fit. Most curve-fitting procedures operate by attempting to minimize a measure of the overall divergence between model and data (e.g., sum of squared errors). Another fundamental criterion is that the values found for parameters be physically realistic (for example, we would reject a model fit that resulted in a positive exponential decline rate, as it would imply that ever-increasing production) (Motulsky and Christopoulos 2004).
- The difference between the value that a model predicts for each year and the actual data value is the *residual*. A primary method of analysis of residuals is visual inspection, as all summary statistics result in loss of some information (NIST/SEMATECH 2008). An ideal model fit results in the residuals 1) being normally distributed; 2) being evenly arrayed above and below the line with few consecutive runs of positive or negative values; and 3) having a consistent spread over time (not more divergence in one time period than other). Formal tests exist to determine whether a fit has each of these characteristics.
- If there is consistent divergence between the model and data (e.g., a positive residual is likely to be followed by another positive residual), then the residuals are said to be *serially correlated* (also known as *autocorrelated*). This most commonly occurs because the data are being generated in part by one or more explanatory variables that are not included in the model formulation (or *omitted variables*) (Pindyck and Rubinfeld 1998). For example, in fitting the logistic model to global production data, predicted values are consistently too high after the 1973 oil crisis. This is because the logistic model does not include variables that allow modeling of an oil embargo.
- Serial correlation is often measured using the Durbin-Watson statistic. A Durbin-Watson value of 2 indicates that the data are not serially correlated, while values below 1 indicate significant positive serial correlation. This test is often performed on econometric models of oil production, but is rarely performed on other oil depletion models despite their often obvious problems with serial correlation of residuals. Kaufman (1991) notes that the residuals from the Hubbert fit to US production data are serially correlated, as are the fits to many other simple functions.

model, the fitting process can degrade quickly into "data dredging." Data dredging occurs when analysts fit a number of models to a dataset and study the ones that fit best, without justification of the model form. This results in misleading statistics of fit. For example, p-values from such efforts can be misleading because of the discarded fits to other poorly fitting models.[71] To avoid this problem, models should be selected from a group of models that is chosen based on the scientific nature of the problem, *before* any significant fitting is performed (Burnham and Anderson 2002).

7.2.2 Empirical comparisons of model fit

Given the complexities involved, it is not surprising that empirical analyses of model fit were not performed during the initial controversy surrounding Hubbert's work. Kaufman

[71] One goal of statistical analysis is to eliminate false positives, also known as *Type I* errors. These occur when one finds a relationship or effect when no genuine relationship exists. An example would be finding that a given dataset is best described by the Hubbert curve and concluding that Hubbert-like phenomena generated the data, when in fact it was simply chance that the Hubbert model fit better than another model. Probabilities of a false positive for a single comparison (*comparisonwise* error rate) are different than overall probabilities of error for multiple comparisons (*experimentwise* error rate). These error rates can be very different, particularly if data dredging is performed. This is because a number of models might be fit to a dataset, out of which a small number of models with "good-looking" fits might be selected for formal testing. This informal initial rejection of some models is actually an implicit test that strongly affects the overall probability of obtaining a false positive (Kirchner 2001). For this reason, formal comparisons of model fit should be applied.

(1983) notes that in the 1950s and 1960s "[no] comparison [was] made, based on accepted statistical principles, of the relative qualities of fit to the data or of the predictive accuracies of possible alternative models."

The first such analysis was by Wiorkowski (1981), who compared the ability of curve-fitting models to fit cumulative production and discoveries data. His study focused on resource estimates derived from observed data, a different focus than the models described here. He developed a Generalized Richards function which can take the form of an exponential, logistic, or Gompertz model depending on the value of a single parameter. He then compares this flexible function to the Weibull function and finds that: 1) their quality of fit is almost identical when applied to cumulative production data, and 2) despite this similar quality of fit, the two models result in significantly different projected resource quantities. These results suggested that production data could not, on their own, suggest which of the models is better for use in projecting resource availability.

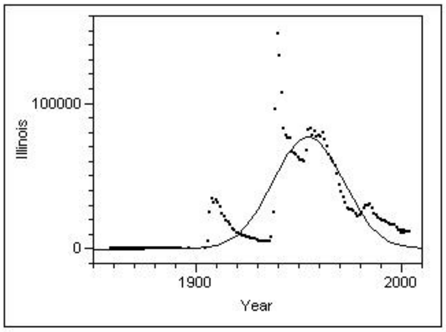
The Energy Modeling Forum of Stanford University sponsored two structured comparisons of economic and econometric models of oil supply in the 1980s. This effort compared the difference between model predictions when differing models are given identical input assumptions (e.g., GDP growth and oil prices) (EMF 1982; 1992). No post-hoc analyses of model accuracy were found.

As described above in Section 3.3, Brandt (2007) compared the fit of six simple (3 and 4 parameter) curve-fitting models to 139 oil production curves at a variety of scales (US states and regions, countries and multi-country regions) He used AIC to compare symmetric and asymmetric versions of a Gaussian bell-shaped model, a linear model and an exponential model. He did not find strong evidence to choose one functional form over another, and each model type was useful in some regions.

No empirical comparisons between broad model types (e.g., curve-fitting vs. econometric) were found in the published literature. However, one general conclusion can easily be drawn from the discussion above: more complex models fit historical data more closely than simpler models. One example is the model of Kaufmann and Cleveland, shown in Figure 24. As discussed above, this increased fidelity is due largely to the additional degrees of freedom of more complex models. Despite this better fit to historical data, it is unclear that complex models are more useful for projecting future production. We address this question next.

7.2.3 Complexity and the purpose of modeling

Once a model surpasses a relatively low level of complexity (e.g., ceases to be a simple curve-fitting model), there is a natural tendency to address shortcomings in its behavior by adding additional complexity to the model (e.g., additional functions or modules). This approach is doomed to failure as a means of producing more accurate forecasts (Smil 2003). Complexity fails because of two fundamental and unavoidable difficulties of mathematical modeling in general: many aspects of the real world must, by necessity, be



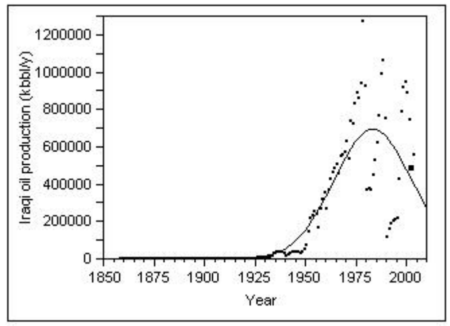


Figure 27. Examples of "out of model" impacts overwhelming bell-shaped production profiles. Left: Oil production in the state of Illinois, fit with the Gaussian model. Note that the best fitting Gaussian model has poor explanatory power due to the strong second cycle of production induced by the introduction of seismography. Right: Oil production in Iraq, with best-fitting Guassian model. Again, the fit of the Gaussian model is poor, due to chaotic political and economic history of the region.

left out of any model, and what is included in a model must necessarily be greatly simplified.[72]

Out-of-model effects – features of the world that are not included in the equations of a model – are technically infinite in number. Though most such effects will be unimportant, in even the most complex model there are numerous that are important enough to cause significant deviations between model predictions and observed behavior.[73]

Critics of curve-fitting models use technological change as the most often cited out-of-model effect, and it serves as a useful example. As an example of technological change, Figure 27 (left) shows the case of Illinois. Hubbert argued that this state exhibited early and late production cycles caused by the introduction of seismography. While these early and late cycles are clear in retrospect, an observer from 1915 could be forgiven for imagining that the oil industry in that state had run its course. Even if seismography were foreseen by this observer, it is fanciful to believe that they could have reliably estimated the timing and overall magnitude of the increase in production resulting from this technological advance.

Thankfully, we are better prepared than such an observer because our knowledge of what is technically and physically possible is significant and is improving over time. But there is still great uncertainty with respect to how future technologies will affect the oil industry. Given the difficulty of defining functions to represent any single type of

[72] This is a very general problem that goes far beyond oil depletion modeling. And models of oil depletion are not significantly less reliable than other types of energy models (Smil 2000; Smil 2003). For example, Smil (2008) calls the IPCC integrated emissions models "computerized fairy tales".
[73] What I call "out-of-model" effects are caused by missing variables, or omitted variables (as they are called in the econometrics literature). They are effects that cannot be predicted by the model because the model contains no representation that they exist.

technological development, adding numerous technologies in any detail would add a tremendous amount of complexity and uncertainty to any model. This is humbling because technological change is in many ways a well-contained problem: it tends to be cumulative (technologies are not often lost in the modern world) and to move in one direction (technologies are not often adopted that result in worse recovery factors or higher production costs).

A better strategy than adding complexity to attempt to forecast the progress of technology is to estimate bounds on the amount of technical change that could occur with respect to a single resource type. This process can be aided by the relevant physical aspects of petroleum production. For example, one might estimate a maximum recovery factor more easily than guessing at the exact technologies to be applied to oil recovery. Another oft-cited limit is that the energy expended in producing crude oil should not exceed the energy content of the oil itself, placing limits on the intensity of extraction technologies.[74]

Far more difficult to model are "human factors" out-of-model effects, such as political disruption, war and conflict, changes in institutional structure or regulation, or demand discontinuities. Who in 1965 would have predicted the recent rise of capitalist China or the Iran-Iraq war of the 1980s? What about the magnitude and timing of the impact of these events on world oil supply and demand? Many such effects are smoothed by the actions of the market, but significant deviations from bell-shaped profiles have occurred often in the past and are certain to happen again (see Figure 27, right). Even if we understood how to quantify such stochastic social phenomena, the complexity required to address even a fraction of these societal or political phenomena in detail would presumably be far beyond that of any existing model.

The second fundamental problem with complexity as a modeling strategy is that as functions are added to a model, data limitations often require them to be in highly simplified form. For example, in the model of Green *et al.*, which is as complex as any model reviewed here, the rate of increase of extraction costs as a function of the depletion level (β, in eq. 27) is equal for all regions and all fuels (Greene, Hopson *et al.* 2003). A model built by this author adopts the same assumption (Brandt and Farrell 2008). This is not because it is a good assumption, but because data on which to base a better assumption are not available.[75]

These problems of complexity are discussed to illustrate a key point: increasing model specificity and complexity is an activity subject to rapidly diminishing returns. It generally does little to improve the reliability of predictions, and can have significant detrimental impacts on the other key goal of model building: increasing our understanding of the behavior of the global oil production system.

[74] There are some difficulties with these arguments, as more than 1 unit of a low-quality energy resource (e.g., waste heat from a power plant) might readily be used to gain 1 unit of high-quality energy.
[75] Oil companies most certainly have data that would allow much better assumptions about the variation of production cost with depletion level, but these data are not publically available.

7.2.4 Making forecasts: Promise and pitfalls

To introduce the problem of forecasting global oil production, we will use the case of Hubbert's 1956 prediction of US oil production as an example. First, while Hubbert's analysis is well-remembered, no fewer than 7 papers published in the 1950s predicted a peak in US production within an approximately 10 year period surrounding the actual peak (1962-1973). These predictions are listed in Table 3.

There are two ways to view the fact that multiple authors made quite similar predictions. One could argue that this proves that Hubbert's method was not extraordinary, as other authors using other methods came just as close to predicting the peak date.[76] Alternatively, one could argue that a variety of authors using a variety of methods produced estimated peak dates clustered in a reasonably tight band around the actual peak date. This interpretation suggests that the future of US oil production could be generally intuited 10-15 years beforehand using quantitative methods.

Another important lesson from these projections is that while a number of them had the peak date approximately correct, all of them underestimated URR. Cumulative production already exceeds 200 Gbbl and significant reserves still remain. Thus, production has not dropped as quickly as Hubbert (or the other authors of his time) predicted (Jackson 2006).

Table 3. 1950s predictions of peak US oil production.

Author	Year of estimate	Early peak date	Late peak date	URR (Gbbl)
PMPC	1952	1963	1967[a]	NA
Ayres	1952	1962	1964[b]	100
Ayres	1953	1960	1968-1970	100 / 200
Hubbert	1956	1965	1970[c]	150 / 200
Ion	1956	1965	1970[d]	NA
Pogue	1956	1970	1972[b]	165
Davis	1958	1964	1973	NA

a – Figure presented on p. 103. These peaks are quite angular, rising and then falling in linear segments with discontinuities in the slopes. Three scenarios were presented: 1 - production grows until 1975; 2 - production has shallow peak in 1967; 3 - production has sharp peak in 1963.
b – These dates come from a single prediction, but it is difficult to determine the precise year of peak production from the figure.
c – In the article text, Hubbert states, "about 1965" and "about 1970", so we use these dates. It is difficult to determine precise peak years from the figure.
d – Ion presents a table (p. 83) which shows production in 1955, 1965, and 1975. Production peaks in the West Coast and "Other States" regions in 1965, while it continues to growth in the Rockies and Gulf Coast region until 1975. Total production is equal in 1965 and 1970, implying that the overall peak is somewhere in this time period. He additionally suggests the possibility of a flat peak: "production in the U.S.A. might well reach 400 million tons p.a. (8.3 million b.d.) within a few years and then flatten there for twenty years."

[76] Note that while Hubbert's prediction of 1970 pinpointed the exact year of peak production, this was his upper range estimate. When comparing his 5 year range to other author's predictions, it appears that some of the other predictions could be considered just as accurate.

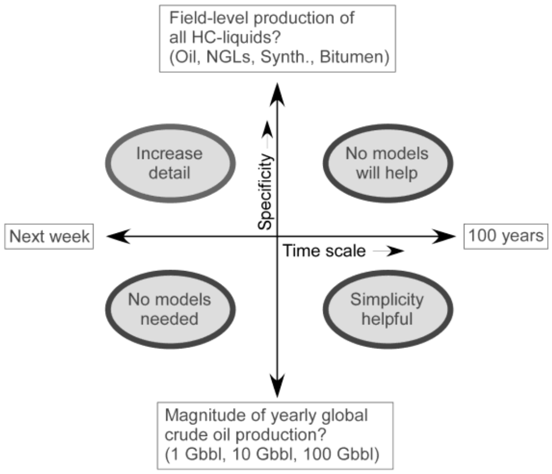


Figure 28. Schematic of the variation in predictions with time scale and specificity. Note that the appropriate response to a modeling problem varies depending on the quadrant in which the question of interest lies.

With this earlier case in mind, we now discuss the problem of predicting global oil production. Of key importance is that the viability of forecasts varies with the desired specificity and time scale of the forecast. This is illustrated by Figure 28, which shows how forecasts vary with these two parameters.[77] For example, if one wishes to predict the order of magnitude of global crude oil production next year (e.g., will it be approximately 1 Gbbl, 10 Gbbl, or 100 Gbbl?), no mathematical model is needed: barring a global catastrophe, crude oil production will not change significantly over the course of a year. A similar case exists if we only wish to know the century of peak oil production. If, instead, one wishes to predict basin-level production of multiple hydrocarbon products in the year 2050, no model can provide a useful answer.

It is the other cases in Figure 28 that are more interesting. These cases involve using simple models to predict general behavior over longer time periods, or using more complex models to make near-term predictions.

Simple models can be used to generate estimates of general long-term behavior. For example, such models tell us that conventional production of hydrocarbons of the scale of 50 Gbbl/y in 2100 is extremely unlikely, because all reasonable estimates of URR suggest that this is not possible.

If future profiles are even approximately bell-shaped, simple mathematics allows these types of predictions: the long periods of rapid growth seen in bell-shaped or exponential-like profiles result in very high consumption rates near the peak of production. The exact shape of the curve is relatively unimportant (Bartlett 1978). This is the reason for the

[77] Many of the disputes in the literature surrounding depletion modeling likely result from a misunderstanding (or poor communication) regarding which of these quadrants the predictions from a particular model reside in.

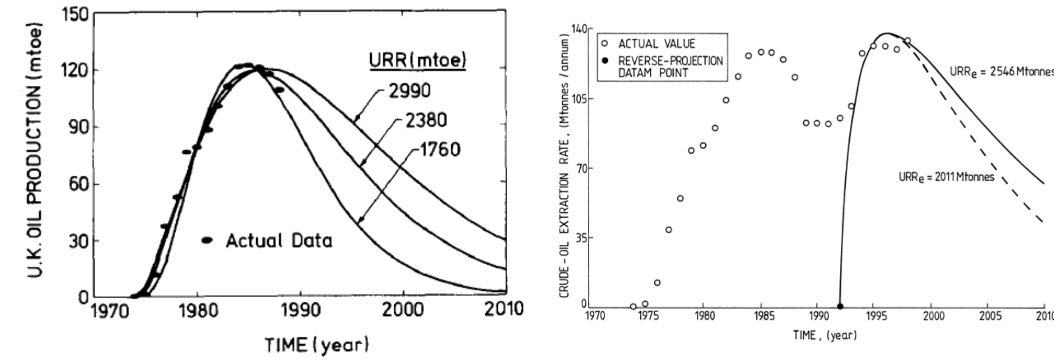


Figure 29. Comparison of projections of UK oil production from Hammond and Mackay (1993) (left) and Mackay and Probert (2001) (right), both generated with skew normal production profile (SNPP) model.

general agreement between the estimates of Hubbert and his contemporaries (noted above in Table 3).

Note that there are numerous caveats and reasons for skepticism here: using a bell-shaped model for a well-defined resource type (e.g., onshore light oil) is reasonable if demand continues to grow, and if production is unhindered by political interference or economic disruption. This is, of course, not strictly the case on the global stage. Also, recall that the predictions of Hubbert and his contemporaries underestimated URR (Jackson 2006), and that evidence suggests that production declines tend to be less steep than inclines (Brandt 2007), suggesting that declines after the peak could be more shallow than currently thought.

Complex mechanistic models have clear advantages in reproducing short-term fluctuations in historical oil production because they have more freedom to allow the fitting algorithm to more closely match predictions to data. Predictions made with such models will tend to be reasonable for periods in which underlying variables that are omitted from the model itself stay reasonably constant, and therefore the fitted model formulation still holds.

Unfortunately, it is not clear that complex models have advantages for long-run prediction. This is because the uncertainty in a forecast necessarily increases as the time scale of the prediction increases. This necessarily increases the model's reliance on assumptions. In a simple model, these assumptions are few in number, simple to communicate, and open to ready critique. The more complex the model, the more opaque and difficult this process becomes, and the more these proliferating assumptions counteract the benefit of the model's increased detail. In the case where a modeler is making hundreds of long-range assumptions about future discoveries or recovery rates for a field-level bottom-up model, prudence and parsimony suggest that a simpler model might be more appropriate.

An example shows the futility of trying to use these models in an inappropriate context. Curve-fitting models are poor at making precise short-term predictions because they can relentlessly fit one short-term trend after another, even if supplied with consistent estimates of URR (see Figure 29). Hammond and Mackay (1993) used their SNPP model to predict UK oil production of 15 to 70 Mtonne in the year 2000. Unfortunately, the curve-fitting algorithm had locked onto what was only a temporary decline. Mackay and Probert (2001) later used the same model, showing that actual production in the year 2000 was above 130 Mtonne, higher even than the peak modeled in the previous paper. Inexplicably, the authors call this model "well tested and tried" in the 2001 paper, despite its very poor prior performance. This model is neither alone nor exceptional in its poor predictive ability. Nearly all simple models would do the same.

A complex field-level bottom-up model could have noted that the decline in UK production shown in Figure 29 (right) was due to the Piper Alpha disaster, and predicted that production would likely increase after this temporary problem, due to the underlying reserve base and installed production capacity. But if the modeled region were less mature than the UK continental shelf, the use of a bottom-up model would require a large number of assumptions about future discoveries and reserve growth. This would reduce its advantages over a curve-fitting model for long-term predictions.

Note that the above discussion has focused on modeling the *path* of extraction of a known resource base, neglecting the uncertainty caused because we do not know the appropriate value of URR. While URR is discussed in detail in a companion report (Sorrell and Speirs 2009), one point is worth emphasizing here: estimates of URR are simultaneously subject to geological, technological, and economic uncertainty. It would be very difficult to build a model that accounts for all of these factors in any detail. For this reason, the models generally rely on an exogenous value or probabilistic ranges of values for URR, such as the USGS estimates (U. S. Geological Survey 2000).

In summary, this author's judgment with respect to the predictive value of oil depletion models is that:

1. Simple curve-fitting models can provide a rough outline of future production, *for a given level of URR*. The uncertainties involved limit the precision of such estimates to the scale of a decade.
2. More-detailed mechanistic models (e.g., bottom-up models), exhibit greater fidelity in reproducing historical data and are therefore more useful for near-term predictions. Their insight into development projects (e.g., Skrebowski's work) in principle allows good predictive ability for a 3-5 year time period, *excluding the potential for external shocks*. This advantage wanes for long-term forecasts because complexity makes them over-specified and "brittle" with respect to the uncertainties of future decades.
3. All models are vulnerable to any number of unforeseeable external shocks (whether technological, economic, or political in nature), meaning that *none* of the reviewed models can provide estimates of great precision. Increasing model complexity does little to address this problem.

7.3 Moving forward: Improving oil depletion modeling

A number of ways to improve oil depletion models have been implied by the above discussion. Here we will discuss one important opportunity: further integrating the views of physical scientists and economists on the problem of resource depletion. A useful way of understanding the need for such integration arises from asking a simple question: *which fuels will be produced after the peak in output of conventionally produced petroleum?*

Most models outlined above, particularly the curve-fitting models, leave this question aside entirely. Some analysts make explicit that they are only projecting output of conventional oil (e.g., Campbell's recent models are very specific about which resources are included). Others do not explicitly outline which resources are included within their projections. And often analysts blur the distinction between a decline in conventionally-produced petroleum and a decline in overall production of liquid hydrocarbons, implicitly assuming that a decline in the former is equivalent to a decline in the latter — which need not be the case.

This focus on conventionally produced petroleum (regardless of how one defines "conventional") misses a crucial aspect of the energy supply system: we already do not produce oil from a single, well-defined resource with an easily-definable resource base. Instead, hydrocarbons are produced from a wide range of deposits using a variety of technologies. And the diversity of production strategies will increase significantly in the face of a peak of conventional oil output.

Economists have paid significant attention to this issue. They argue that what is important for understanding oil depletion is not how much hydrocarbon remains in the ground, but the characteristics of hydrocarbon resources and our ability to substitute other resources for conventional hydrocarbons.[78] This is a key point because a decline in conventional oil output will not occur in a vacuum. Instead, it will likely result in significant oil price increases, which will induce investment in a wide variety of alternative resources. This has happened before. As Deming (2001) argues, "The history of energy use is one of substitution."

Some analysts have taken this economic argument to an extreme, committing the opposite error of those who focus too exclusively on conventional oil. Van der Veen (2006) quotes Linden, who argued that the US peak "had nothing to do with any geologic factors, but was merely a rational reaction to the realities of the global oil market." This is nonsense. The United States peak was the combination of a geologic reality (that resource depletion caused the cost of adding domestic production capacity to rise to prohibitive levels) and an economic reality (that cheap foreign oil was available as a substitute).

[78] McCabe (1998) argued that "the total amount of any fossil fuel present in the Earth's crust (its crustal abundance) is an astronomic figure that, while of possible academic interest, is a number that has no economic significance."

Simplistic arguments on either side of this debate fail because resource depletion is not an “all or nothing” phenomenon. As oil resources in a region become depleted, the average quality of the resource produced will tend to decline because producers will preferentially extract the best resources first if they are able. This decline in quality will cause costs to increase.[79] Producers will make compensating investments to mitigate these cost increases, but eventually they will induce substitution with a now-cheaper alternative energy source. We do not live in a depletion-or-substitution world, but a depletion-*and*-substitution world.[80]

Models constructed without attention to this substitution effect are missing a trend already well underway. Hydrocarbon substitutes for conventionally-produced petroleum (SCPs) are already produced in significant quantities.[81] Enhanced oil recovery technologies in the US produced about 0.6 million barrels per day (Mbbl/d) in 2006, mostly steam-induced heavy oil production in California and CO_2-EOR in the Permian Basin of Texas (Moritis 2006). Production of oil from Canada’s tar sands reached 1.25 Mbbl/d in 2006 (ADE 2007). Production from Venezuela’s extra-heavy oil resources was about 0.6 Mbbl/d in 2000 (Williams 2003), and has been relatively steady since. In addition, approximately 0.15 Mbbl/d of synthetic fuels are produced from other fossil fuel feedstocks, primarily from coal (Fleisch, Sills *et al.* 2002). And oil shale is produced in minor quantities around the world in small facilities, with total world output estimated at 10,000 to 15,000 bbl/d (Bartis, LaTourrette *et al.* 2005). In total, SCP production is currently above 2 M crude-oil-equivalent bbl per day, or more than 2.5% of total liquids production.

In a world with depletion and substitution, we will likely witness a decline in production of conventionally-produced crude oil coupled with a simultaneous increase output of SCPs. In this case, we should be concerned with the characteristics of the SCPs that we might use instead of conventional oil.

Of particular importance is the *rate* at which production capacity for SCPs can be built relative to the rate of decline of conventionally-produced petroleum. As Hirsch *et al.* (2005) cogently argue, these rates will strongly affect the amount of economic disruption that will result from a peak in conventional oil production. This is because the long lead-times and delays associated with introduction of new technologies can result in shortfalls of supply (and therefore oil price increases) if conventional oil production drops more quickly than SCP production capacity for can be built.

---

[79] For example, heavy oil is viscous and hydrogen-deficient compared to conventional crude oil. A depletion-induced shift to heavy oil will result in increased extraction costs (due to steam injection, which is used to reduce the viscosity of oil to enable it to flow in the reservoir) and increased refining costs (more hydrogen input required or expansion of coking capacity).

[80] McCabe (1998) made a similar argument in contrasting “closed market” and “open market” views of the oil depletion problem.

[81] Here we consider conventionally-produced petroleum as petroleum produced from a well-bore using primary and secondary recovery technologies. Tertiary recovery technologies and other means of hydrocarbon extraction (e.g., tar sands mining) are considered unconventional technologies. All definitions of “unconventional” are, of course, somewhat subjective and can change over time.

Another important factor is how intrinsically costly it is to access, extract, and upgrade SCPs into refined fuels. These characteristics are usefully described as the "quality" of the hydrocarbon resource. The quality of a resource stems from its physical characteristics, such as the carbon/hydrogen ratio, which affects upgrading energy intensity, or the sulfur content, which affects desulfurization hydrogen demand. These intrinsic physical characteristics affect both the energetic value of the product (often quantified using the energy return on investment, or EROI) and, of great importance, the resulting environmental impacts from producing fuels from SCPs. Because of the lower quality of SCP resources, both the energy intensity and environmental impacts of producing fuels from them are likely to be significantly greater than those from conventional fuels. (Farrell and Brandt 2006). This is the reasoning Dusseault (1997) uses when he argues that "limitations on oil use are therefore more logically related to environmental issues such as global warming and urban pollution."

Because this transition to oil substitutes is already underway, models that do not account for production of SCPs are sure to be pessimistic with respect to total liquid fuel output. Such models are lacking because they do not account for the adaptive ability of our energy system. Thus they cannot analyze the already-growing economic, social, and environmental impacts of the transition to substitutes for conventional oil. This is unfortunate because these impacts are, in fact, the first effects of the peak in conventional oil output.

This need to account for *both* depletion and substitution in oil depletion models points to a clear strategy for improving our depletion modeling: include both physical and economic aspects of the resource extraction process in our models. Such efforts have been made in a number of previous models (see Table 2 above for models that combine physical and economic insights), but additional work must continue in this area. While such collaboration across disciplines is not easy, the potential benefits to the improvement of oil depletion models are large.

8 Conclusions

Miller (2005) provides a single sentence summary of nearly all that can be said with certainty about this complex topic: "The eventual peak of oil production will be determined by *geology*, *economics* and *politics*." Geology, by fixing the size and physical qualities of deposits of hydrocarbons, sets the limits to what we might extract. Economics, by defining how much consumers are willing to sacrifice in order to receive a barrel of oil, limits the amount of labor, energy, steel, and other commodities that producers will invest into its extraction. And politics will augment or hinder the economic returns from finding and extracting oil, resulting in production profiles that could differ greatly from those we might expect based on geologic and economic factors alone.

Given the many difficulties with our methods of modeling oil production, can these models tell us about the future of oil production in future decades? At a gross level, and with assumptions and limitations accepted, the answer is "perhaps yes." But for questions of any precision, the answer is a firm and clear "no." But in either case we should heed the advice of McKelvey (1966), who responded wisely to Ryan's early criticisms of Hubbert's models: "Recognizing their inadequacy, however, it is nevertheless important not to loose sight of their value." Therefore, a better question might be, "What value do these models provide?"

For one, even the simplest models remind us of the truth presented by Hubbert repeatedly in his graph of fossil fuel production over millennia: the era of exponentially growing consumption of fossil fuels will, with all certainty, be short. Or, as Ayres (1952) argued more poetically some years before Hubbert's famous prediction,

> We seem to have a choice between prediction of expansion for the indefinite future, based upon hope, and prediction of the more probable shape of things to come, based upon reason. Reason may well prove to be inadequate and therefore misleading, but hope would seem to be an even more slim reed to lean upon.

And while simple models cannot show us the future of oil production in detail, they do provide as good a means as any other to make approximations of the production of a defined resource base under stable market conditions. They might allow, for example, useful estimates of the decade of peak production for a given estimate of URR. Such decadal accuracy is likely within the bounds of uncertainty placed on the oil market by the vagaries of politics or conflict. But, because URR is a complex quantity with both physical and economic underpinnings, it will remain uncertain in the real world. Analysts should therefore be modest in claims made with such models.

Perhaps most importantly, more complex models that include substitution and competition between various energy resources provide us with a framework for studying the scale of the economic and environmental impacts of the inevitable transition to substitutes for conventional oil. Given that the potential impacts of such a transition are profound, this is clearly a contribution of value.

9 References

ADE (2007). Alberta's oil sands 2006, Alberta Department of Energy.

Adelman, M. A. and M. C. Lynch (1997). "Fixed view of resource limits creates undue pessimism." Oil and Gas Journal **95**(14): 4.

Anonymous (1920). Asserts americans face oil shortage. New York Times. New York, NY.

Arnold, R. (1916). "The petroleum resources of the United States." Annual report of the Board of Regents of the Smithsonian Institution **1916**: 273-287.

Arps, J. J. (1945). "Analysis of decline curves." Transactions of the American Institute of Mining Engineers **160**: 228-247.

Arps, J. J. and T. J. Roberts (1958). "Economics of drilling for Cretaceous oil on the east flank of Denver-Julesberg basin " AAPG Bulletin **42**(11): 2549-2566.

Ayres, E. (1952). "Synthetic liquid fuels - When and how?" Petroleum Processing **7**(1): 41-44.

Ayres, E. (1953). "U.S. oil outlook: How coal fits in." Coal Age **1953**(August): 70-73.

Babusiaux, D., S. Barreau, et al. (2004). Oil and gas production: Reserves, costs, and contracts. Paris, Editons TECHNIP.

Bakhtiari, A. M. S. and F. Shahbudaghlou (2001). "IEA, OPEC oil supply forecasts challenged." Oil & Gas Journal **99**(18): 24-26.

Bardi, U. (2005). "The mineral economy: A model for the shape of oil production curves." Energy Policy **33**(1): 53-61.

Bartis, J. T., T. LaTourrette, et al. (2005). Oil shale development in the United States: Prospects and policy issues. RAND: Infrastructure, Safety and Environment. Santa Monica, CA, RAND**:** 68.

Bartlett, A. A. (1978). "Forgotten fundamentals of the energy crisis." American Journal of Physics **46**(9): 876-888.

Bartlett, A. A. (2000). "An analysis of US and world oil production patterns using Hubbert-style curves." Mathematical Geology **32**(1): 1-17.

Basile, P. S. and A. Papin (1981). "World oil: A long-term look." Energy **6**(6): 529-541.

Behrens, W. W. (1973). The dyanmics of natural resource utilization. Toward global equilibrium: Collected papers. D. L. Meadows and D. H. Meadows. Cambridge, MA, Wright-Allen Press inc.

Bentley, R. and G. Boyle (2007). "Global oil production: forecasts and methodologies." Environment and Planning B: Planning and Design **34**: 000-000.

Bentley, R. W., R. H. Booth, et al. (2000). "Perspectives on future of oil." Energy Exploration & Exploitation **18**(2-3): 147-206.

Bentley, R. W., S. A. Mannan, et al. (2007). "Assessing the date of the global oil peak: the need to use 2P reserves." Energy Policy **35**(12): 6364-82.

Berg, P. and S. Korte (2008). "Higher-order Hubbert models for world oil production." Petroleum Science and Technology **26**(2): 217-230.

Brandt, A. R. (2007). "Testing Hubbert." Energy Policy **35**(5): 3074-3088.

Brandt, A. R. and A. E. Farrell (2008). Dynamics of the oil transition: Modeling capacity, costs, and emissions. UCEI Energy Policy and Economics Working Paper. U. o. C. E. Institute. Berkeley, CA, Univeristy of California Energy Institute**:** 51.

Burnham, K. P. and D. R. Anderson (2002). Model selection and multimmodel inference: A practical information-theoretic approach. New York, NY, Springer.

Caithamer, P. (2007). "Regression and time series analysis of the world oil peak of production: Another look." Mathematical Geosciences.

Campbell, C. (1998). "The future of oil." Energy exploration & exploitation **16**(2-3): 125-152.

Campbell, C. and C. J. Campbell (2004). Essence of Oil & Gas Depletion, Multi-Science Publishing Co. Ltd.

Campbell, C. and J. Laherrere (1995). The world's supply of oil 1930-2050. Geneva, Petroconsultants SA.

Campbell, C. J. (1995). "The next oil price shock - the worlds remaining oil and its depletion." Energy Exploration & Exploitation **13**(1): 19-46.

Campbell, C. J. (1996). "Status of world oil depletion at the end of 1995." Energy Exploration & Exploitation **14**(1): 63-81.

Campbell, C. J. (1997). "Better understanding urged for rapidly depleting reserves." Oil and Gas Journal **95**(14): 3.

Campbell, C. J. (2000). "A new energy crisis: When will we ever learn?" Energy Exploration and Exploitation **18**(5): 569-571.

Campbell, C. J. (2003). "Industry urged to watch for regular oil production peaks, depletion signals." Oil and Gas Journal **101**(27): 38-45.

Cavallo, A. J. (2002). "Predicting the peak in world oil production." Natural Resources Research (International Association for Mathematical Geology) **11**(3): 187-195.

Charpentier, R. R. (2003). The future of petroleum: Optimism, pessimism, or something else? USGS Open-File Report. Washington, D.C., United States Geological Survey.

Cleveland, C. J. (1991). "Physical and economic aspects of resource quality: The cost of oil supply in the lower 48 United States, 1936-1988." Resources and Energy **13**(2): 163-188.

Cleveland, C. J. and R. K. Kaufmann (1991). "Forecasting ultimate oil recovery and its rate of production: incorporating economic forces into the models of M. King Hubbert." The Energy Journal **12**(2): 17-46.

Conrad, J. M. and C. W. Clark (1987). Natural resource economics: Notes and problems. Cambridge, Cambridge University Press.

Dahl, C. and T. E. Duggan (1998). "Survey of price elasticities from economic exploration models of US oil and gas supply." Journal of Energy Finance & Development **3**(2): 129-169.

Davidsen, P. I., J. D. Sterman, et al. (1990). "A petroleum life cycle model for the United States with endogenous technology, exploration, recovery, and demand." System Dynamics Review **6**(1): 66-93.

Davis, W. (1958). "A study of the future productive capacity and probable reserves of the U.S." Oil & Gas Journal **1958**(February 24th): 105-118.

Day, D. T. (1909). "The petroleum resources of the United States." The American Review of Reviews **39**(1): 49-56.

Day, D. T. (1909). Report of the National Convservation Commission: Volume III - The petroleum resources of the United States. U. S. Congress, Government Printing Office**:** 446-464.

Deffeyes, K. S. (2003). Hubbert‘s Peak: The Impending World Oil Shortage, Princeton University Press.

Deming, D. (2001). "Oil; are we running out?; Petroleum provinces of the twenty-first century." AAPG Memoir **74**: 45-55.

Dittrick, P. (2006). "Comment: CERA study disputes peak-oil capacity growth arguments." Oil and Gas Journal **104**(31): 20-22.

Drew, L. J. and J. H. Schuenemeyer (1993). "The evolution and use of discovery process models at the US Geological Survey." AAPG Bulletin **77**(3): 467-478.

Dusseault, M. B. (1997). "Flawed reasoning about oil and gas." Nature **386**(March 6): 12.

Edmonds, J. and J. Reilly (1983). "Global energy production and use to the year 2050." Energy **8**(6): 419-432.

EMF (1982). World oil: Summary report. Stanford, CA, Energy Modeling Forum, Stanford University.

EMF (1992). International oil supplies and demands. Stanford, CA, Energy Modeling Forum, Stanford University.

Epple, D. (1983). Econometrics of Exhaustible Resource Supply: A Theory and an Application. United States**:** 73p.

Erickson, E. W. and R. M. Spann (1971). "Supply response in a regulated industry: the case of natural gas." The Bell Journal of Economics and Management Science **2**(1): 94-121.

Farrell, A. E. and A. R. Brandt (2006). "Risks of the oil transition." Environmental Research Letters **1**(1).

Feng, L., J. Li, et al. (2008). "Peak oil models forecast China's oil supply, demand." Oil & Gas Journal **106**(2): 43-47.

Fisher, A. C. (1979). Measures of natural resource scarcity. Scarcity and Growth Reconsidered. V. K. Smith. Baltimore, MD, Johns Hopkins University Press.

Fisher, F. M. (1964). Supply and costs in the United Sates petroleum industry. Baltimore, MD, Johns Hopkins University Press (for Resources for the Future).

Fleisch, T. H., R. A. Sills, et al. (2002). "2002 - Emergence of the gas-to-liquids industry: A review of global GTL developments." Journal of Natural Gas Chemistry **2002**(11): 1-14.

Greene, D. L., J. L. Hopson, et al. (2006). "Have we run out of oil yet? Oil peaking analysis from an optimist's perspective." Energy Policy **34**(5): 515-31.

Greene, D. L., J. L. Hopson, et al. (2003). Running into and out of oil: Analyzing global oil depletion and transition through 2050. Oak Ridge, Tennessee, Oak Ridge National Laboratory**:** 124.

Greene, D. L., J. L. Hopson, et al. (2004). "Running Out of and Into Oil: Analyzing Global Oil Depletion and Transition Through 2050." Transportation Research Record **1880**(1880): 1-9.

Guseo, R., A. Dalla Valle, et al. (2007). "World Oil Depletion Models: Price effects compared with strategic or technological interventions." Technological Forecasting and Social Change **74**(4): 452-469.

Hakes, J. E. (2000). "Long-term oil and gas supply; American Association of Petroleum Geologists 2000 annual meeting." American Association of Petroleum Geologists 2000 annual meeting, New Orleans, LA, United States, Apr. 16-19, 2000 **2000**: 62.

Hallock Jr, J. L., P. J. Tharakan, et al. (2004). "Forecasting the limits to the availability and diversity of global conventional oil supply." Energy **29**(11): 1673-1696.

Hammond, G. P. and R. M. Mackay (1993). "Projections of UK oil and gas supply and demand to 2010." Applied Energy **44**(2): 93-112.

Hartigan, J. A. (1981). "Estimating Volumes of Remaining Fossil Fuel Resources: A Critical Review: Comment." Journal of the American Statistical Association **76**(375): 548.

Hewett, D. F. (1929). Cycles in metal production, The American Institute of Mining and Metalurgical Engineers**:** 31.

Hirsch, R. L. (2005). "Shaping the peak of world oil production." World Oil **226**(10).

Hirsch, R. L., R. Bezdek, et al. (2005). Peaking of world oil production: Impacts, mitigation, & risk management, US Department of Energy, National Energy Technologies Laboratory**:** 91.

Hoel, M. (1978). "Resource extraction when a future substitute has an uncertain cost." Review of Economic Studies **45**(3): 637-644.

Holland, S. P. (2008). "Modeling peak oil." Energy Journal **29**(2): 61-79.

Hotelling, H. (1931). "The economics of exhaustible resources." The Journal of Political Economy **39**(2): 137-175.

Howarth, R. B. and R. B. Norgaard (1990). "Intergenerational resource rights, efficiency, and social optimality." Land Economics **66**(1): 1-11.

Hubbert, M. K. (1949). "Energy from fossil fuels." Science **109**(2823): 103-109.

Hubbert, M. K. (1956). Nuclear energy and the fossil fuels. Meeting of the Southern Dsitrict, Division of production, American Petroleum Institute, San Antonio, Texas, Shell Development Company.

Hubbert, M. K. (1967). "Degree of Advancement of Petroleum Exploration in United States." AAPG Bulletin **51**: 2207-2227.

Hubbert, M. K. (1969). Energy resources. Resources and Man. N. A. o. S. C. o. R. a. Man. San Francisco, W.H. Freeman and Company.

Hubbert, M. K. (1972). Estimation of oil and gas resources. Workshop on tecniques of mineral resource appraisal: Talks, questions and candid thoughts by geologists of the USGS. Denver, CO, United States Geological Survey.

Hubbert, M. K. (1980). Techniques of prediction as applied to the production of oil and gas. Symposium on Oil and Gas Supply Modeling, Department of Commerce, Washington, D.C., National Bureau of Standards.

Huber, P. W. and M. P. Mills (2005). The bottomless well. New York, Basic Books.

Ion, D. C. (1956). Oil resources in the next half century. Essential factors in the future development of the oil industry. G. Sell. London, The Institute of Petroleum.

Jackson, P. (2006). "Why the “Peak Oil” theory falls down myths, legends, and the future of oil resources." Cambridge Energy Research Associates, Nov.

Kaufman, G. M. (1983). Oil and gas: Estimation of undiscovered resources. Energy Resources in an Uncertain Future: Coal, Gas, Oil and Uranium Supply Forecasting. M. A. Adelman, J. C. Houghton, G. M. Kaufman and M. B. Zimmerman. Cambridge, MA, Ballinger Publishing Company.

Kaufmann, R. K. (1991). "Oil production in the lower 48 states : Reconciling curve fitting and econometric models." Resources and Energy **13**(1): 111-127.

Kaufmann, R. K. and C. J. Cleveland (2001). "Oil production in the lower 48 states: Economic, geological, and institutional determinants." Energy Journal **22**(1): 27-49.

Kaufmann, R. K. and L. D. Shiers (2008). "Alternatives to conventional crude oil: When, how quickly, and market driven." Ecological Economics **67**(3): 405-411.

Kirchner, J. (2001). Data Analysis Toolkit #9: Experimentwise error rates and statistical fishing expeditions. Berkeley, CA, Department of Earth and Planetary Sciences, University of California, Berkeley.

Krautkraemer, J. and M. Toman (2003). Fundamental economics of depleteable energy supply. Discussion Paper. Washington, D.C., Resources for the Future**:** 30.

Krautkraemer, J. A. (1998). "Nonrenewable resource scarcity." Journal of Economic Literature **36**(4): 2065-2107.

Laherrere, J. (2003). Modeling future oil production, population and the economy. ASPO Second International Workshop on Oil and Gas, Paris.

Laherrère, J. H. (1999). "World oil supply - what goes up must come down, but when will it peak?" Oil and Gas Journal **97**(5): 57-64.

Laherrère, J. H. (2000). "Learn strengths, weaknesses to understand Hubbert curve." Oil and Gas Journal **98**(16): 63-64.

Livernois, J. R. and R. S. Uhler (1987). "Extraction costs and the economics of nonrenewable resources." The Journal of Political Economy **95**(1).

Lohrenz, J. and E. A. Monash (1980). Some Modern Notions on Oil and Gas Reservoir Production. Symposium on Oil and Gas Supply Modeling, Department of Commerce, Washington, D.C., National Bureau of Standards.

Lynch, M. C. (1999). "The wolf at the door or crying wolf: Fears about the next oil crisis." Advances in the Economics of Energy and Resources **11**: 117-142.

Lynch, M. C. (2002). "Forecasting oil supply: theory and practice." The Quarterly Review of Economics and Finance **42**(2): 373-389.

Lynch, M. C. (2003). "Petroleum resources pessimism debunked in Hubbert model and Hubbert modelers' assessment." Oil and Gas Journal **101**(27): 38-47.

Mackay, R. M. and S. D. Probert (2001). "Forecasting the United Kingdom's supplies and demands for fluid fossil-fuels." Applied Energy **69**(3): 161-89.

McCabe, P. J. (1998). "Energy resources - Cornucopia or empty barrel?" AAPG Bulletin **82**(11): 2110-2134.

McCarthy, T. (2001). "The coming wonder? Foresight and early concerns about the automobile." Environmental History **6**(1): 46-74.

McKelvey, V. E. (1966). "V.E. McKelvey's reply to J.M. Ryan." Journal of Petroleum Technology **1966**(March): 287.

Meadows, D. H., J. Randers, et al. (2004). Limits to growth: The 30-year update. White River Junction, VT, Chelsea Green Pub.

Meng, Q. Y. and R. W. Bentley (2008). "Global oil peaking: Responding to the case for 'abundant supplies of oil'." Energy **33**(8): 1179-1184.

Miller, R. G. (2005). "Global oil supply to 2030." Unpublished Manuscript.

Mohr, S. and G. Evans (2008). "Peak oil: Testing Hubbert’s curve via theoretical modeling." Natural Resources Research **17**(1): 1-11.

Mohr, S. H. and G. M. Evans (2007). "Mathematical model forecasts year conventional oil will peak." Oil and Gas Journal **105**(17): 45-46.

Moore, C. L. (1962). Method for evaluating U.S. crude oil resources and projecting domestic crude oil availability. Washington, D.C., United States Department of the Interior**:** 112.

Moore, C. L. (1966). Analysis and projections of the historic patterns of U.S. domestic supply of crude oil, natural gas, and natural gas liquids. Washigton, D.C., United States Departement of the Interior, Office of Oil and Gas**:** 82.

Moritis, G. (2006). "Special report: EOR / Heavy oil survey." Oil & Gas Journal **2006**(April 17th): 37-57.

Moroney, J. R. and M. D. Berg (1999). "An integrated model of oil production." The Energy Journal **20**(1): 105-124.

Motulsky, H. and A. Christopoulos (2004). Fitting models to biological data using linear and non-linear regression: A practical guide to curve fitting. New York, New York, Oxford University Press.

Naill, R. F. (1973). The discovery life cycle of a finite resource: A case study in U.S. natural gas. Toward global equilibrium: collected papers. D. L. Meadows and D. H. Meadows. Cambridge, MA, Wright-Allen Press inc.

NIST/SEMATECH. (2008). "e-Handbook of Statistical Methods." Retrieved August 20, 2008, from http://www.itl.nist.gov/div898/handbook/.

Nordhaus, W. D. (1973). "Allocation of energy resources." Brookings Papers on Economic Activity **3**: 529-570.

Norgaard, R. B. (1971). Output, input and productivity change in U.S. petroleum development: 1939-1968. Department of Economics. Chicago, IL, University of Chicago. **Ph.D.**

Norgaard, R. B. and G. J. Leu (1986). "Petroleum accessibility and drilling technology - An analysis of United-States development costs from 1959 to 1978." Land Economics **62**(1): 14-25.

Olien, D. D. and R. M. Olien (1993). "Running out of oil: Discourse and public policy 1909-1929." Business and Economic History **22**(2): 36-66.

Patzek, T. W. (2008). "Exponential gowth, energetic Hubbert cycles, and the advancement of technology." Archives of Mining Sciences of the Polish Academy of Sciences(Accepted for publication May 3, 2008): 22.

Pesaran, M. H. (1990). "An econometric analysis of exploration and extraction of crude oil in the U.K. continental shelf." The Economic Journal **100**(401): 367-390.

PFC. (2004). "PFC Energy's global crude oil and natural gas liquids supply forecast." 2005.

Pickering, A. (2002). "The Discovery Decline Phenomenon: Microeconometric evidence from the UK Continental Shelf." Energy Journal **23**(1): 57-71.

Pickering, A. (2008). "The oil reserves production relationship." Energy Economics **30**(2): 352-370.

Pindyck, R. S. (1978). "Optimal exploration and production of non-renewable resources." Journal of Political Economy **86**(5): 841-861.

Pindyck, R. S. and D. L. Rubinfeld (1998). Econometric models and economic forecasts. Boston, MA, McGraw-Hill

Pindyk, R. S. and D. L. Rubinfeld (1998). Econometric models and economic forecasts. Boston, MA, McGraw-Hill

PMPC (1952). Resources for freedom: Volume I - Foundations for growth and security. Washington, D.C., The President's Materials Policy Commission.

Pogue, J. E., K. E. Hill, et al. (1956). Future growth and financial requirements of the world petroleum industry. New York, Chase Manhattan Bank, Petroleum Department**:** 40.

Ramsey, J. B. (1980). Models, understanding and reliable forecasts. Symposium on Oil and Gas Supply Modeling, Department of Commerce, Washington, D.C., National Bureau of Standards.

Rehrl, T. and R. Friedrich (2006). "Modelling long-term oil price and extraction with a Hubbert approach: The LOPEX model." Energy Policy **34**(15): 2413-2428.

Reynolds, D. B. (1999). "The mineral economy: how prices and costs can falsely signal decreasing scarcity." Ecological Economics **31**(1): 155-166.

Rogner, H. H. (1997). "An assessment of world hydrocarbon resources." Annual Review of Energy and the Environment **22**: 217-262.

Ruth, M. and C. J. Cleveland (1993). "Nonlinear dynamic simulation of optimal depletion of crude oil in the lower 48 United States." Computers, Environment and Urban Systems **17**(5): 425-35.

Schuenemeyer, J. H. (1981). "Estimating volumes of remaining fossil fuel resources: A critical review: Comment." Journal of the American Statistical Association **76**(375): 554-558.

Simon, J. L. (1996). The ultimate resource 2. Princeton, NJ, Princeton University Press.

Skrebowski, C. (2004). "Oil field mega projects 2004." Petroleum Review **2004**(January): 18-20.

Skrebowski, C. (2005). "Prices set firm, despite massive new capacity." Petroleum Review **2005**(October): 36-40.

Skrebowski, C. (2006). "Prices holding steady, despite massive planned capacity additions." Petroleum Review **2006**(April): 28-31.

Skrebowski, C. (2007). "New capacity fails to boost 2006 production - Delays or depletion?" Petroleum Review **2007**(February): 40-42.

Slade, M. E. (1982). "Trends in natural-resource commodity prices: An analysis of the time domain." Journal of Environmental Economics and Management **9**(June): 122-137.

Smil, V. (2000). "Perils of long-range energy forecasting: Reflections on looking far ahead." Technological Forecasting and Social Change **65**: 251-264.

Smil, V. (2003). Energy at the crossroads: Global perspectives and uncertaintites. Cambrige, Massachusetts, MIT Press.

Smil, V. (2008). "Long-range energy forecasts are no more than fairy tales." Nature **Vol. 253**(8 May): 154.

Smith, M. R. (2006). The future for global oil supply: The size of the supply gap, EnergyFiles Ltd.

Smith, M. R. (2008). EnergyFiles Forecasting Model - Oil production, EnergyFiles Ltd.

Smith, M. R. (2008). Personal communication. A. Brandt. Berkeley, CA.

Sneddon, J. W., F. Sarg, et al. (2003). "Exploration play analysis from a sequence stratigraphic perspective." Search and Discovery(Article 40079).

Sorrell, S. and J. Speirs (2009). Methods for estimating ultimately recoverable resources. London, UK Energy Research Centre.

Sterman, J. D. (1983). "Economic vulnerability and the energy transition." Energy Systems and Policy **7**(4): 259-301.

Sterman, J. D. and G. P. Richardson (1983). An experiment to evaluate methods for estimating fossil fuel resources. Cambridge, MA, Sloan School of Management, MIT**:** 68.

Sterman, J. D. and G. P. Richardson (1985). "An experiment to evaluate methods for estimating fossil fuel resources." Journal of Forecasting **4**(2): 197-226.

Sterman, J. D., G. P. Richardson, et al. (1988). "Modelling the estimation of petroleum resources in the United States." Technological Forecasting and Social Change **33**(3): 219-249.

Tahvonen, O. (1997). "Fossil fuels, stock externalities, and backstop technology." The Canadian Journal of Economics **30**(4a): 855-874.

Taylor, P. J. (1998). "Author's reply to discussion of modeling the U.S. oil industry: How much oil is left." Journal of Petroleum Technology **50**(11): 82-83.

Thompson, E., S. Sorrell, et al. (2009). The nature and importance of reserve growth. London, UK Energy Research Centre.

U. S. Geological Survey, U. S. (2000). USGS world petroleum assessment 2000; new estimates of undiscovered oil and natural gas, including reserve growth, outside the United States. United States (USA), U. S. Geological Survey, Reston, VA, United States (USA)**:** 2.

Uhler, R. S. (1976). "Costs and supply in petroleum exploration: The case of Alberta." Canadian Journal of Economics **9**(1): 72-90.

Uri, N. D. (1982). "Domestic crude oil resource appraisal." Applied Mathematical Modelling **6**(2): 119-123.

van der Veen, C. J. (2006). "Reevaluating Hubbert's prediction of U.S. peak oil." EOS, Transactions, American Geophysical Union **87**(20): 199.

Walls, M. A. (1989). Forecasting oil market behavior: Rational expectations analysis of price shocks. Washington, D.C., Resources for the Future.

Walls, M. A. (1992). "Modeling and forecasting the supply of oil and gas: A survey of existing approaches." Resources and Energy **14**(3): 287-309.

Walls, M. A. (1994). "Using a 'hybrid' approach to model oil and gas supply: A case study of the gulf of mexico outer continental shelf." Land Economics **70**(1): 1-19.

Watkins, G. C. (2006). "Oil scarcity: What have the past three decades revealed?" Energy Policy **34**(5): 508-14.

White, D. (1920). "The petroleum resources of the world." Annals of the American Academy of Political and Social Science **89**: 111-134.

White, D. (1922). "The oil supply of the world." Engineering and Mining Journal **1922**(March 18th): 455-456.

Wiener, R. J. and D. M. Abrams (2007). "A physical basis for Hubbert's decline from the midpoint empirical model of oil production." Energy and Sustainability **105**: 377-383.

Williams, B. (2003). "Heavy hydrocarbons playing key role in peak-oil debate, future energy supply." Oil & Gas Journal. **101**(29): 20.

Wiorkowski, J. J. (1981). "Estimating volumes of remaining fossil fuel resources: a critical review." Journal of the American Statistical Association **76**.

Withagen, C. (1998). "Untested hypotheses in non-renewable resource economics." Environmental & Resource Economics **11**(3-4): 623-634.

Wood, J. H., G. Long, et al. (2000). Long term oil supply scenarios: The future is neither as rosy or as bleak as some assert. Washington DC, Energy Information Administration**:** 7.

www.hubbertpeak.com. (2008). "System dynamics and energy modeling." Retrieved July 23, 2008, from http://www.hubbertpeak.com/hubbert/SystemDynamicsEnergyModeling/.